Physics Letters B 736 (2014) 317–320

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Planck-scale dimensional reduction without a preferred frame

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ARTICLE INFO

Article history:
Received 18 June 2014
Received in revised form 16 July 2014
Accepted 17 July 2014
Available online 23 July 2014
Editor: M. Cvetič

ABSTRACT

Several approaches to quantum gravity suggest that the standard description of spacetime as probed at low-energy, with four dimensions, is replaced in the Planckian regime by a spacetime with a spectral dimension of two. The implications for relativistic symmetries can be momentous, and indeed the most tangible picture for "running" of the spectral dimension, found within Horava–Lifshitz gravity, requires the breakdown of relativity of inertial frames. In this Letter we incorporate running spectral dimensions in a scenario that does not require the emergence of a preferred frame. We consider the best studied mechanism for deforming relativistic symmetries whilst preserving the relativity of inertial frames, based on a momentum space with curvature at the Planck scale. We show explicitly how running of the spectral dimension can be derived from these models.

1. Introduction

One of the most robust predictions of quantum-gravity is that spacetime itself should acquire quantum properties, when probed at the Planck scale ($\simeq 10^{28}$ eV). Over the past decade it was gradually appreciated that two key issues deserve priority in investigations of quantum models of spacetime: 1) What is the fate of relativistic symmetries in the Planck-scale description of spacetime? 2) Is spacetime still four-dimensional in the Planck-scale regime? The second question may appear to be ill-defined, since our intuitive notion of spacetime dimensionality is based on properties of purely classical geometries. The most intuitive such notion is the Hausdorff dimension, captured by the scaling exponent of the volume of a sphere. Unlike the case of a smooth classical spacetime, in a quantum or discrete geometry the notion of Hausdorff dimension is technically more challenging to define. This has led quantum-gravity researchers to employ an alternative definition: the spectral dimension. This concept is encoded in the spectral properties of the scalar Laplacian for the theory of interest. For smooth classical spacetimes the spectral dimension coincides with the Hausdorff dimension. In a quantum geometry the latter is in general inapplicable, but the spectral dimension of spacetime is still well-defined.

Interestingly, as the spectral dimension criterion became adopted in a growing number of approaches, it emerged that rather generically the spectral dimension in the UV regime is smaller than 4 (e.g. [1] and references therein). It is particularly intriguing that some of the most studied, but ostensibly very different, quantum gravity theories predict that the value of the spectral dimension in the UV is 2. This conclusion finds support in the CDT (Causal-Dynamical-Triangulation) approach [2], Asymptotic Safety [3], Horava–Lifshitz (HL) gravity [4], and Loop Quantum Gravity (LQG) [5].

Irrespective of the alleged UV dimensional reduction phenomenon, the fate of relativistic symmetries in the Planckian regime has attracted interest from other angles (see e.g. [6–8]). Relativistic symmetries may be left unscathed by the new structures at the Planck scale (e.g. [9]), but there are at least two other possibilities. Planck-scale effects may *break* relativistic invariance, introducing a preferred-frame [10–14]; or they may *deform* the relativistic symmetry transformations, preserving the relativity of inertial frames [15–20]. In this Letter we contribute to the understanding of the interplay between spectral dimensional reduction and the fate of relativistic symmetries at the Planck scale.

It is evident that any model of spacetime with dimensional reduction must bring relativistic transformations under scrutiny [21,22]. Yet, in most studies the analysis is confined to the perspective of a single observer, without mention of how a boosted observer would describe the same phenomenon. An exception is found in HL gravity, where an explicit breakdown of the equivalence of inertial observers is vividly manifest [4,22]. The fate of relativistic invariance in CDT, Asymptotic Safety and LQG remains the subject of a lively debate (e.g. [12,23,24]). We hope to contribute to this debate by showing that the phenomenon of running spectral dimension arises naturally within the most studied mechanism

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http://dx.doi.org/10.1016/j.physletb.2014.07.030
0370-2693/

for deformation of relativistic symmetries, preserving the relativity of inertial frames. The mechanism assumes that momentum space is curved at the Planck scale, and remarkably it can easily describe the topical case of a two-dimensional UV regime. In our closing remarks we discuss the significance of these findings.

2. Preferred-frame scenarios for running spectral dimension

We start by summarizing a few facts about preferred-frame scenarios. The results collected here are all essentially known ([25] and references therein), but some have not been previously spelled out as explicitly as we shall do. They will be useful to contrast preferred-frame and frame-invariant models, the latter being our main focus of interest.

A useful starting point is a modified-Laplacian with Euclideanized ("Wick-rotated") form:

$$D_E = -\partial_t^2 - \nabla^2 - \ell_t^{2\gamma_t}\partial_t^{2(1+\gamma_t)} - \ell_x^{2\gamma_x}\nabla^{2(1+\gamma_x)} \qquad (1)$$

where γ_t and γ_x are dimensionless parameters, and ℓ_t and ℓ_x are parameters with dimensions of length (usually assumed to be of the order of the inverse of the Planck-scale). Modified Laplacians with Euclidean form given by (1) are relevant in scenarios where Planck-scale effects break relativistic invariance, often labeled LIV (Lorentz Invariance Violating). The transition from IR to UV in the chosen Laplacians is not relevant for this paper, only the asymptotic forms.

