



Building up the elliptic flow: Analytical insights



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ABSTRACT

In this paper, we present a fully analytical description of the early-stage formation of elliptic flow in relativistic viscous hydrodynamics. We first construct an elliptic deformation of Gubser flow which is a boost invariant solution of the Navier–Stokes equation with a nontrivial transverse profile. We then analytically calculate the momentum anisotropy of the flow as a function of time and discuss the connection with the empirical formula by Bhalerao et al. regarding the viscosity dependence of elliptic flow.

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1. Introduction

One of the most intriguing results of ultrarelativistic heavy ion collisions at the RHIC and the LHC is the strong collectivity of the created hot and dense matter, especially the considerable elliptic flow [1–6]. In non-central collisions, the overlapping region of the colliding nuclei approximately has the shape of an ellipse in the transverse plane. This region expands hydrodynamically, and the initial anisotropy in the pressure gradient gets converted into momentum space anisotropy, resulting in the modulation of the azimuthal angle distribution of charged particles in the final state [7]:

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi. \quad (1)$$

The coefficient v_2 is called the elliptic flow parameter and is one of the central objects of experimental and theoretical study in heavy-ion physics because it is a sensitive probe of the equilibrium/non-equilibrium properties of the created matter.

While the mechanism to generate v_2 is well understood, little is known about its analytical details. Theoretically, the extraction of v_2 relies heavily on numerical (viscous) hydrodynamic simulations supplemented with some initial condition and the equation of state (see, e.g., [8–17]). In this paper, we provide a fully analytical description of the early-stage formation of v_2 by deriving and utilizing an approximate elliptic solution of the relativistic Navier–Stokes equation. Such an analysis has long been infeasible due to the difficulty of constructing realistic elliptically-shaped solutions of hydrodynamic equations for which only a few attempts have

been made in the literature [18–21]. The new solution we are able to present here has been achieved along the line of the recent progress in constructing exact solutions of viscous hydrodynamic equations using conformal symmetry [22–26]. Within the region of validity of the solution, one can study explicitly how the elliptic flow develops as a function of proper time and how the shear viscosity affects this evolution.

In Section 2, we review the Gubser flow [22,23] which is an exact boost-invariant solution of the relativistic Navier–Stokes equation with nontrivial radial flow velocities. In Section 3, we use the so-called Zhukovsky transform to elliptically deform the Gubser flow in the transverse plane and construct an approximate solution. We then calculate in Section 4 the momentum space anisotropy of this flow and discuss the connection to the empirical formula proposed by Bhalerao et al. [27] regarding the shear viscosity dependence of the elliptic flow. In the end, we summarize in Section 5.

2. Gubser flow

In this section, we briefly review the exact boost-invariant solution of the relativistic Navier–Stokes equation found by Gubser [22, 23]. The solution is naturally explained by rewriting the Minkowski metric as

$$\begin{aligned} d\hat{s}^2 &= -d\hat{t}^2 + d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2 \\ &= -d\hat{t}^2 + d\hat{x}_\perp^2 + \hat{x}_\perp^2 d\hat{\phi}^2 + \hat{t}^2 d\hat{\zeta}^2, \end{aligned} \quad (2)$$

where $\hat{t} \equiv \sqrt{\hat{t}^2 - \hat{z}^2}$ is the proper time and $\hat{\zeta} \equiv \tanh^{-1} \hat{z}/\hat{t}$ is the space-time rapidity. In this coordinate system, the four-velocity $\hat{u}^\mu$ of the fluid (normalized as $\hat{u}_\mu \hat{u}^\mu = -1$) reads

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$$\begin{aligned}\hat{u}_\tau &= -\cosh\left[\tanh^{-1}\frac{2\hat{\tau}\hat{x}_\perp}{L^2 + \hat{\tau}^2 + \hat{x}_\perp^2}\right], \\ \hat{u}_\perp &= \sinh\left[\tanh^{-1}\frac{2\hat{\tau}\hat{x}_\perp}{L^2 + \hat{\tau}^2 + \hat{x}_\perp^2}\right],\end{aligned}\quad (3)$$

with $\hat{u}_\phi = \hat{u}_\zeta = 0$ and L is roughly the initial transverse size of the fluid. The $\hat{\zeta}$ -independence of Eq. (3) and $\hat{u}_\zeta = 0$ mean that the flow expands in the longitudinal ($\hat{z}$) direction in a boost-invariant way. It also expands in the transverse direction with the transverse velocity

$$\hat{v}_\perp = -\frac{\hat{u}_\perp}{\hat{u}_\tau} = \frac{2\hat{\tau}\hat{x}_\perp}{L^2 + \hat{\tau}^2 + \hat{x}_\perp^2}. \quad (4)$$

