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Gauge origin of discrete flavor symmetries in heterotic orbifolds

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ABSTRACT

We show that non-Abelian discrete symmetries in orbifold string models have a gauge origin. This can be understood when looking at the vicinity of a symmetry enhanced point in moduli space. At such an enhanced point, orbifold fixed points are characterized by an enhanced gauge symmetry. This gauge symmetry can be broken to a discrete subgroup by a nontrivial vacuum expectation value of the Kähler modulus T. Using this mechanism it is shown that the $\Delta(54)$ non-Abelian discrete symmetry group originates from a $SU(3)$ gauge symmetry, whereas the D_4 symmetry group is obtained from a $SU(2)$ gauge symmetry.

1. Introduction

It is important to understand the flavor structure of the standard model of particle physics. Quark and lepton masses are hierarchical. Two of the mixing angles in the lepton sector are large, while the mixing angles in the quark sector are suppressed, except for the Cabibbo angle. Non-Abelian discrete flavor symmetries may be useful to understand this flavor structure. Indeed, many works have considered field-theoretical model building with various non-Abelian discrete flavor symmetries (see [1–3] for reviews).

Understanding the origin of non-Abelian flavor symmetries is an important issue we have to address. It is known that several phenomenologically interesting non-Abelian discrete symmetries can be derived from string models.[1] In intersecting and magnetized D-brane models, the non-Abelian discrete symmetries D_4, $\Delta(27)$ and $\Delta(54)$ can be realized [5–8]. Also, their gauge origins have been studied [6]. In heterotic orbifold compactifications [9–17] (also see a review [18]), non-Abelian discrete symmetries appear due to geometrical properties of orbifold fixed points and certain properties of closed string interactions [19]. First, there are permutation symmetries of orbifold fixed points. Then, there are string selection rules which determine interactions between orbifold sectors. The combination of these two kinds of discrete symmetries leads to a non-Abelian discrete symmetry. In particular, it is known that the D_4 group emerges from the one-dimensional orbifold S^1/Z_2, and that the $\Delta(54)$ group is obtained from the two-dimensional orbifold T^2/Z_3. The phenomenological applications of the string-derived non-Abelian discrete symmetries are analyzed e.g. in [20].

In this paper we point out that these non-Abelian discrete flavor symmetries originate from a gauge symmetry. To see this, we consider a heterotic orbifold model compactified on some six-dimensional orbifold. The gauge symmetry G_{gauge} of this orbifold model is, if we do not turn on any Wilson lines, a subgroup of $E_8 \times E_8$ which survives the orbifold projection. In addition, from the argument in [19], we can derive a non-Abelian discrete symmetry G_{discrete}. Then, the effective action of this model can be derived from $G_{\mathrm{gauge}} \times G_{\mathrm{discrete}}$ symmetry invariance.[2] However, this situation slightly changes if we set the model to be at a symmetry enhanced point in moduli space. At that special point, the gauge symmetry of the model is enlarged to $G_{\mathrm{gauge}} \times G_{\mathrm{enhanced}}$, where G_{enhanced} is a gauge symmetry group. The maximal rank of the enhanced gauge symmetry G_{enhanced} is six, because we compactify six internal dimensions. At this specific point in moduli space, orbifold fixed points are characterized by gauge charges of G_{enhanced}, and the spectrum is extended by additional massless fields charged under G_{enhanced}. Furthermore, the Kähler moduli fields T in the untwisted sector obtain G_{enhanced}-charges and a non-zero vacuum expectation value (VEV) of T corresponds to a movement away

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[1] In [4], field theoretical models where non-Abelian discrete groups are embedded into non-Abelian gauge groups are considered.

[2] Here we do not consider the R-charge invariance since this is not relevant to our discussion.

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from the enhanced point. This argument suggests the possibility that the non-Abelian discrete symmetry G_{discrete} is enlarged to a continuous gauge symmetry G_{enhanced} at the symmetry enhanced point. In other words, it suggests a gauge origin of the non-Abelian discrete symmetry. Moreover, the group G_{enhanced} originates from a larger non-Abelian gauge symmetry that exists before the orbifolding. We will show this explicitly in the following.

2. Gauge origin of non-Abelian discrete symmetry

In this section we demonstrate the gauge origin of non-Abelian discrete symmetries in heterotic orbifold models. We concentrate on the phenomenologically interesting non-Abelian discrete symmetries D_4 and $\Delta(54)$ which are known to arise from orbifold models.

2.1. D_4 non-Abelian discrete symmetry

First, we study a possible gauge origin of the D_4 non-Abelian discrete symmetry. This symmetry is associated with the one-dimensional S^1/Z_2 orbifold. Here, we consider the heterotic string on a S^1/Z_2 orbifold, but it is straightforward to extend our argument to T^2/Z_2 or $T^6/(Z_2 \times Z_2)$. The coordinate corresponding to the one dimension of S^1 is denoted by X. It suffices to discuss only the left-movers in order to develop our argument. Let us start with the discussion on S^1 without the Z_2 orbifold. There is always a $U(1)$ symmetry associated with the current $H = i\partial X$. At a specific point in the moduli space, i.e. at a certain radius of S^1, two other massless vector bosons appear and the gauge symmetry is enhanced from $U(1)$ to $SU(2)$. Their currents are written as

$$E_\pm = e^{\pm i\alpha X}, \tag{1}$$

where $\alpha = \sqrt{2}$ is a simple root of the $SU(2)$ group. These currents, H and $E_\pm$, satisfy the $su(2)$ Kac–Moody algebra.

