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How (non-)linear is the hydrodynamics of heavy ion collisions?

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ABSTRACT

We provide evidence from full numerical solutions that the hydrodynamical evolution of initial density fluctuations in heavy ion collisions can be understood order-by-order in a perturbative series in deviations from a smooth and azimuthally symmetric background solution. To leading linear order, modes with different azimuthal wave numbers do not mix. When quadratic and higher order corrections are numerically sizable, they can be understood as overtones with corresponding wave numbers in a perturbative series. Several findings reported in the recent literature result naturally from the general perturbative series formulated here.

In recent years, fluid dynamic simulations of relativistic heavy ion collisions have provided strong evidence for a picture according to which the momentum distributions of soft hadrons result from a fluid dynamic evolution of initial density fluctuations, see Refs. [1–4] for recent reviews. The research focuses now on understanding in detail the mapping from fluctuations in the initial state to experimentally accessible observables in the final state [5–16]. While hydrodynamic evolution always shows non-linearities of some size, we ask here whether the hydrodynamics of heavy ion collisions is sufficiently weakly non-linear to be described by a perturbative series. We shall demonstrate that the fluid dynamic response to initial perturbations obeys a general ordering principle in that it can be organized in terms of a perturbative expansion in powers of the amplitudes of initial fluctuations around a background, see Eq. (2) below. This perturbative series will be shown to apply also in cases where non-linearities are sizable or dominant. This is of interest since a mapping in which non-linearities are organized as corrections to a linear response provides a particularly simple and thus particular powerful tool for relating experimental observables to the initial conditions of heavy ion collisions and to those properties of matter that govern their fluid dynamic evolution [17].

We consider initial conditions of heavy ion collisions, specified in terms of fluctuating fluid dynamic fields h_i on a hyper surface at fixed initial time τ_0. Here, the index i runs over all independent fields,

$$h_i(\tau, r, \varphi, \eta) = \left(w, u^r, u^\phi, u^\eta, \pi_{\text{bulk}}, \pi^{\eta\eta}, \ldots\right), \quad (1)$$

including e.g. the enthalpy density $h_1 = w$, three independent fluid velocity components, the bulk viscous tensor, the independent components of the shear viscous tensor, etc. In the following we assume Bjorken boost invariance and drop the rapidity-argument η in the hydrodynamical fields. Following Refs. [17,18], we express h_i in terms of a background component h_i^{BG} and an appropriately normalized perturbation $\tilde{h}_i$. The background is taken to be a solution of the non-linear hydrodynamic equations initialized at τ_0 with an azimuthally symmetric average over many events. It is evolved with the fluid dynamic solver ECHO-QGP [19]. For any sample of events, this background needs to be determined only once. We ask to what extent the time evolution of the $\tilde{h}_i$, obtained from the full numerical ECHO-QGP solutions without any linearized approximation, can be understood in terms of a perturbative series on top of the background fields,

$$\tilde{h}_i(\tau, r, \varphi) = \int_{r', \varphi'} \mathcal{G}_{ij}(\tau, \tau_0, r, r', \varphi - \varphi')\tilde{h}_j(\tau_0, r', \varphi')$$

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$$+\frac{1}{2}\int_{r',r'',\varphi',\varphi''}\mathscr{H}_{ijk}(\tau,\tau_0,r,r',r'',\varphi-\varphi',\varphi-\varphi'')$$
$$\times\tilde{h}_j(\tau_0,r',\varphi')\tilde{h}_k(\tau_0,r'',\varphi'')+\mathcal{O}(\tilde{h}^3), \qquad (2)$$

where $\int_r=\int_0^\infty dr\,r$, $\int_\varphi=\int_0^{2\pi}d\varphi$ etc. The kernels $\mathscr{G}_{ij}$, $\mathscr{H}_{ijk}$ (and corresponding terms for higher orders in $\tilde{h}_i$) depend on the time-evolved background h_l^{BG} only. Due to the azimuthal rotation symmetry of the background, $\mathscr{G}_{ij}$ depends on the angles φ and φ' only via the difference $\varphi-\varphi'$ and similarly for $\mathscr{H}_{ijk}$. The question we raise in the title can now be made more precise: We ask whether the expansion (2) is possible for a suitably chosen background,[1] in which range it is dominated by the first linear term, and whether non-linearities even if large can be understood perturbatively on the basis of (2).

