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Spin–orbit correlations in the nucleon

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ABSTRACT

We investigate the correlations between the quark spin and orbital angular momentum inside the nucleon. Similarly to the Ji relation, we show that these correlations can be expressed in terms of specific moments of measurable parton distributions. This provides a whole new piece of information about the partonic structure of the nucleon.

1. Introduction

One of the key questions in hadronic physics is to unravel the spin structure of the nucleon, a very interesting playground for understanding many non-pertubative aspects of quantum chromodynamics (QCD). So far, most of the efforts have focused on the proper decomposition of the nucleon spin into quark/gluon and spin/orbital angular momentum (OAM) contributions (see Ref. [1] for a detailed recent review) and their experimental extraction. The spin structure is however richer than this.

Since the spin and OAM have negative intrinsic parity, the only non-vanishing single-parton ($a = q, G$) longitudinal correlations allowed by parity invariance are $\langle S^a_z S^N_z\rangle$, $\langle L^a_z S^N_z\rangle$ and $\langle L^a_z S^a_z\rangle$, where $\langle\ \rangle$ denotes the appropriate average, $S^{q,G}_z$ is the quark/gluon longitudinal spin, $L^{q,G}_z$ is the quark/gluon longitudinal OAM and S^N_z is the nucleon longitudinal spin. Since we are interested in the intrinsic correlations only, the global orbital motion of the system L^N_z is not considered. The first two kinds of correlation are usually just called spin and OAM contributions of parton a to the nucleon spin. The last type is simply the parton spin–orbit correlation.

Even though generalized parton distributions (GPDs) and transverse-momentum dependent parton distributions (TMDs) are naturally sensitive to the parton spin–orbit correlations, no quantitative relation between them has been derived so far. The only quantitative relation we are aware of has been obtained in Ref. [2] at the level of generalized TMDs (GTMDs) [3,4], also known as unintegrated GPDs (uGPDs), which are unfortunately not yet related to any experimental process.

In this Letter we provide the relation between the quark spin–orbit correlation and measurable parton distributions. Our approach is similar to the one used in Ref. [5] in the case of quark OAM, but this time in the parity-odd sector and with an asymmetric tensor. The Letter is organized as follows: In Section 2, we define the quark spin–orbit correlation operator and express the corresponding expectation value in terms of form factors. In Section 3 we relate these form factors to moments of measurable parton distributions. In Section 4, we provide an estimate of the various contributions, and conclude the paper with Section 5.

2. Quark spin–orbit correlation

It is well known that the light-front operator giving the total number of quarks can be decomposed into the *sum* of right- and left-handed quark contributions

$$\hat N^q = \int d^3x\,\overline{\psi}\gamma^+\psi \tag{1}$$

$$= \underbrace{\int d^3x\,\overline{\psi}_R\gamma^+\psi_R}_{\hat N^{q_R}} + \underbrace{\int d^3x\,\overline{\psi}_L\gamma^+\psi_L}_{\hat N^{q_L}}, \tag{2}$$

where $\psi_{R,L} = \frac{1}{2}(\mathbb{1}\pm\gamma_5)\psi$, $a^\pm = \frac{1}{\sqrt{2}}(a^0 + a^3)$ for a generic four-vector a, and $d^3x = dx^- d^2x_\perp$. The quark longitudinal spin operator simply corresponds to half of the *difference* between right- and left-handed quark numbers

$$\hat S^q_z = \int d^3x\,\frac{1}{2}\overline{\psi}\gamma^+\gamma_5\psi \tag{3}$$

$$= \frac{1}{2}\left(\hat N^{q_R} - \hat N^{q_L}\right). \tag{4}$$

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Similarly, we decompose the local gauge-invariant light-front operator for the quark longitudinal OAM [5] into the *sum* of right- and left-handed quark contributions

$$\hat{L}_z^q = \int \mathrm{d}^3x \, \frac{1}{2}\overline{\psi}\gamma^+(\boldsymbol{x} \times i\overleftrightarrow{\boldsymbol{D}})_z\psi \tag{5}$$

$$= \hat{L}_z^{q_R} + \hat{L}_z^{q_L}, \tag{6}$$

where $\overleftrightarrow{\boldsymbol{D}} = \overrightarrow{\boldsymbol{\partial}} - \overleftarrow{\boldsymbol{\partial}} - 2ig\boldsymbol{A}$ is the symmetric covariant derivative, and $\hat{L}_z^{q_{R,L}} = \int \mathrm{d}^3x \, \frac{1}{2}\overline{\psi}_{R,L}\gamma^+(\boldsymbol{x} \times i\overleftrightarrow{\boldsymbol{D}})_z\psi_{R,L}$. The *difference* between these right- and left-handed quark contributions will be referred to as the quark longitudinal spin–orbit correlation operator which reads