Now, let us study the Z_2 orbifolding $X \to -X$. The current $H = i\partial X$ is not invariant under this reflection and the corresponding $U(1)$ symmetry is broken. However, the linear combination $E_+ + E_-$ is Z_2-invariant and the corresponding $U(1)$ symmetry remains on S^1/Z_2. Thus, the $SU(2)$ group breaks down to $U(1)$ by orbifolding. Note that the rank is not reduced by this kind of orbifolding. It is convenient to use the following basis,

$$H' = i\partial X' = \frac{1}{\sqrt{2}}(E_+ + E_-), \tag{2}$$

$$E'_\pm = e^{\pm i\alpha X'} = \frac{1}{\sqrt{2}}H \mp \frac{1}{2}(E_+ - E_-). \tag{3}$$

The introduction of the boson field X' is justified because H' and $E'_\pm$ satisfy the same operator product expansions (OPEs) as the original currents H and $E_\pm$. The invariant current H' corresponds to the $U(1)$ gauge boson. The $E'_\pm$ transform as

$$E'_\pm \to -E'_\pm \tag{4}$$

under the Z_2 reflection and correspond to untwisted matter fields U_1 and U_2 with $U(1)$ charges $\pm\alpha$. In addition, there are other untwisted matter fields U which have vanishing $U(1)$ charge, but are charged under an unbroken subgroup of $E_8 \times E_8$.

From (4), it turns out that the Z_2 reflection is represented by a shift action in the X' coordinate,

$$X' \to X' + 2\pi \frac{w}{2}, \tag{5}$$

where $w = 1/\sqrt{2}$ is the fundamental weight of $SU(2)$. That is, the Z_2-twisted orbifold on X is equivalent to a Z_2-shifted orbifold on X' with the shift vector $s = w/2$ (see e.g., [21]). In the twist representation, there are two fixed points on the Z_2 orbifold, to each of which corresponds a twisted state. Note that the one-dimensional bosonic string X with the Z_2-twisted boundary condition has a contribution of $h = 1/16$ to the conformal dimension. In the shift representation, the two twisted states can be understood as follows. Before the shifting, X' also represents a coordinate on S_1 at the enhanced point, so the left-mover momenta p_L lie on the momentum lattice

Table 1
Field contents of $U(1) \rtimes Z_2$ model from Z_2 orbifold. $U(1)$ charges are shown. Charges under the Z_4 unbroken subgroup of the $U(1)$ group are also shown.

Sector	Field	$U(1)$ charge	Z_4 charge
U	U	0	0
U	U_1	α	0
U	U_2	$-\alpha$	0
T	M_1	$\frac{\alpha}{4}$	$\frac{1}{4}$
T	M_2	$-\frac{\alpha}{4}$	$-\frac{1}{4}$

$$\Gamma_{SU(2)} \cup (\Gamma_{SU(2)} + w), \tag{6}$$

where $\Gamma_{SU(2)}$ is the $SU(2)$ root lattice, $\Gamma_{SU(2)} \equiv n\alpha$ with integer n. Then, the left-mover momenta in the Z_2-shifted sector lie on the original momentum lattice shifted by the shift vector $s = w/2$, i.e.

$$\left(\Gamma_{SU(2)} + \frac{w}{2}\right) \cup \left(\Gamma_{SU(2)} + \frac{3w}{2}\right). \tag{7}$$

Thus, the shifted vacuum is degenerate and the ground states have momenta $p_L = \pm\alpha/4$. These states correspond to charged matter fields M_1 and M_2. Note that $p_L^2/2 = 1/16$, which is exactly the same as the conformal dimension $h = 1/16$ of the twisted vacuum in the twist representation. Indeed, the twisted states can be related to the shifted states by a change of basis [21]. Notice that the twisted states have no definite $U(1)$ charge, but the shifted states do. Table 1 shows corresponding matter fields and their $U(1)$ charges.

From Table 1, we find that there is an additional Z_2 symmetry of the matter contents at the lowest mass level (in a complete model, these can correspond to massless states): Transforming the $U(1)$-charges q as

$$q \to -q, \tag{8}$$

while at the same time permuting the fields as $U_1 \leftrightarrow U_2$ and $M_1 \leftrightarrow M_2$ maps the spectrum onto itself. The action on the U_i and M_i fields is described by the 2×2 matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{9}$$

This Z_2 symmetry does not commute with the $U(1)$ gauge symmetry and it turns out that one obtains a symmetry of semi-direct product structure, $U(1) \rtimes Z_2$.

In the twist representation, this model contains the Kähler modulus field T, which corresponds to the current H and is charged under the $U(1)$ group. In the shift representation, the field T is described by the fields U_i as

$$T = \frac{1}{\sqrt{2}}(U_1 + U_2). \tag{10}$$

Now we consider the situation where our orbifold moves away from the enhanced point by taking a specific VEV of the Kähler modulus field T which corresponds to the VEV direction

$$\langle U_1 \rangle = \langle U_2 \rangle. \tag{11}$$