Physics Letters B 672 (2009) 77–81

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quantum lift of non-BPS flat directions

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ARTICLE INFO

Article history:
Received 16 December 2008
Accepted 20 December 2008
Available online 31 December 2008
Editor: L. Alvarez-Gaumé

ABSTRACT

We study $N=2$, $d=4$ attractor equations for the quantum corrected two-moduli prepotential $\mathcal{F}=st^2+i\lambda$, with λ real, which is the only correction which preserves the axion shift symmetry and modifies the geometry.

In the classical case the black hole effective potential is known to have a flat direction. We found that in the presence of $D0$–$D6$ branes the black hole potential exhibits a flat direction in the quantum case as well. It corresponds to non-BPS $Z\neq 0$ solutions to the attractor equations. Unlike the classical case, the solutions acquire non-zero values of the axion field.

For the cases of $D0$–$D4$ and $D2$–$D6$ branes the classical flat direction reduces to separate critical points which turn out to have a vanishing axion field.

1. Introduction

The attractor mechanism was firstly described in the seminal papers [1–5] and is now the object of intense studies (for a comprehensive list of references, see e.g. [6]). While originally this mechanism was discovered in the context of extremal BPS black holes, later it was found to be present even for non-BPS ones. Differently from the BPS black holes, such new attractors do not saturate the BPS bound and thus, when considering a supergravity theory, they break all supersymmetries at the black hole event horizon [7].

Attractor mechanism equations are given by the condition of extremality [5]

$$\phi_H(p,q): \quad \left.\frac{\partial V_{\mathrm{BH}}(\phi,p,q)}{\partial\phi^a}\right|_{\phi=\phi_H(p,q)}=0 \tag{1}$$

of the so-called black hole potential V_{BH}, which is a real function of the moduli ϕ^a and magnetic p^Λ and electric q_Λ charges.

The crucial condition for a critical point $\phi_H(p,q)$ to be an attractor in the strict sense is that the Hessian matrix

$$\mathcal{H}_{ab}(p,q)=\nabla_a\nabla_b V_{\mathrm{BH}}|_{\phi=\phi_H}=\partial_a\partial_b V_{\mathrm{BH}}|_{\phi=\phi_H} \tag{2}$$

of V_{BH} evaluated at the critical point (1) be positive definite.

In $N=2$, $d=4$ Maxwell–Einstein supergravities based on homogeneous scalar manifolds, the Hessian matrix has in general either positive or zero eigenvalues. The latter ones correspond to massless Hessian modes, which have been proven to be flat directions of V_{BH} [8,9].

The presence of flat directions does not contradict the essence of the attractor mechanism: although the moduli might not be stabilized, the value of the entropy does not change when the moduli change along the flat directions of V_{BH}. Indeed, in $N=2$, $d=4$ supergravity, the black hole entropy is related to its potential through the formula [5]

$$S_{\mathrm{BH}}(p,q)=\pi V_{\mathrm{BH}}(\phi,p,q)|_{\phi=\phi_H}. \tag{3}$$

Therefore, whether the flat directions are present or not, it does not affect the value of the entropy. Consequently, one may allow the eigenvalues of the Hessian matrix to be zero, as well.

Actually, this phenomenon always occurs in $N>2$-extended, $d=4$ supergravities, also for $\frac{1}{N}$-BPS configurations, and it can be understood through an $N=2$ analysis, as being due to $N=2$ hypermultiplets always present in these theories [6,8].

In $N=2$, $d=4$ supergravity with more than one vector multiplet coupled to the supergravity one, the black hole potential V_{BH} has flat directions provided that the critical points exist [9,10]. They correspond to non-BPS states with non-vanishing central charge.

The simplest model possessing a flat direction is that with two vector multiplets, i.e. the so-called st^2 model. The latter we treat in this Letter which might be thought of as a continuation of the investigation started in an earlier Letter [11], where we found an

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0370-2693/$ – see front matter © 2008 Published by Elsevier B.V.
doi:10.1016/j.physletb.2008.12.059

effect of multiplicity of the attractors in the presence of quantum corrections. This effect is related to the fact that when quantum corrections are introduced, the scalar manifold is not simply connected any more.

