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Induced vacuum currents in anti-de Sitter space with toral dimensions

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ABSTRACT

We investigate the Hadamard function and the vacuum expectation value of the current density for a charged massive scalar field on a slice of anti-de Sitter (AdS) space described in Poincaré coordinates with toroidally compact dimensions. Along compact dimensions periodicity conditions are imposed on the field with general phases. Moreover, the presence of a constant gauge field is assumed. The latter gives rise to Aharonov–Bohm-like effects on the vacuum currents. The current density along compact dimensions is a periodic function of the gauge field flux with the period equal to the flux quantum. It vanishes on the AdS boundary and, near the horizon, to the leading order, it is conformally related to the corresponding quantity in Minkowski bulk for a massless field. For large values of the length of the compact dimension compared with the AdS curvature radius, the vacuum current decays as power-law for both massless and massive fields. This behavior is essentially different from the corresponding one in Minkowski background, where the currents for a massive field are suppressed exponentially.

1. Introduction

In quantum field theory on curved backgrounds, the anti-de Sitter (AdS) spacetime is interesting from several points of view. The corresponding metric tensor is a maximally symmetric solution of the Einstein equations in the presence of a negative cosmological constant and, because of the high degree of symmetry, numerous physical problems are exactly solvable on its background. These investigations may help to gain deeper understanding of the influence of gravity on quantum matter. The early interest in the dynamics of quantum fields in AdS spacetime was motivated by principal questions of the quantization on curved backgrounds. The presence of both regular and irregular modes and the possibility of getting an interesting causal structure, lead to a number of interesting features that have no analogs in Minkowski spacetime field theory. Being a constant negative curvature manifold, AdS space provides a convenient infrared regulator in interacting quantum field theories [1], cutting infrared divergences without reducing the symmetries. The importance of AdS spacetime as a gravitational background increased after the discovery that it generically arises as a ground state in extended supergravity and in string theories, and also as the near-horizon geometry of the extremal black holes and branes.

More recent interest in this subject was generated in connection with two types of models where AdS geometry plays a crucial role. The first one, called AdS/CFT correspondence [2] (for a review see [3]), represents a realization of the holographic principle and provides a mapping between string theories or supergravity in the AdS bulk and a conformal field theory living on its boundary. In this mapping the solutions in AdS space play the role of classical sources for the correlators in the boundary field theory. The AdS/CFT correspondence has many interesting consequences and provides an opportunity to investigate non-perturbative field-theoretical effects at strong couplings. Among the recent developments of such a holography is the application to strong-coupling problems in condensed matter physics (holographic superconductors, quantum phase transitions, topological insulators). The second type of model with AdS spacetime as a background geometry is a realization of a braneworld scenario with large extra dimensions, and provides a solution to the hierarchy problem between the gravitational and electroweak mass scales (for reviews see [4]). Braneworlds naturally appear in string/M-theory context and give a novel setting for discussing phenomenological and cosmological issues related to extra dimensions, including the problem of the cosmological constant.

The presence of extra compact dimensions is an inherent feature of all these models. In quantum field theory, the boundary

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conditions imposed on the field operator along compactified dimensions lead to a number of interesting physical effects that include topological mass generation, instabilities in interacting field theories and symmetry breaking. These boundary conditions modify the spectrum of the zero-point fluctuations, as a result the vacuum energy density and the stresses are changed. This is the well-known topological Casimir effect. This effect has been investigated for large number of geometries and has important implications on all scales, from mesoscopic physics to cosmology (for reviews see [5]). The vacuum energy depends on the size of extra dimensions and this provides a stabilization mechanism for moduli fields in Kaluza–Klein-type models and in braneworld scenario. In particular, motivated by the problem of radion stabilization in Randall–Sundrum-type braneworlds, the investigations of the Casimir energy on AdS bulk have attracted a great deal of attention.[1] The Casimir effect in AdS spacetime with compact internal spaces has been considered in [7]. The vacuum energy generated by the compactification of extra dimensions can also serve as a model of dark energy needed for the explanation of the present accelerated expansion of the universe.

An important characteristic associated with charged fields is the vacuum expectation value (VEV) of the current density. Although the corresponding operator is local, because of the global nature of the quantum vacuum, this expectation value carries information about both the geometry and topology of the background space. Moreover, this VEV acts as the source in the semiclassical Maxwell equations and therefore plays an important role in modeling a self-consistent dynamics involving the electromagnetic field. The VEV of the current density for a fermionic field in flat spaces with toral dimensions has been investigated in [8]. Applications were given to the electrons of a graphene sheet rolled into cylindrical and toroidal shapes and described in terms of an effective Dirac theory in a two-dimensional space. Combined effects of the compactification and boundaries are discussed in [9]. The finite temperature effects on the current densities for scalar and fermionic fields in topologically nontrivial spaces have been studied in [10]. The VEV of the current density for charged scalar and Dirac spinor fields in de Sitter spacetime with toroidally compact spatial dimensions are considered in [11]. The effects of nontrivial topology induced by the compactification of a cosmic string along its axis have been discussed in [12].

