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Exact third-order density perturbation and one-loop power spectrum in general dark energy models

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ABSTRACT

Under the standard perturbation theory (SPT), we obtain the fully consistent third-order density fluctuation and kernels for the general dark energy models without using the Einstein–de Sitter (EdS) universe assumption for the first time. We also show that even though the temporal and spatial components of the SPT solutions cannot be separable, one can find the exact solutions to any order in general dark energy models. With these exact solutions, we obtain the less than % error correction of one-loop matter power spectrum compared to that obtained from the EdS assumption for $k = 0.1\ \mathrm{h\,Mpc^{-1}}$ mode at $z = 0$ (1, 1.5). Thus, the EdS assumption works very well at this scale. However, if one considers the correction for P_{13}, the error is about 6 (9, 11)% for the same mode at $z = 0$ (1, 1.5). One absorbs P_{13} into the linear power spectrum in the renormalized perturbation theory (RPT) and thus one should use the exact solution instead of the approximation one. The error on the resummed propagator N of RPT is about 14 (8, 6)% at $z = 0$ (1, 1.5) for $k = 0.4\ \mathrm{h\,Mpc^{-1}}$. For $k = 1\ \mathrm{h\,Mpc^{-1}}$, the error correction of the total matter power spectrum is about 3.6 (4.6, 4.5)% at $z = 0$ (1, 1.5). Upcoming observation is required to archive the sub-percent accuracy to provide the strong constraint on the dark energy and this consistent solution is prerequisite for the model comparison.

The standard perturbation theory (SPT) has been widely used to investigate the correction to the linear power spectrum in a quasi-nonlinear regime. The recent progress and the development of alternative analytical methods have been made [1,2]. The approximate recursion relations for the Fourier components of the n-th order matter density fluctuation $\hat{\delta}_n(\tau, \vec{k})$ and the divergence of the peculiar velocity $\hat{\theta}_n(\tau, \vec{k})$ has been obtained for the Einstein–de Sitter (EdS) universe [3,4]. When one extends the SPT to the general background universe, one uses the assumption that the dependence of the SPT solutions on the cosmological parameters is encoded in the linear growth factor, $D_1(a)$ [1]. This is also confirmed for the dark energy models [5,6]. However, this argument is partly correct because one also needs to investigate the error on the power spectrum induced from EdS assumption (*i.e.* the value of the linear growth rate is equal to that of the square root of the matter energy density contrast, $f_1 \equiv \frac{d \ln D_1}{d \ln a} = \sqrt{\Omega_m}$). We obtain the exact kernels for $\hat{\delta}_n$ and $\hat{\theta}_n$ without using EdS assumption and study its effect on the power spectrum.

The renormalized perturbation theory (RPT) tries to reorganize the perturbative series expansion of SPT and resums some of the terms into a function that can be factorized out of the series [7,8]. This function is called as the resummed propagator and referred as N. All the kernels of the higher order power spectrum terms must be expressed as a product of kernels that correspond to full mode coupling terms and full propagator terms in order to make the resummation possible. If the kernels are approximated as a product of one-loop propagator kernels, then the resummed propagator is given by $N(k) \equiv \exp[P_{13}(k)/P_{\rm lin}(k)]$. We find that $P_{13}(k)$ using EdS assumption causes 6–11% errors for $k = 0.5\ \mathrm{h\,Mpc^{-1}}$ mode at $z = 0$–1.5 and these induce errors on N about 11–20%.

In addition to SPT, the Lagrangian perturbation theory (LPT) is an another widely used analytic technique for the quasi-linear perturbative expansion. There also have been studies to investigate the dark energy dependence on the linear growth factor in LPT [9,10]. Recently, we also obtain the kernels in the recursion relations without using EdS assumption in the LPT and investigate its consequences on the one-loop power spectrum [11].

In this *Letter*, we obtain the exact relations for the temporal and spatial components of the SPT solutions in general dark energy models up to third order. When we obtain the kernels, we remove the EdS assumption in the derivation and investigate the its effects on the observable quantities.

The equations of motion of $\hat{\delta}(\tau, \vec{k})$ and $\hat{\theta}(\tau, \vec{k})$ in the Fourier space are given by