Plugging this velocity profile into the relativistic Navier–Stokes equation and assuming conformal symmetry, Gubser obtained the following exact solution for the energy density

$$\begin{aligned}\hat{\mathcal{E}} &= \frac{1}{\hat{\tau}^4} \frac{C^4}{(\cosh \rho)^{8/3}} \\ &\times \left[1 + \frac{\eta_0}{9C} (\sinh \rho)^3 {}_2F_1\left(\frac{3}{2}, \frac{7}{6}, \frac{5}{2}; -\sinh^2 \rho\right) \right]^4, \end{aligned}\quad (5)$$

where $C > 0$ is a constant and the shear viscosity $\hat{\eta}$ has been made dimensionless by factoring out the corresponding power of the energy density $\eta_0 \equiv \hat{\eta}/\hat{\mathcal{E}}^{3/4}$. In Eq. (5), we have defined

$$\sinh \rho \equiv -\frac{L^2 - \hat{\tau}^2 + \hat{x}_\perp^2}{2L\hat{\tau}}. \quad (6)$$

The following components of the shear tensor will be needed for a later calculation.

$$\begin{aligned}\hat{\sigma}_{\tau\perp} &= \frac{2}{3} \frac{\hat{x}_\perp (L^2 - \hat{\tau}^2 + \hat{x}_\perp^2) (L^2 + \hat{\tau}^2 + \hat{x}_\perp^2)}{((L^2 + \hat{\tau}^2 - \hat{x}_\perp^2)^2 + (2L\hat{x}_\perp)^2)^{3/2}}, \\ \hat{\sigma}_{\perp\perp} &= -\frac{1}{3\hat{\tau}} \frac{(L^2 + \hat{\tau}^2 + \hat{x}_\perp^2)^2 (L^2 - \hat{\tau}^2 + \hat{x}_\perp^2)}{((L^2 + \hat{\tau}^2 - \hat{x}_\perp^2)^2 + (2L\hat{x}_\perp)^2)^{3/2}}, \\ \hat{\sigma}_{\perp\phi} &= 0, \quad \hat{\sigma}_{\phi\phi} = -\frac{1}{3\hat{\tau}} \frac{\hat{x}_\perp^2 (L^2 - \hat{\tau}^2 + \hat{x}_\perp^2)}{\sqrt{(L^2 + \hat{\tau}^2 - \hat{x}_\perp^2)^2 + (2L\hat{x}_\perp)^2}}.\end{aligned}\quad (7)$$

As already pointed out by Gubser, the solution has a pathological behavior at large negative values of ρ corresponding to large $\hat{x}_\perp$ and/or small $\hat{\tau}$. Indeed, when $|\sinh \rho| \gg 1$ one can approximate

$${}_2F_1\left(\frac{3}{2}, \frac{7}{6}, \frac{5}{2}; -\sinh^2 \rho\right) \approx \frac{9}{2} (-\sinh \rho)^{-7/3} + \mathcal{O}(\sinh^{-3} \rho), \quad (8)$$

so that

$$\hat{\mathcal{E}} \approx \frac{C^4}{\hat{\tau}^4 (\cosh^2 \rho)^{4/3}} \left[1 - \frac{\eta_0}{2C} \{(-\sinh \rho)^{2/3} + \mathcal{O}(1)\} \right]^4. \quad (9)$$

The quantity inside the square brackets is proportional to the temperature $\hat{T} \propto \hat{\mathcal{E}}^{1/4}$ and this becomes negative for sufficiently negative values of ρ . This is actually not surprising since the relativistic (first-order) Navier–Stokes equation is known to have unphysical features.¹ As demonstrated in Ref. [24], the problem can be cured by switching to the (second-order) Israel–Stewart equation. For the present purpose, we are not concerned about this issue since one can consider η_0 to be arbitrarily small (or C arbitrarily large) so

that the temperature remains positive in a parametrically large region of $\hat{x}_\perp$ and $\hat{\tau}$.

$$\frac{\hat{\tau}L}{L^2 \text{ or } \hat{x}_\perp^2} \gg \left(\frac{\eta_0}{C}\right)^{3/2}. \quad (10)$$

In this region, the solution is well-behaved and offers an attractive model for the studies of strongly interacting matter created in heavy-ion collisions as discussed in Refs. [22,23].

