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Sketching the pion's valence-quark generalised parton distribution

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ABSTRACT

In order to learn effectively from measurements of generalised parton distributions (GPDs), it is desirable to compute them using a framework that can potentially connect empirical information with basic features of the Standard Model. We sketch an approach to such computations, based upon a rainbow-ladder (RL) truncation of QCD's Dyson–Schwinger equations and exemplified via the pion's valence dressed-quark GPD, $H_\pi^{\mathrm{v}}(x,\xi,t)$. Our analysis focuses primarily on $\xi=0$, although we also capitalise on the symmetry-preserving nature of the RL truncation by connecting $H_\pi^{\mathrm{v}}(x,\xi=\pm 1,t)$ with the pion's valence-quark parton distribution amplitude. We explain that the impulse-approximation used hitherto to define the pion's valence dressed-quark GPD is generally invalid owing to omission of contributions from the gluons which bind dressed-quarks into the pion. A simple correction enables us to identify a practicable improvement to the approximation for $H_\pi^{\mathrm{v}}(x,0,t)$, expressed as the Radon transform of a single amplitude. Therewith we obtain results for $H_\pi^{\mathrm{v}}(x,0,t)$ and the associated impact-parameter dependent distribution, $q_\pi^{\mathrm{v}}(x,|\vec{b}_\perp|)$, which provide a qualitatively sound picture of the pion's dressed-quark structure at a hadronic scale. We evolve the distributions to a scale $\zeta=2$ GeV, so as to facilitate comparisons in future with results from experiment or other nonperturbative methods.

1. Introduction

Quarks were discovered in a series of deep inelastic scattering (DIS) experiments at the Stanford Linear Accelerator Center [1–3]. When analysed in the infinite momentum frame, i.e., treating the target as an extremely rapidly moving object, such experiments yield parton distribution functions (PDFs). PDFs are probability densities, which reveal how partons within the speeding target share the bound-state's gross properties; e.g., there are PDFs that describe the distributions over the target's constituent partons of the total longitudinal momentum and helicity. Crucially, this probability interpretation is only valid in the infinite-momentum frame owing to its connection with quantisation on the light-front [4–6], a procedure that ensures, *inter alia*, particle number conservation.

A good deal is known about hadron light-front structure after more than forty years of studying PDFs. Notwithstanding that, much more needs to be understood, particularly, e.g., in connection with the distribution of helicity [7,8]. Moreover, PDFs only describe hadron light-front structure incompletely because inclusive DIS measurements do not yield information about the distribution of partons in the plane perpendicular to the bound-state's total momentum, i.e., within the light front. Such information is expressed in generalised parton distributions (GPDs) [9–12], which are accessible via deeply virtual Compton scattering on a target hadron, T; viz., $\gamma^*(q)T(p)\to\gamma^*(q')T(p')$, so long as at least one of the photons $[\gamma^*(q),\ \gamma^*(q')]$ possesses large virtuality, and in the analogous process of deeply virtual meson production: $\gamma^*(q)T(p)\to M(q')T(p')$. Importantly [see Section 2], GPDs connect PDFs with hadron form factors because any PDF may be recovered as a forward limit of the relevant GPD and any hadron elastic form factors can be expressed via a GPD-based sum rule. The potential that GPDs hold for providing manifold insights into hadron structure has led to intense experimental and theoretical activity [13–17].

Most of the constraints that apply to GPDs are fulfilled when the GPD is written as a double distribution [10,18,19], which is equivalent to expressing the GPD as a Radon transform [20]:

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$$H(x,\xi,t) = \int\limits_{|\alpha|+|\beta|\le 1} d\alpha\, d\beta\, \delta(x-\alpha-\xi\beta)\big[F(\alpha,\beta,t)+\xi G(\alpha,\beta,t)\big], \tag{1}$$

where the variables x, ξ, t are defined following Eq. (2) and, at leading-twist, F, G have operator definitions analogous to the GPD itself. In order to obtain insights into the nature of hadron GPDs, it has been common to model the Radon amplitudes, F, G, following Ref. [21]. This approach has achieved some phenomenological success [17,22]; but more flexible parameterisations enable a better fit to data [23]. Such fits played a valuable role in establishing the GPD framework; but if one wishes to use measured GPDs as a means by which to validate our basic perception of strong interactions in the Standard Model, then data fitting is inadequate. Instead, it is necessary to compute GPDs using a framework that possesses a direct connection with QCD. This observation is highlighted by experience drawn from the simpler case of the pion's valence-quark PDF [24]. Herein, we therefore adopt a different approach, sketching a procedure for the computation of hadron GPDs based on the example provided by the pion's valence-quark PDF.

2. General features of pion GPDs

From a quark model perspective, in the isospin symmetric limit, the pion is a quantum mechanical bound-state of two equal-mass constituents and it is therefore the simplest hadronic bound-state. That is a misapprehension, however. Owing to the connection between pion properties and dynamical chiral symmetry breaking (DCSB), i.e., its dichotomous nature as a Goldstone mode and relativistic bound-state [25,26], a veracious description of the pion is only possible within a framework that faithfully expresses symmetries and their breaking patterns. The Dyson–Schwinger equations (DSEs) fulfil this requirement [27–29] and hence we employ that framework to compute pion properties herein.