For the initial conditions, we make assumptions that are widely spread in the phenomenological literature. The initial transverse velocity components vanish, the longitudinal velocity is Bjorken boost invariant, the shear stress tensor is initialized by its Navier–Stokes value, and the bulk viscous pressure is neglected. Initial fluctuations reside then only in the initial enthalpy density $w(\tau,\vec{r})$, that we parametrize in terms of an azimuthally averaged background $w_{BG}(\tau,r)$ and the weights $\tilde{w}_l^{(m)}$ of the azimuthal (m) and radial (l) wave numbers of a discrete orthonormal Bessel–Fourier decomposition [17]

$$w(\tau_0,r,\varphi)=w_{BG}(\tau_0,r)\left(1+\sum_{m=-\infty}^{\infty}\tilde{w}^{(m)}(\tau_0,r)\,e^{im\varphi}\right),$$
$$\tilde{w}^{(m)}(\tau_0,r)=\sum_{l=1}^{\infty}\tilde{w}_l^{(m)}J_m\big(k_l^{(m)}r\big). \qquad (3)$$

Here $k_l^{(m)}=z_l^{(m)}/R$, where $z_l^{(m)}$ is the l-th zero of the modified Bessel function J_m and $R=8$ fm throughout this work. Since $\tilde{w}(\tau,r,\varphi)$ is real, we have $\tilde{w}^{(m)}(\tau,r)=\tilde{w}^{(-m)*}(\tau,r)$. In the following, we take the weights with $m\geq 0$ as the independent ones and write

$$\tilde{w}_l^{(m)}=\big|\tilde{w}_l^{(m)}\big|e^{-im\psi_l^{(m)}}. \qquad (4)$$

The corresponding modes with $m<0$ are then not independent and are defined by the condition $|\tilde{w}_l^{(m)}|=|\tilde{w}_l^{(-m)}|$ with azimuthal angle $\psi_l^{(-m)}=\psi_l^{(m)}\pm\pi$.

We next discuss the physically relevant range of $|\tilde{w}_l^{(m)}|$. For central heavy ion collisions, the event averaged weights $\langle|\tilde{w}_l^{(m)}|\rangle\simeq O(0.1)$ and the tails of event distributions satisfy $|\tilde{w}_l^{(m)}|\lesssim 0.5$, see e.g. Fig. 13 of Ref. [18]. In peripheral collisions, event distributions shift to larger $|\tilde{w}_l^{(m)}|$ with increasing impact parameter b, since the expansion (3) is around an azimuthally symmetric background. Also in these non-central collisions, $\langle|\tilde{w}_l^{(m)}|\rangle$ is much smaller than unity (e.g. $\langle|\tilde{w}_l^{(m)}|\rangle\simeq 0.5$ at $b=6$ fm for the model in Ref. [18]). For an intuitive understanding of these values, one may consider the case for which one single fluctuating basis mode, say the mode with the weight $\tilde{w}_1^{(2)}$, is embedded on top of $w_{BG}(\tau_0,r)$

$$w(\tau_0,\vec{r})=w_{BG}(\tau_0,r)\big[1+2\big|\tilde{w}_1^{(2)}\big|J_2\big(k_1^{(2)}r\big)\cos\big(2\big(\varphi-\psi_1^{(2)}\big)\big)\big]. \qquad (5)$$

For one single mode, we can set without loss of generality $\psi_1^{(2)}=0$. The Bessel function J_2 takes a maximal value $\max[J_2(r)]=0.4865$, and the enthalpy density (5) is therefore positive definite at all transverse positions only for $|\tilde{w}_1^{(2)}|<1/(2\max[J_2(r)])=1.028$. Larger values for $|\tilde{w}_1^{(2)}|$ can arise only if the presence of additional modes $|\tilde{w}_l^{(m)}|$ ensures that the negative contribution from $|\tilde{w}_1^{(2)}|$ to $w(\tau_0,\vec{r})$ is canceled everywhere. The larger the cancellation needed, the smaller the probability that it arises in an event sample. This may provide an intuitive understanding for why even the tails of event distributions of peripheral collisions are confined to values $|\tilde{w}_1^{(2)}|$ smaller than 1.5, and why the most likely initial conditions show values that are $O(0.5)$ or smaller. Numerically similar constraints are obtained for other basis modes.