$$\hat{C}_z^q = \int \mathrm{d}^3x \, \frac{1}{2}\overline{\psi}\gamma^+\gamma_5(\boldsymbol{x} \times i\overleftrightarrow{\boldsymbol{D}})_z\psi \tag{7}$$

$$= \hat{L}_z^{q_R} - \hat{L}_z^{q_L}. \tag{8}$$

The quark spin and OAM operators attracted a lot of attention because they enter the Ji decomposition of the total angular momentum operator in QCD [5]

$$\hat{J}_z = \hat{S}_z^q + \hat{L}_z^q + \hat{J}_z^G. \tag{9}$$

Though, as we have seen, a complete characterization of the nucleon longitudinal spin structure requires us to go beyond this and to consider the quark number and spin–orbit correlation operators as well. Contrary to the quark number, the quark spin–orbit correlation defined by Eq. (7) has, to the best of our knowledge, never been studied so far. The purpose of this Letter is to fill this gap and to show that such a quantity is actually related to measurable quantities.

We basically follow the same strategy as Ji in Ref. [5], except for the fact that we directly consider the more general asymmetric gauge-invariant energy–momentum tensor instead of the symmetric gauge-invariant (or Belinfante) one. We postpone the discussion of this particular point to Section 4. The quark OAM operator can then conveniently be expressed as follows

$$\hat{L}_z^q = \int \mathrm{d}^3x \left(x^1\hat{T}_q^{+2} - x^2\hat{T}_q^{+1}\right), \tag{10}$$

where $\hat{T}^{\mu\nu}$ is the quark energy–momentum tensor operator [1]

$$\hat{T}_q^{\mu\nu} = \frac{1}{2}\overline{\psi}\gamma^\mu i\overleftrightarrow{D}^\nu\psi \tag{11}$$

$$= \hat{T}_{q_R}^{\mu\nu} + \hat{T}_{q_L}^{\mu\nu} \tag{12}$$

with $\hat{T}_{q_{R,L}}^{\mu\nu} = \frac{1}{2}\overline{\psi}_{R,L}\gamma^\mu i\overleftrightarrow{D}^\nu\psi_{R,L}$. Similarly, we rewrite the quark spin–orbit operator as

$$\hat{C}_z^q = \int \mathrm{d}^3x \left(x^1\hat{T}_{q5}^{+2} - x^2\hat{T}_{q5}^{+1}\right), \tag{13}$$

where $\hat{T}_{q5}^{\mu\nu}$ can be considered as the parity-odd partner of the quark energy–momentum tensor operator

$$\hat{T}_{q5}^{\mu\nu} = \frac{1}{2}\overline{\psi}\gamma^\mu\gamma_5 i\overleftrightarrow{D}^\nu\psi \tag{14}$$

$$= \hat{T}_{q_R}^{\mu\nu} - \hat{T}_{q_L}^{\mu\nu}. \tag{15}$$

Just like in the case of the generic asymmetric energy–momentum tensor [1,6], we find that the non-forward matrix elements of $\hat{T}_{q5}^{\mu\nu}$ can be parametrized in terms of five form factors (FFs)

$$\langle p', \boldsymbol{s}'|\hat{T}_{q5}^{\mu\nu}|p, \boldsymbol{s}\rangle = \overline{u}(p', \boldsymbol{s}')\Gamma_{q5}^{\mu\nu}u(p, \boldsymbol{s}) \tag{16}$$

with

$$\begin{aligned}\Gamma_{q5}^{\mu\nu} &= \frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2}\tilde{A}_q(t) + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{4M}\tilde{B}_q(t) \\ &\quad + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{C}_q(t) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M}\tilde{D}_q(t) \\ &\quad + Mi\sigma^{\mu\nu}\gamma_5\tilde{F}_q(t),\end{aligned} \tag{17}$$

where M is the nucleon mass, $\boldsymbol{s}$ and $\boldsymbol{s}'$ are the initial and final rest-frame polarization vectors satisfying $\boldsymbol{s}^2 = \boldsymbol{s}'^2 = 1$, $P = \frac{p'+p}{2}$ is the average four-momentum, and $t = \Delta^2$ is the square of the four-momentum transfer $\Delta = p' - p$. For convenience, we used the notations $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ and $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$.