In the present paper we investigate the VEV of the current density for a charged scalar field in a slice of AdS spacetime covered by Poincaré coordinates assuming that a part of spatial coordinates are compactified to a torus. In addition to the background gravitational field, we also assume the presence of a constant gauge field interacting with the field. Though the corresponding field strength vanishes, the nontrivial spatial topology gives rise Aharonov–Bohm-like effect on the current density along compact dimensions. This current is a source of magnetic fields in the uncompactified subspace, in particular, on the branes in braneworld scenario. The problem under consideration is also of separate interest as an example with gravitational and topological polarizations of the vacuum for charged fields, where one-loop calculations can be performed in closed form.

The outline of the paper is as follows. In the next section we describe the geometry of the problem and evaluate the Hadamard function for a charged massive scalar field obeying general quasiperiodicity conditions along compact dimensions. By using this function, in Section 3, the VEV of the current density is investigated. The behavior of this VEV in various asymptotic regions of the parameters is discussed. The main results of the paper are summarized in Section 4. The main steps for the transformation of the Hadamard function to its final expression are described in Appendix A.

[1] See references in [6].

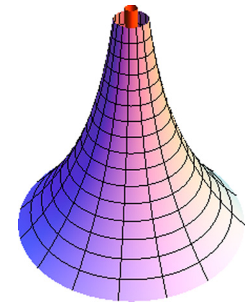

Fig. 1. The spatial section of the geometry under consideration for $D = 2$ embedded into a 3-dimensional Euclidean space.

2. Hadamard function

As a background geometry we consider $(D + 1)$-dimensional AdS spacetime. In Poincaré coordinates the corresponding line element is expressed as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{-2y/a}\eta_{ik}dx^i dx^k - dy^2, \tag{1}$$

where $\eta_{ik} = \mathrm{diag}(1, -1, \dots, -1)$, $i, k = 0, 1, \dots, D - 1$, is the metric tensor for D-dimensional Minkowski spacetime and $-\infty < y < +\infty$. The parameter a is the AdS curvature radius and is related with the cosmological constant. Note that the Poincaré coordinates cover a part of the AdS manifold and there is a horizon corresponding to the hypersurface $y = +\infty$. In what follows we assume that the coordinates x^l, with $l = p + 1, \dots, D - 1$, are compactified to circles with the lengths L_l, so $0 \leqslant x^l \leqslant L_l$. For the coordinates x^l, with $l = 1, 2, \dots, p$, one has $-\infty < x^l < +\infty$. Hence, the subspace perpendicular to the y-axis has a topology $R^p \times (S^1)^q$, where $q + p = D - 1$. The coordinates in uncompactified and compactified subspaces will be denoted by $\mathbf{x}_p = (x^1, \dots, x^p)$ and $\mathbf{x}_q = (x^{p+1}, \dots, x^{D-1})$, respectively.

Introducing a new coordinate

$$z = ae^{y/a}, \quad 0 \leqslant z < \infty, \tag{2}$$

the line element is presented in a conformally-flat form

$$ds^2 = (a/z)^2\big(\eta_{ik}dx^i dx^k - dz^2\big). \tag{3}$$

The hypersurfaces identified by $z = 0$ and $z = \infty$ correspond to the AdS boundary and horizon, respectively. In Fig. 1 we have displayed the spatial geometry corresponding to (1) for $D = 2$ embedded into the 3-dimensional Euclidean space. The compact dimension corresponds to the circles. We have also displayed the flux of the gauge field strength which threads the compact dimension (see below). Note that for a given z, the proper length of the lth compact dimension is given by $L_{(p)l} = aL_l/z$ and it decreases with increasing z.

In this paper we are interested in the evaluation of the VEV of the current density

$$j_\mu(x) = ie\big[\varphi^+(x)D_\mu\varphi(x) - \big(D_\mu\varphi^+(x)\big)\varphi(x)\big], \tag{4}$$

associated with a massive charged scalar field, $\varphi(x)$, in the presence of an external classical gauge field, A_μ. In (4), $D_\mu = \nabla_\mu + ieA_\mu$, with ∇_μ being the standard covariant derivative. The corresponding equation of motion is given by

$$\big(g^{\mu\nu}D_\mu D_\nu + m^2 + \xi R\big)\varphi(x) = 0, \tag{5}$$