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$$\frac{\partial \hat{\delta}}{\partial \tau} + \hat{\theta} = -\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \alpha(\vec{k}_1, \vec{k}_2) \times \hat{\theta}(\tau, \vec{k}_1) \hat{\delta}(\tau, \vec{k}_2), \tag{1}$$

$$\frac{\partial \hat{\theta}}{\partial \tau} + \mathcal{H}\hat{\theta} + \frac{3}{2}\Omega_m \mathcal{H}^2 \hat{\delta} = -\frac{1}{2}\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \beta(\vec{k}_1, \vec{k}_2) \hat{\theta}(\tau, \vec{k}_1) \hat{\theta}(\tau, \vec{k}_2), \tag{2}$$

where τ is the conformal time, $\vec{k}_{12} \equiv \vec{k}_1 + \vec{k}_2$, δ_D is the Dirac delta function, $\mathcal{H} \equiv \frac{1}{a}\frac{\partial a}{\partial \tau}$, Ω_m is the matter energy density contrast, $\alpha(\vec{k}_1, \vec{k}_2) \equiv \frac{\vec{k}_{12}\cdot\vec{k}_1}{k_1^2}$, and $\beta(\vec{k}_1, \vec{k}_2) \equiv \frac{k_{12}^2(\vec{k}_1\cdot\vec{k}_2)}{k_1^2 k_2^2}$.

Due to the mode coupling of the nonlinear terms shown in the right hand side of Eqs. (1)–(2), one needs to make a perturbative expansion in $\hat{\delta}$ and $\hat{\theta}$ [1]. One can introduce the proper perturbative series of solutions for the fastest growing mode D_n

$$\hat{\delta}(\tau, \vec{k}) \equiv \sum_{n=1}^{\infty} \hat{\delta}^{(n)}(\tau, \vec{k}), \tag{3}$$

$$\hat{\theta}(\tau, \vec{k}) \equiv \sum_{n=1}^{\infty} \hat{\theta}^{(n)}(\tau, \vec{k}), \tag{4}$$

where one can define the each order solution as

$$\hat{\delta}^{(1)}(a, \vec{k}) \equiv D_1(a)\delta_1(\vec{k}), \tag{5}$$

$$\hat{\theta}^{(1)}(a, \vec{k}) \equiv D_{\theta 1}(a)\theta_1(\vec{k}) \equiv -a\mathcal{H}\frac{dD_1}{da}\delta_1(\vec{k}), \tag{6}$$

$$\hat{\delta}^{(2)}(a, \vec{k}) \equiv \sum_{i=1}^{2} D_{2i}(a)K_{2i}(\vec{k}) \equiv D_1^2(a)\sum_{i=1}^{2} c_{2i}(a)K_{2i}(\vec{k}), \tag{7}$$

$$\hat{\theta}^{(2)}(a, \vec{k}) \equiv \sum_{i=1}^{2} D_{\theta 2i}(a)K_{2i}(\vec{k}) \equiv a\mathcal{H}D_1\frac{dD_1}{da}\sum_{i=1}^{2} c_{\theta 2i}(a)K_{2i}(\vec{k}), \tag{8}$$

$$\hat{\delta}^{(3)}(a, \vec{k}) \equiv \sum_{i=1}^{6} D_{3i}(a)K_{3i}(\vec{k}) \equiv D_1^3(a)\sum_{i=1}^{6} c_{3i}(a)K_{3i}(\vec{k}), \tag{9}$$

$$\hat{\theta}^{(3)}(a, \vec{k}) \equiv \sum_{i=1}^{6} D_{\theta 3i}(a)K_{3i}(\vec{k}) \equiv a\mathcal{H}D_1^2\frac{dD_1}{da}\sum_{i=1}^{6} c_{\theta 3i}(a)K_{3i}(\vec{k}). \tag{10}$$

To be consistent with the current observation, we consider the dark energy dominated flat universe as a background model. It has been known that the n-th order fastest growing mode solutions are proportional to the n-th power of the linear growth factor D_1 (*i.e.* $D^n \propto D_1^n$) for the EdS universe. And this is not true for the general background models. There have been the investigations of the validity of these ansatz (3) and (4) by using the different growth rates for $\hat{\delta}$ and $\hat{\theta}$ [5,6]. However, the improper decomposition of fastest mode solutions and the incorrect initial conditions are used for the n-th order growth rate in both cases (see Appendix A).

If one takes a derivatives of Eq. (1) and replace Eq. (2) into it, then one obtains

$$\frac{\partial^2 \hat{\delta}}{\partial \tau^2} + \mathcal{H}\frac{\partial \hat{\delta}}{\partial \tau} - \frac{3}{2}\Omega_m \mathcal{H}^2 \hat{\delta} = -\mathcal{H}\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \alpha(\vec{k}_1, \vec{k}_2) \hat{\theta}(\tau, \vec{k}_1) \hat{\delta}(\tau, \vec{k}_2) - \int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \alpha(\vec{k}_1, \vec{k}_2) \times \left[\frac{\partial \hat{\theta}(\tau, \vec{k}_1)}{\partial \tau}\hat{\delta}(\tau, \vec{k}_2) + \hat{\theta}(\tau, \vec{k}_1)\frac{\partial \hat{\delta}(\tau, \vec{k}_2)}{\partial \tau}\right] + \frac{1}{2}\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \beta(\vec{k}_1, \vec{k}_2) \times \hat{\theta}(\tau, \vec{k}_1)\hat{\theta}(\tau, \vec{k}_2). \tag{11}$$