Notwithstanding the complex nature of the pion bound-state, it is still a $J=0$ system and hence for a vector probe there is only one GPD associated with a quark q in the pion ($\pi^\pm$, π^0). It is defined by the matrix element

$$\begin{aligned} H_\pi^q(x,\xi,t) = \int \frac{d^4z}{4\pi} e^{ixP\cdot z}\delta(n\cdot z)\delta^2(z_\perp) \\ \times \big\langle \pi(P_+)\big|\bar q(-z/2) n\cdot\gamma q(z/2)\big|\pi(P_-)\big\rangle, \end{aligned} \tag{2}$$

where: k, n are light-like four-vectors, satisfying $k^2=0=n^2$, $k\cdot n=1$; $z_\perp$ represents that two-component part of z annihilated by both k, n; and $P_\pm = P\pm\Delta/2$. In Eq. (2), $\xi = -n\cdot\Delta/[2n\cdot P]$ is the "skewness", $t=-\Delta^2$ is the momentum transfer, and $P^2 = t/4 - m_\pi^2$, $P\cdot\Delta = 0$. The GPD also depends on the resolving scale, ζ. Within the domain upon which perturbation theory is valid, evolution to another scale ζ' is described by the ERBL equations [30,31] for $|x|<\xi$ and the DGLAP equations [32–35] for $|x|>\xi$, where $\xi\ge 0$.

In order to produce quantities that are gauge invariant for all values of z, Eq. (2) should contain a Wilson line, $\mathcal{W}[-z/2,z/2]$, between the quark fields. Notably, for any light-front trajectory, $\mathcal{W}[-z/2,z/2]\equiv 1$ in light-cone gauge: $n\cdot A=0$, and hence the Wilson line does not contribute in this case. On the other hand, light-cone gauge is seldom practicable in either model calculations or quantitative nonperturbative analyses in continuum QCD. Indeed, herein, as typical of nonperturbative DSE studies, we employ Landau gauge because, *inter alia* [36,37]: it is a fixed point of the renormalisation group; and a covariant gauge, which is readily implemented in numerical simulations of lattice-QCD. It is therefore significant that $\mathcal{W}[-z/2,z/2]$ is not quantitatively important in the calculation of the leading-twist contributions to numerous matrix elements [38].

It is worth recapitulating here upon some general properties of GPDs. Most generally, Poincaré covariance entails that GPDs are only nonzero on $x\in(-1,1)$. Moreover, owing to time-reversal invariance, $H^q(x,\xi,t) = H^q(x,-\xi,t)$. Kinematically, the skewness is bounded: $\xi\in[-1,1]$, but $\xi\in[0,1]$ for all known processes that provide empirical access to GPDs.

Focusing on the pion, a charge conjugation mapping between charged states entails $H^{u,d}_{\pi^+}(x,\xi,t) = -H^{u,d}_{\pi^-}(-x,\xi,t)$; and consequently, in the isospin symmetric limit:

$$H^u_{\pi^+}(x,\xi,t) = -H^d_{\pi^+}(-x,\xi,t). \tag{3}$$

It follows that the isospin projections:

$$H^I(x,\xi,t) := H^u_{\pi^+}(x,\xi,t) + (-1)^I H^d_{\pi^+}(x,\xi,t), \quad I=0,1, \tag{4}$$

have well-defined symmetry properties under $x\leftrightarrow -x$; viz., H^0 is odd and H^1 is even.

Returning to the definition in Eq. (2), it is plain that if one considers the forward limit: $\xi=0$, $t=0$, then x is Bjorken-x and the GPD reduces to a PDF; viz.,

$$H_\pi^q(x,0,0) = \begin{cases} q^\pi(x), & x>0 \\ -\bar q^\pi(-x), & x<0. \end{cases} \tag{5}$$

Moreover, irrespective of the value of ξ, the electromagnetic pion form factor may be computed as

$$F_{\pi^+}(\Delta^2) = \int_{-1}^{1} dx\big[e_u H^u_{\pi^+}(x,\xi,-\Delta^2) + e_d H^d_{\pi^+}(x,\xi,-\Delta^2)\big] \tag{6}$$

$$=: e_u F^u_{\pi^+}(\Delta^2) + e_d F^d_{\pi^+}(\Delta^2) = F^u_{\pi^+}(\Delta^2), \tag{7}$$

where $e_{u,d}$ are the quark electric charges in units of the positron charge and we have used Eq. (3) to show $F^d_{\pi^+}(\Delta^2) = -F^u_{\pi^+}(\Delta^2)$. Additional information may be found elsewhere [39].

3. Heuristic example

Imagine a bound-state of two scalar particles with effective mass σ and suppose that the interaction between them is such that it produces a light-front wave function of the form ($\bar x = 1-x$):

$$\psi(x,k_\perp^2) = \sqrt{\frac{15}{2\pi\sigma^2}}\frac{\sqrt{x\bar x}}{1+k_\perp^2/(4\sigma^2 x\bar x)}\theta(x)\theta(\bar x). \tag{8}$$

(A merit of considering a bound-state of scalar constituents is that in describing the wave function of the composite system one avoids the complication of Melosh rotations, which arise in treating spin states in light-front quantum mechanics [5].) If the skewness is zero, in which case the momentum transfer is purely light-front transverse, then the GPD for this system can be written as a wave function overlap [13,14,40,41]:

$$H_\sigma(x,0,-\Delta_\perp^2) = \int d^2k_\perp \psi\big(x,k_\perp+(1-x)\Delta_\perp\big)\psi(x,k_\perp). \tag{9}$$

This entails

$$\big\{H_\sigma(x,0,-\Delta_\perp^2) > 0 : x\in[-1,1], \Delta_\perp^2 \ge 0\big\}. \tag{10}$$

Owing to the simplicity of the starting point, Eqs. (8) and (9) allow one to obtain an algebraic expression for the GPD; viz., with $z^2 = \Delta_\perp^2(1-x)/4x\sigma^2$, then