Since we are interested in the matrix element of Eq. (13) which involves only one explicit power of $\boldsymbol{x}$, we need to expand Eq. (16) only up to linear order in Δ [1,6]. Considering initial and final nucleon states with the same rest-frame polarization $\boldsymbol{s}' = \boldsymbol{s} = (\boldsymbol{s}_\perp, s_z)$ and using the light-front spinors (see *e.g.* Appendix A of Ref. [7]), we arrive at the following expression

$$\begin{aligned}\langle p', \boldsymbol{s}|\hat{T}_{q5}^{\mu\nu}|p, \boldsymbol{s}\rangle &= \left[P^{\{\mu}S^{\nu\}} - \frac{P^{\{\mu}i\epsilon^{\nu\}+\Delta P}}{2P^+}\right]\tilde{A}_q \\ &\quad \left[P^{[\mu}S^{\nu]} - \frac{P^{[\mu}i\epsilon^{\nu]+\Delta P}}{2P^+}\right](\tilde{C}_q - 2\tilde{F}_q) \\ &\quad + i\epsilon^{\mu\nu\Delta P}\tilde{F}_q + \mathcal{O}(\Delta^2)\end{aligned} \tag{18}$$

with $\epsilon_{0123} = +1$ and the covariant spin vector $S^\mu = [s_zP^+, -s_zP^- + \frac{\boldsymbol{P}_\perp}{P^+}\cdot(M\boldsymbol{s}_\perp + \boldsymbol{P}_\perp s_z), M\boldsymbol{s}_\perp + \boldsymbol{P}_\perp s_z]$ satisfying $P \cdot S = 0$ and $S^2 = -M^2 - s_z^2(P^2 - M^2)$. For convenience, we removed the argument of the FFs when evaluated at $t = 0$, *i.e.* $\tilde{X}_q = \tilde{X}_q(0)$.

Substituting the expansion (18) into the matrix element of Eq. (13) and working in the symmetric light-front frame, *i.e.* with $\boldsymbol{P}_\perp = \boldsymbol{0}_\perp$, we find

$$C_z^q \equiv \frac{\langle P, \boldsymbol{e}_z|\hat{C}_z^q|P, \boldsymbol{e}_z\rangle}{\langle P, \boldsymbol{e}_z|P, \boldsymbol{e}_z\rangle} = \frac{1}{2}(\tilde{A}_q + \tilde{C}_q). \tag{19}$$

Thus, to determine the quark spin–orbit correlation, one has to measure the $\tilde{A}_q(t)$ and $\tilde{C}_q(t)$ FFs, which are analogous to the axial-vector FF $G_A^q(t)$. The $\tilde{B}_q(t)$ and $\tilde{D}_q(t)$ FFs, which are analogous to the induced pseudoscalar FF $G_P^q(t)$, are not needed since they contribute only to higher x-moments of $\hat{T}_{q5}^{\mu\nu}$, as one can see from the expansion $\overline{u}(p', \boldsymbol{s})\frac{P^\mu\Delta^\nu\gamma_5}{4M}u(p, \boldsymbol{s}) = \mathcal{O}(\Delta^2)$.

3. Link with parton distributions

Like in the case of the energy–momentum tensor, there is no fundamental probe that couples to $\hat{T}_{q5}^{\mu\nu}$ in particle physics. However, it is possible to relate the corresponding various FFs to specific moments of measurable parton distributions. From the component $\hat{T}_{q5}^{++}$, we find

$$\int \mathrm{d}x \, x\tilde{H}_q(x, \xi, t) = \tilde{A}_q(t), \tag{20}$$

$$\int \mathrm{d}x \, x\tilde{E}_q(x, \xi, t) = \tilde{B}_q(t), \tag{21}$$

where $\tilde{H}_q(x, \xi, t)$ and $\tilde{E}_q(x, \xi, t)$ are the GPDs parametrizing the non-local twist-2 axial-vector light-front quark correlator [8]

$$\begin{aligned}&\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}e^{ixP^+z^-}\langle p', \boldsymbol{s}'|\overline{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\gamma_5\mathcal{W}\psi\left(\frac{z^-}{2}\right)|p, \boldsymbol{s}\rangle \\ &= \frac{1}{2P^+}\overline{u}(p', \boldsymbol{s}')\Gamma_{qA}^+u(p, \boldsymbol{s})\end{aligned} \tag{22}$$