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Perturbation of the metric around a spherical body from a nonminimal coupling between matter and curvature

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ABSTRACT

In this work, the effects of a nonminimally coupled model of gravity on a perturbed Minkowski metric are presented. The action functional of the model involves two functions, $f^1(R)$ and $f^2(R)$, of the Ricci scalar curvature R: the former extends the usual linear term found in the Einstein–Hilbert Lagrangian, while the latter is multiplied by the matter Lagrangian density, thus introducing an explicit nonminimal coupling.

Based upon a Taylor expansion around $R = 0$ for both functions, we find that the metric around a spherical object is a perturbation of the weak-field Schwarzschild metric: the perturbation of the tt component of the metric tensor is shown to be a Newtonian plus Yukawa term, which can be constrained using the available experimental results. It is shown that this effect can be canceled or made arbitrarily small when the characteristic mass scales of the two functions are similar. We conclude that the Starobinsky model for inflation complemented with a generalized preheating mechanism is not experimentally constrained by observations. The geodetic precession effects of the model are also shown to be of no relevance for the constraints.

1. Introduction

Modern physics uses the concepts of dark matter and dark energy to advance an explanation for the astrophysical problem of the flattening of galactic rotation curves and the cosmological problem of the accelerated expansion of the universe, respectively. Dark energy, which is supposed to account for 74% of all the matter of the universe, has many theories on its basis, as the so-called "quintessence" models [1–3] and the existence of scalar fields that account for both dark matter and dark energy [4].

More recent approaches start from the idea of the incompleteness of the fundamental laws of General Relativity (GR), involving, for example, corrections to the Einstein–Hilbert action. Such theories involve a non-linear correction to the geometry part of the action, being thus called $f(R)$ theories. In the last decade, work on $f(R)$ theories has been very profitable, as thoroughly discussed in Ref. [5]. These can be extended to also include a nonminimum coupling (NMC) between the scalar curvature and the matter Lagrangian density.

Indeed, these NMC theories have many interesting features, as can be seen by several studies, such as the impact on stellar observables [6], the energy conditions [7], the equivalence with multi-scalar–tensor theories (with only one degree of freedom arising from the $f(R)$ term, as the NMC gives rises to an auxiliary scalar field with no kinetic term) [8], the possibility to account for galactic [9] and cluster [10] dark matter, cosmological perturbations [11], a mechanism for mimicking a Cosmological Constant at astrophysical scales [12], post-inflationary reheating [13] or the current accelerated expansion of the universe [14,15], the dynamical impact of the choice of the Lagrangian density of matter [16,17], gravitational collapse [18], its Newtonian limit [19], the existence of closed timelike curves [20] and, the most recent one, a determination of Solar System constraints to a cosmological NMC [21].

For other NMC gravity theories and their potential applications, see *e.g.* [22–26].

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One of the first motivations that brought $f(R)$ theories into the physicists daily work was the Starobinsky inflation model, where $f(R) = R + R^2/(6m^2)$ was considered [27,13], with WMAP normalization of the CMB temperature anisotropies indicating that $m \sim 3 \times 10^{-6} M_P$, where M_P is the Planck mass [28].

Without directly mentioning the Starobinsky inflation, Ref. [29] considers a quadratic $f(R)$ function and develops an expansion in powers of $(1/c)$ of an asymptotically flat Minkowski metric, showing the presence of a Yukawa correction to the tt component of the latter [29]. Following the equivalence between scalar–tensor and $f(R)$ theories [30–32], this can be interpreted as due to the additional gravitational contribution of the massive degree of freedom embodied in a non-linear $f(R)$ function.

In this work we follow a similar procedure of Ref. [29] where we instead consider a NMC model. We consider that the additional degree of freedom arising from a non-trivial $f(R)$ function is sufficiently massive so that its effects are not extremely long-ranged, and as such we can neglect the background cosmological setting — this point (which shall be developed in the following) shows that this work is complementary to the recent study on the compatibility between cosmological and Solar System dynamics of a NMC model [21].

In Section 2 such a model is presented and in Section 3 the solution of the linearized field equations is computed. We obtain the solutions for the perturbative potentials $\Psi(r)$ and $\Phi(r)$ of the metric, which contain a form factor specific of the Yukawa potential that is addressed in Section 4. The tt component of the metric yields the modified gravitational potential, which includes a Newtonian plus a Yukawa contribution. The comparison of these results with available experimental constrains is presented in Section 5. This section also addresses the radial potential through the constraints obtained to the geodetic precession values. Finally, conclusions are drawn.

2. The model

The action functional of gravity for the NMC case is of the form [33]

$$S = \int \left[\frac{1}{2} f^1(R) + \left[1 + f^2(R)\right] \mathcal{L}_m \right] \sqrt{-g}\, d^4x, \tag{1}$$

where $f^i(R)$ $(i = 1, 2)$ are functions of the Ricci scalar curvature R, $\mathcal{L}_m$ is the Lagrangian density of matter and g is the metric determinant. The standard Einstein–Hilbert action is recovered by taking

$$f^1(R) = 2\kappa(R - 2\Lambda), \qquad f^2(R) = 0, \tag{2}$$

where $\kappa = c^4/16\pi G$, G is Newton's gravitational constant and Λ is the Cosmological Constant.