Even in the classical case, solutions for the attractor equations are known just for quite a few models. For example, in the framework of special Kähler d-geometry, supersymmetric attractor equations are solved in [12]. Non-supersymmetric ones are solved completely both for the t^3 model [13] and for the stu one [14], taking advantage of the presence of a large duality symmetry. States with vanishing central charge are investigated in [14,15].

As it has been already mentioned, in the paper [11] we began the study of a quantum t^3 model of $N=2$, $d=4$ supergravity with the prepotential[1]

$$F(X)=\frac{(X^1)^3}{X^0}+i\lambda\left(X^0\right)^2=\left(X^0\right)^2\left(t^3+i\lambda\right),\quad \lambda\in\mathbb{R}.$$

There it was argued that this is the only possible correction preserving the axion shift symmetry and that it cannot be reabsorbed by a field redefinition [16,17]. The black hole potential of this model does not possess any flat direction, nevertheless, the appearance of the quantum contribution reveals an effect of multiplicity of the attractors. This effect is similar to that observed in [18]. Due to this effect other ones arise such as “transmutations” and “separation” of attractors. In st^2 model they appear as well, but here we are mostly concerned with another phenomenon, not present in t^3 model — namely, how the flat direction of the st^2 model undergoes the insertion of quantum corrections.

The quantum corrected st^2 model that we consider is based on the holomorphic prepotential

$$F(X)=\frac{X^1(X^2)^2}{X^0}+i\lambda\left(X^0\right)^2=\left(X^0\right)^2\left(st^2+i\lambda\right),\quad \lambda\in\mathbb{R}.$$

The complex moduli s and t span the rank-2 special Kähler manifold $(SU(1,1)/U(1))^2$. When $\lambda=0$ this formula gives classical expression for the prepotential, which we start the next section with.

Knowing the superpotential, one may easily calculate the corresponding black hole potential[2] [5]

$$V_{\mathrm{BH}}=e^K\left[W\bar{W}+g^{a\bar{b}}\nabla_a W\bar{\nabla}_{\bar{b}}\bar{W}\right]\tag{4}$$

in terms of the superpotential W and the Kähler potential K

$$W=q_\Lambda X^\Lambda+p^\Lambda F_\Lambda,\qquad K=-\ln\left[-i\left(X^\Lambda\bar{F}_\Lambda-\bar{X}^\Lambda F_\Lambda\right)\right].\tag{5}$$

2. *D*0–*D*4 branes

This brane configuration corresponds to vanishing charges q_a and p^0. The quartic invariant in this case is given by

$$I_4=4q_0p^1\left(p^2\right)^2.\tag{6}$$

When it is negative, the classical black hole potential possesses a non-compact flat direction related to the $SO(1,1)$ manifold [9]

$$\begin{aligned}
&\operatorname{Im}s=\pm\sqrt{-\frac{p^1q_0}{(p^2)^2}\frac{(\operatorname{Re}t)^2+\frac{q_0}{p^1}}{(\operatorname{Re}t)^2-\frac{q_0}{p^1}}},\\
&\operatorname{Re}s=\frac{p^1q_0}{p^2}\frac{2\operatorname{Re}t}{(\operatorname{Re}t)^2-\frac{q_0}{p^1}},\\
&\operatorname{Im}t=\pm\sqrt{-\frac{q_0}{p^1}-(\operatorname{Re}t)^2}
\end{aligned}\tag{7}$$

parameterized, for instance, by the real part of the modulus t. Naturally, it solves the criticality condition of the black hole potential (4) evaluated when $\lambda=0$

$$\frac{\partial V_{\mathrm{BH}}}{\partial s}=0,\qquad \frac{\partial V_{\mathrm{BH}}}{\partial t}=0\tag{8}$$

and corresponds to a non-BPS state. The black hole entropy (3) turns out not to depend on $\operatorname{Re}t$

$$S_{\mathrm{BH}}=\pi\sqrt{-I_4}=2\pi\sqrt{-q_0p^1\left(p^2\right)^2}\tag{9}$$

in complete agreement with the attractor mechanism paradigm.