In Fig. 1, we test the fluid dynamic response to a single fluctuating basis mode (5) for the entire physically relevant parameter range $|\tilde{w}_l^{(m)}|<1.5$. Assuming for simplicity a Bjorken-boost invariant longitudinal dependence, we evolve these initial conditions with the $2+1$ dimensional version of the hydrodynamical code ECHO-QGP [19] with a value $\eta/s=1/4\pi$ for the ratio of shear viscosity to entropy density.[2] Following Ref. [20], we use the equation of state s95p-PCE which combines lattice QCD results at high temperatures with a hadron resonance gas at low temperatures. The background w_{BG} used throughout this paper is initialized at $\tau_0=0.6$ fm/c with an azimuthally symmetric average of Glauber model initial conditions for Pb+Pb collisions at the LHC, described in Ref. [18]. The time evolution of w_{BG} determined from ECHO-QGP is shown in Fig. 1. The time-evolved fluctuation $\tilde{w}^{(2)}(\tau,r)$ is determined from the full hydrodynamic evolution via Fourier analysis.[3] The main observation from Fig. 1 is that the fluid dynamic response to (5) scales approximately linearly with the initial amplitude $\tilde{w}_1^{(2)}$ of the perturbation. This scaling, expected from the linear term in Eq. (2), is very good for values $\tilde{w}_1^{(2)}<0.6$. We observe this linear dependence with similar accuracy also for other basic modes (data not shown). More sizable deviations from linear scaling are observed for $\tilde{w}_1^{(2)}>0.6$, see Fig. 1.

The main message of this work is that even for the largest physically relevant amplitudes $\tilde{w}_l^{(m)}$, where non-linear contributions can dominate the hydrodynamic response, the observed non-linearities can be understood in terms of the perturbative ansatz (2). To explain this point, we note first that the dominant non-linearity in the hydrodynamic response to a fluctuation with weights $w_1^{(2)}$ is in modes $m\neq 2$. Consider the Fourier series $\tilde{h}_i(\tau,r,\varphi)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{im\varphi}\,\tilde{h}_i^{(m)}(\tau,r)$, where $\tilde{h}_i^{(m)}(\tau,r)$ are in general complex expansion coefficients, but $\tilde{h}_i^{(m)}(\tau,r)=\tilde{h}_i^{(-m)*}(\tau,r)$ since $\tilde{h}(\tau,r,\varphi)\in\mathbb{R}$. Since the kernels in (2) depend only on the background field, they are invariant under azimuthal rotation and their Fourier expansions read

$$\mathscr{G}_{ij}\big(\tau,\tau_0,r,r',\Delta\varphi\big)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{im\Delta\varphi}\mathscr{G}_{ij}^{(m)}\big(\tau,\tau_0,r,r'\big),$$

[1] Hydrodynamic evolution is governed by non-linear partial differential equations and it may be chaotic or it may contain terms that are non-analytic in the initial fluid fields $\tilde{h}_j$. Hence, the validity of the expansion (2) is not guaranteed. Also, it will depend on the choice of the background h^{BG} and on the strength of the perturbations $\tilde{h}$.

[2] For these $2+1$ dimensional simulations in Bjorken coordinates, we adopt a uniform grid in x and y with a spatial resolution of 0.2 fm, whereas the time-step is set to 10^{-3} fm/c, with a Courant number of 0.2 to ensure stability. Spatial reconstruction is achieved by employing the MPE5 scheme, the most accurate one available in ECHO-QGP (fifth order for smooth flows). For further technical details, see Ref. [19].

[3] Fluctuations at time τ_0 are cut-off in the region of very low background density, see e.g. $\tilde{w}^{(2)}(\tau_0,r)$ in Fig. 1. Also, for the extreme weights $\tilde{w}_1^{(2)}=1.2$ and 1.4, the distribution (5) is cut off in the regions in which it would turn negative. For values $|\tilde{w}_1^{(2)}|<1/(2\max[J_2(r)])=1.028$, the single mode can be propagated without such an ad hoc modification. We have checked that these cuts do not affect our conclusions.