From Eqs. (1) and (11), one obtains the expressions for the higher order solutions of $\hat{\delta}^{(2)}$, $\hat{\theta}^{(2)}$, and $\hat{\delta}^{(3)}$ as

$$\begin{aligned}\hat{\delta}^{(2)}(a, \vec{k}) &\equiv D_{21}(a)K_{21}(\vec{k}) + D_{22}(a)K_{22}(\vec{k}) \\ &\equiv D_1^2\big[c_{21}(a)K_{21}(\vec{k}) + c_{22}(a)K_{22}(\vec{k})\big] \\ &\equiv D_1^2(a)\delta_2(a, \vec{k}) \\ &\equiv D_1^2\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \\ &\quad \times F_2^{(s)}(a, \vec{k}_1, \vec{k}_2)\delta_1(\vec{k}_1)\delta_1(\vec{k}_2),\end{aligned} \tag{12}$$

$$\begin{aligned}\hat{\theta}^{(2)}(a, \vec{k}) &\equiv D_{\theta 21}(a)K_{21}(\vec{k}) + D_{\theta 22}(a)K_{22}(\vec{k}) \\ &\equiv D_1\frac{\partial D_1}{\partial \tau}\big[c_{\theta 21}(a)K_{21}(\vec{k}) + c_{\theta 22}(a)K_{22}(\vec{k})\big] \\ &\equiv D_1\frac{\partial D_1}{\partial \tau}\theta_2(a, \vec{k}) \\ &\equiv -D_1\frac{\partial D_1}{\partial \tau}\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \\ &\quad \times G_2^{(s)}(a, \vec{k}_1, \vec{k}_2)\delta_1(\vec{k}_1)\delta_1(\vec{k}_2),\end{aligned} \tag{13}$$

$$\begin{aligned}\hat{\delta}^{(3)}(a, \vec{k}) &\equiv D_{31}(a)K_{31}(\vec{k}) + \cdots + D_{36}(a)K_{36}(\vec{k}) \\ &\equiv D_1^3(a)\big[c_{31}(a)K_{31}(\vec{k}) + \cdots + c_{36}(a)K_{36}(\vec{k})\big] \\ &\equiv D_1^3(a)\int d^3k_1 d^3k_2 d^3k_3 \delta_D(\vec{k}_{123} - \vec{k}) \\ &\quad \times F_3^{(s)}(a, \vec{k}_1, \vec{k}_2, \vec{k}_3)\delta_1(\vec{k}_1)\delta_1(\vec{k}_2)\delta_1(\vec{k}_3),\end{aligned} \tag{14}$$

where

$$c_{2i} = \frac{D_{2i}}{D_1^2}, \qquad c_{\theta 2i} = \frac{D_{\theta 2i}}{D_1}\left(\frac{\partial D_1}{\partial \tau}\right)^{-1}, \qquad c_{3i} = \frac{D_{3i}}{D_1^3}, \tag{15}$$

$$K_{21}(\vec{k}) = -\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \alpha(\vec{k}_1, \vec{k}_2) \times \theta_1(\vec{k}_1)\delta_1(\vec{k}_2), \tag{16}$$

$$K_{22}(\vec{k}) = -\int d^3k_1 \int d^3k_2 \delta_D(\vec{k}_{12} - \vec{k}) \beta(\vec{k}_1, \vec{k}_2) \times \theta_1(\vec{k}_1)\theta_1(\vec{k}_2), \tag{17}$$

$$\begin{aligned}F_2^{(s)}(a, \vec{k}_1, \vec{k}_2) &= \frac{1}{2}\left[c_{21}\left(\frac{\vec{k}_{12}\cdot\vec{k}_1}{k_1^2} + \frac{\vec{k}_{12}\cdot\vec{k}_2}{k_2^2}\right) - 2c_{22}\frac{k_{12}^2(\vec{k}_1\cdot\vec{k}_2)}{k_1^2 k_2^2}\right] \\ &= c_{21} - 2c_{22}\left(\frac{\vec{k}_1\cdot\vec{k}_2}{k_1 k_2}\right)^2 + \frac{1}{2}(c_{21} - 2c_{22})\vec{k}_1\cdot\vec{k}_2\left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right),\end{aligned} \tag{18}$$

$$G_2^{(s)}(a, \vec{k}_1, \vec{k}_2) = \frac{1}{2}\left[-c_{\theta 21}\left(\frac{\vec{k}_{12}\cdot\vec{k}_1}{k_1^2} + \frac{\vec{k}_{12}\cdot\vec{k}_2}{k_2^2}\right) + 2c_{\theta 22}\frac{k_{12}^2(\vec{k}_1\cdot\vec{k}_2)}{k_1^2 k_2^2}\right]$$