The spectral dimension is the effective dimension probed by a fictitious diffusion process governed by a Laplacian operator. The core features are encoded in the average "return probability", given by

$$P(s) \propto \int dE\, dp\, p^{D-1} e^{-s\Omega(E,p)}, \qquad (2)$$

where s is a fictitious "diffusion time", p is the modulus of the spatial momentum, D is the number of spatial dimensions in the IR regime, for which we shall often assume $D=3$, and $\Omega(p)$ is the momentum-space representation of the Laplacian operator, which for (1) is

$$\Omega_{\gamma_t\gamma_x}(E,p) = E^2 + p^2 + \ell_t^{2\gamma_t}E^{2(1+\gamma_t)} + \ell_x^{2\gamma_x}p^{2(1+\gamma_x)}. \qquad (3)$$

The spectral dimension in the UV regime, $d_S(0)$, is obtained from the $P(s)$ by computing

$$d_S(0) = -2\lim_{s\to 0}\frac{d\ln P(s)}{d\ln(s)}, \qquad (4)$$

and the spectral dimension "runs" whenever $d_S(0) \neq D+1$. The presence of a running spectral dimension at short diffusion scales signals an "anomalous" diffusion which can be seen as a reflection of the quantum properties of spacetime. In the IR regime, where quantum effects of the geometry are switched off, we recover standard diffusion on a smooth flat geometry and the limit $s\to\infty$ leads to an IR spectral dimension coinciding with the Hausdorff dimension $d_S(\infty) = D+1$. We stress that we will not assume at any stage that $P(s)$ in (2) is a probability: rather, we use it as a technical expedient for computing the UV dimension. This is particularly important since recently Ref. [1] exposed noteworthy examples in which $P(s)$ cannot be interpreted as a return probability. Yet, even in such instances, the UV spectral dimension (i.e. when $s\to 0$) as inferred from (2) is correct.

To compute $d_S(0)$ for the LIV models characterized by Eq. (3) we can use [4,25]

$$f(z) \propto \int dx\, x^n e^{-z(x^2+\alpha x^{2\beta})} \quad\Longrightarrow\quad \lim_{z\to 0}\frac{d\ln f(z)}{d\ln(z)} = -\frac{n+1}{2\beta} \qquad (5)$$

so that:

$$d_S(0) = \frac{1}{1+\gamma_t} + \frac{D}{1+\gamma_x}. \qquad (6)$$

The LIV model with $\gamma_t = 0$ and $\gamma_x = 2$ describes HL gravity [4,22], and indeed gives $d_S(0) = 2$ for $D = 3$. Eq. (6) generalizes this result.

In Ref. [26] we established a correspondence between the UV spectral dimension of spacetime and the UV Hausdorff dimension of momentum space for LIV models with $\gamma_t = 0$ and general γ_x. We now prove that the argument applies for arbitrary values of γ_t and γ_x. For this purpose we adopt a change of integration variables which in the UV takes the form:

$$\tilde{E} \propto E^{1+\gamma_t}$$
$$\tilde{p} \propto p^{1+\gamma_x} \qquad (7)$$

whereas in IR it leaves momentum space unchanged (the transition between the two regimes is irrelevant for the argument). Then *any* integral over momentum space involving a function of the UV-modified Laplacian will be converted into an integral of the same function of the unmodified Laplacian, but with a suitably UV-modified integration measure. For example, Eq. (2) becomes

$$P(s) \propto \int d\tilde{E}\, d\tilde{p}\, \tilde{p}^{\frac{D-\gamma_x-1}{\gamma_x+1}}\, \tilde{E}^{-\frac{\gamma_t}{1+\gamma_t}} e^{-s(\tilde{E}^2+\tilde{p}^2)}, \qquad (8)$$

up to terms that are negligible in the UV regime. We notice that the relevant integration measure factorizes in E and p, leading to a valuable intuitive characterization of (6). The energy integration has measure $d\tilde{E}\,\tilde{E}^{\frac{1}{1+\gamma_t}-1}$ suggesting that the effective UV Hausdorff dimension of energy space is $1/(1+\gamma_t)$, whereas the momentum integration measure is $d\tilde{p}\,\tilde{p}^{\frac{D}{1+\gamma_x}-1}$ suggesting $D/(1+\gamma_x)$ Hausdorff dimensions. These match the two terms in (6), thereby generalizing the argument in [26].

3. Running spectral dimension without a preferred frame

Preferred-frame LIV scenarios, such as the one contained in HL gravity, provide a compelling description of Planck-scale dimensional reduction, including the topical case of a 2-dimensional UV regime. We now show that an equally encouraging picture of the same phenomenon can be found in scenarios where the relativistic symmetries are deformed, without spoiling the relativity of inertial frames. These are often dubbed "DSR" (Doubly, or Deformed, Special Relativity). Remarkably, we need look no further than the simplest such scheme [15,20,27], which is based on the assumption that momentum space has de Sitter geometry, with the Planck scale playing the role of its curvature scale.

We start by highlighting the relevant features of the model, referring the interested reader to the copious literature for more detail (e.g. [15,20,27] and Ref. [28] for the Euclideanization prescription). For definiteness let us use a coordinatization such that the (Wick-rotated) de Sitter metric on momentum space is

$$ds^2 = g^{\mu\nu}dp_\mu dp_\nu = dE^2 + e^{2\ell E}\sum_{j=1}^{D} dp_j^2.$$

The fact that these theories do not pick a preferred frame, but do require a deformation of relativistic transformation laws, is a direct consequence of the fact that de Sitter space is a maximally symmetric geometry. One of several equivalent ways of introducing ordinary special relativity starts from the isometries of a Minkowski momentum space and then derives the transformation laws of spacetime coordinates by consistency [27]. The isometries of de Sitter momentum space can be seen as a deformation of the isometries of Minkowski momentum space, and as a result a theory built upon the isometries of de Sitter momentum space is