When switching the quantum parameter λ on, it is convenient to pass to the rescaled moduli y^1, y^2 and the quantum parameter α

$$\begin{aligned}
&s=p^1\sqrt{-\frac{q_0}{p^1(p^2)^2}}y^1,\qquad t=p^2\sqrt{-\frac{q_0}{p^1(p^2)^2}}y^2,\\
&\lambda=q_0\sqrt{-\frac{q_0}{p^1(p^2)^2}}\alpha
\end{aligned}\tag{10}$$

in order to factorize the dependence of W and V_{BH} on the charges

$$\begin{aligned}
&W=q_0\left[1-2y^1y^2-\left(y^2\right)^2\right],\\
&V_{\mathrm{BH}}=\frac{1}{2}\sqrt{-I_4}v(y,\bar{y})=\sqrt{-q_0p^1\left(p^2\right)^2}v(y,\bar{y}).
\end{aligned}\tag{11}$$

The expression for the black hole potential is quite cumbersome and not too illustrative, so we restricted ourselves to writing down explicitly only the superpotential. The function $v(y,\bar{y})$ is a rational one with the numerator being a polynomial of ninth degree and the denominator — of eighth degree on y^a and $\bar{y}^a$. So at the moment it is quite improbable to resolve attractor mechanism equations (8) analytically. Nevertheless, numerical simulations show that all solutions to Eqs. (8) have vanishing values of the axion fields

$$\operatorname{Re}y^1=\operatorname{Re}y^2=0.\tag{12}$$

This result differs from that present in the classical case (7). With this assumption, the attractor mechanism equations become

$$\begin{aligned}
&4\alpha^4-\alpha^3\left[-4t_1^2t_2-2t_2\left(-3+t_2^2\right)+2t_1\left(3+t_2^2\right)\right]\\
&\quad+\alpha^2t_1t_2\left[5+32t_1^2t_2^2+11t_2^4+t_1\left(-6t_2+26t_2^3\right)\right]\\
&\quad-4\alpha t_1^2t_2^3\left[-1+3t_2^2+2t_1^2t_2^2+2t_2^4+t_1t_2\left(9+t_2^2\right)\right]\\
&\quad-8t_1^3t_2^5\left(-1+t_2^4\right)=0,\\
&4\alpha^4+4\alpha^3t_2\left(-3+t_2^2\right)+\alpha^2t_2^2\left[5+\left(-6+32t_1^2\right)t_2^2+32t_1t_2^3+5t_2^4\right]\\
&\quad-4\alpha t_1t_2^4\left[-1+6t_2^2+4t_1^2t_2^2-t_2^4+2t_1t_2\left(3+t_2^2\right)\right]\\
&\quad+8t_1^2t_2^6\left(1-2t_1^2t_2^2+t_2^4\right)=0,
\end{aligned}\tag{13}$$

where for the sake of brevity we denoted $t_a=\operatorname{Im}y^a$. Depending on the value of the parameter α, the number of the solutions to the Eqs. (13) and their stability change. The stable solutions have all eigenvalues of the Hessian matrix positive, while for the unstable ones — one of them is negative. In what follows we consider only stable solutions.

[1] In general, λ is related to perturbative quantum corrections at the level of non-linear sigma model, computed by 2-dimensional CFT techniques on the world-sheet. For instance, in Type IIA CY_3-compactifications [19–21]

$$\lambda=-\frac{\chi\zeta(3)}{16\pi^3},$$

where χ is the Euler character of CY_3, and ζ is the Riemann zeta-function. Within such a framework, it has been shown that λ has a 4-loop origin in the non-linear sigma-model [19,22,23].

[2] Generally, the indices $a,b,c,\dots$ run from 1 to n, while $\Lambda,\Sigma,\dots$ — from 0 to n, with $n=2$ for the st^2 model.