The variation of the action functional with respect to the metric $g_{\mu\nu}$ yields the field equations

$$\begin{aligned} &\left(f^1_R + 2 f^2_R \mathcal{L}_m\right) R_{\mu\nu} - \frac{1}{2} f^1 g_{\mu\nu} \\ &\quad = \left(1 + f^2\right) T_{\mu\nu} + \left(\Box_{\mu\nu} - g_{\mu\nu}\Box\right)\left(f^1_R + 2 f^2_R \mathcal{L}_m\right), \end{aligned} \tag{3}$$

where $f^i_R \equiv df^i/dR$ and $\Box_{\mu\nu} \equiv \nabla_\mu \nabla_\nu$.

In the following we assume that matter behaves as dust, *i.e.* a perfect fluid with negligible pressure and an energy–momentum tensor described by

$$T_{\mu\nu} = \rho c^2 u_\mu u_\nu, \qquad u_\mu u^\mu = -1, \tag{4}$$

where ρ is the matter density and u_μ is the four-velocity vector. The trace of the energy–momentum tensor is $T = -\rho c^2$. We use $\mathcal{L}_m = -\rho c^2$ for the Lagrangian density of matter (see Ref. [16] for a discussion).

We consider a spherically symmetric body with a static radial mass density $\rho = \rho(r)$ and we assume that the function $\rho(r)$ and its first derivative are continuous across the surface of the body,

$$\rho(R_S) = 0 \quad \text{and} \quad \frac{d\rho}{dr}(R_S) = 0, \tag{5}$$

where R_S denotes the radius of the spherical body. These conditions will play a crucial role in the following sections, when integrals that have R_S as an integration limit will appear.

The metric used is one that describes the spacetime around a spherical star like the Sun and it is given by the following perturbation of the Minkowski metric, in spherical coordinates:

$$ds^2 = -\left[1 + 2\Psi(r)\right] c^2 dt^2 + \left[1 + 2\Phi(r)\right] dr^2 + r^2 d\Omega^2, \tag{6}$$

where Ψ and Φ are perturbing functions such that $|\Psi(r)| \ll 1$ and $|\Phi(r)| \ll 1$.

For the purpose of the present paper the functions Ψ and Φ will be computed at order $\mathcal{O}(1/c^2)$.

We assume that the functions $f^i(R)$ admit the following Taylor expansions around $R = 0$, which coincide with the forms used in Ref. [13]:

$$f^1(R) = 2\kappa\left(R + \frac{R^2}{6m^2}\right) + \mathcal{O}\left(R^3\right), \qquad f^2(R) = 2\xi \frac{R}{m^2} + \mathcal{O}\left(R^2\right), \tag{7}$$

where m is a characteristic mass scale and ξ a dimensionless parameter specific of the NMC, indicating the relative strength of the latter with respect to the quadratic term in $f^1(R)$.

Notice also that the Cosmological Constant is dropped, consistent with the assumption that the metric is asymptotically flat — *i.e.* no cosmological background with a time-dependent, non-vanishing curvature $R_0 \neq 0$ is assumed, contrary to what was considered in Ref. [21]. In that study, a set of viability criteria for the form of $f^2(R)$ was developed based upon the compatibility of the large scale effects (*i.e.* description of dark energy) and allowed Solar System impact: it is worth mentioning that the validity of such criteria required a very light additional degree of freedom, $m_0 r \ll 1$, with mass given by

$$\begin{aligned} m_0^2 = \frac{1}{3}\Bigg[&\frac{f^1_{R0} - f^2_{R0}\mathcal{L}_m}{f^1_{RR0} + 2 f^2_{RR0}\mathcal{L}_m} - R_0 \\ &- \frac{3\Box\left(f^1_{RR0} - 2 f^2_{RR0}\rho^{\cos}\right) - 6\rho\Box f^2_{RR0}}{f^1_{RR0} + 2 f^2_{RR0}\mathcal{L}_m} \Bigg], \end{aligned} \tag{8}$$

where the subscript $_0$ indicates that the quantities are evaluated at their background cosmological value $R = R_0$ (*e.g.* $f^i_{R0} \equiv f^i_R(R_0)$) and $\rho^{\cos}$ is the corresponding background cosmological density.

3. Solution of linearized modified field equations

3.1. Solution for the curvature R

The trace of the field equations (3) is

$$\left(f^1_R + 2 f^2_R \mathcal{L}_m\right) R - 2 f^1 = -3\Box\left(f^1_R + 2 f^2_R \mathcal{L}_m\right) + \left(1 + f^2\right) T. \tag{9}$$

After expanding the trace with the respective expressions, the equation is linearized: this is done by neglecting terms of order $\mathcal{O}(1/c^3)$ or smaller. It yields the following equation,