3. Elliptic solution

The Gubser solution described above is cylindrically symmetric around the $\hat{z}$ -axis. Here we relax this restriction and construct an approximate solution which has the shape of an ellipse in the transverse plane. This can be achieved by employing the so-called Zhukovsky (Joukowski) transform² which maps a circle onto an ellipse as follows

$$\begin{aligned}\hat{x} &= \left(x_\perp + \frac{a^2}{x_\perp}\right) \cos \phi = x + \frac{a^2 x}{x^2 + y^2}, \\ \hat{y} &= \left(x_\perp - \frac{a^2}{x_\perp}\right) \sin \phi = y - \frac{a^2 y}{x^2 + y^2},\end{aligned}\quad (11)$$

where a is a constant and we only consider the region $x_\perp > a$. As is manifest in its complex representation $\hat{\omega} = \hat{x} + i\hat{y} = \omega + \frac{a^2}{\omega}$, Eq. (11) is a conformal transformation in two-dimensions and therefore the metric is preserved up to a Weyl factor

$$\begin{aligned}d\hat{x}^2 + d\hat{y}^2 &= \left(1 - \frac{2a^2}{x_\perp^2} \cos 2\phi + \frac{a^4}{x_\perp^4}\right) (dx^2 + dy^2) \\ &\equiv A^2 (dx^2 + dy^2).\end{aligned}\quad (12)$$

Embedding this in four-dimensions, we obtain

$$\begin{aligned}d\hat{s}^2 &= -d\hat{\tau}^2 + d\hat{x}^2 + d\hat{y}^2 + \hat{\tau}^2 d\hat{\zeta}^2 \\ &= A^2 \left[-d\tau^2 + dx_\perp^2 + x_\perp^2 d\phi^2 + \tau^2 d\zeta^2 - \frac{2\tau}{A} d\tau dA \right. \\ &\quad \left. - \frac{\tau^2}{A^2} (dA)^2 \right],\end{aligned}\quad (13)$$

where we have relabeled $\hat{\tau} = A\tau$ and $\hat{\zeta} = \zeta$. If the last two terms were absent, the metric inside the square brackets would be exactly Minkowskian. Let us find the conditions under which these terms can indeed be neglected. More explicitly, we find

$$dA = \frac{2a^2}{Ax_\perp^2} \left[\left(\cos 2\phi - \frac{a^2}{x_\perp^2} \right) \frac{dx_\perp}{x_\perp} + \sin 2\phi d\phi \right]. \quad (14)$$

Since the elliptic deformation $\sim \cos 2\phi$ is an $\mathcal{O}(a^2/x_\perp^2)$ effect, we assume $a^2/x_\perp^2 \ll 1$ and neglect terms of order $\mathcal{O}(a^4/x_\perp^4)$. Then the $(dA)^2$ term in (13) can be safely dropped. In order to drop the cross term $\mathcal{O}(d\tau dA)$ as well, we must additionally assume that $x_\perp \gg \tau$ and neglect terms of order $\mathcal{O}(\tau a^2/x_\perp^3)$ relative to the leading term.

Under these conditions, the coordinate systems $(\hat{\tau}, \hat{x}, \hat{y}, \hat{\zeta})$ and (τ, x, y, ζ) are conformally related, and one can map the Gubser solution expressed in the former coordinates into the latter.³ The transformation rule is given by

² The Zhukovsky transform was originally used to determine the two-dimensional incompressible potential flow around an airfoil.

³ Since Eq. (11) is not an element of the Möbius transformation, it cannot exactly be promoted to a four-dimensional conformal transformation. This is why the solution obtained is an approximate one.

¹ The problem of negative temperature also appears in an exact solution of the Navier–Stokes equation for the Bjorken flow [26].

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