Physics Letters B 729 (2014) 79–84

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Novel non-equilibrium phase transition caused by non-linear hadronic-quark phase structure

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ARTICLE INFO

Article history:
Received 20 July 2013
Received in revised form 2 January 2014
Accepted 3 January 2014
Available online 8 January 2014
Editor: W. Haxton

Keywords:
Deconfinement
Phase transition
Neutron stars
Thermal evolution

ABSTRACT

We consider how the occurrence of first-order phase transitions in non-constant pressure differs from those at constant pressure. The former has shown the non-linear phase structure of mixed matter, which implies a particle number dependence of the binding energies of the two species. If the mixed matter is mixed hadron–quark phase, nucleon outgoing from hadronic phase and ingoing to quark phase probably reduces the system to a non-equilibrium state, in other words, there exists the imbalance of the two phases when deconfinement takes place. This novel non-equilibrium process is very analogous to the nuclear reactions that nuclei emit neutrons and absorb them under appropriate conditions. We present self-consistent thermodynamics in description for the processes and identify the microphysics responsible for the processes. The microphysics is an inevitable consequence of non-linear phase structure instead of the effect of an additional dissipation force. When applying our findings to the neutron star containing mixed hadron–quark matter, it is found that the newly discovered energy release might strongly change the thermal evolution behavior of the star.

1. Introduction

Glendenning [1] had realized the essentially different character of first-order phase transition between the simple system possessing a single conserved quantity and the complex one having more than one conserved charge. One of the most remarkable features of a simple system is the constancy of the pressure during the transition from one homogeneous phase to the other. In fact, this is the typical depiction of first-order phase transition in textbooks. However, the properties of the phase transition in the complex system turn out to be quite different. The pressure varies continuously with the proportion of the two phases, and obviously, some quantities are non-linear functions of the proportion. This so-called non-linear phase structure has been made a systematic exposition by Glendenning in his article and book [1,2]. He also showed a deconfinement case in the core of neutron stars.

For a long time, people only pay attention to the effect of the mixed phase on the structure of neutron stars regardless of the feature of the transition in progress. Perhaps the discussion of such problem is thought to be unnecessary as emphasized by Heiselberg et al. [3]: the two phases are always in balance as transitions from hadron into quarks are governed by strong reactions with extremely short timescales. However this well-known creed should be modified for the phase transitions in varying pressure. In this Letter, we will show that non-linear phase structure may devote to dynamics of phase transition, and it may lead to different dynamical behaviors unlike bare nucleon reactions $n \to 2d + u$ and $p \to d + 2u$, where n, p, u, d respectively denote neutron, proton, u and d quarks.

Our problem begins with a thermodynamical analysis. As we known, the fundamental formula of thermodynamics must hold for any situation. For a system, an effective Hamiltonian or energy depends on phenomenological parameters, which are assumed to be functions of thermodynamical variables, temperature and chemical potential (or density), there exists so-called self-consistency problem of thermodynamics. When studying a plasma, it is common to regard the system of interacting charged particles as an ideal gas of noninteracting quasi-particles, where a temperature-dependent mass is applied to the effective Hamiltonian of ideal gas. The system of the mixed phase with non-linear phase structure can be treated in the same way. Since particle density and energy density aren't linear functions of proportion, binding energy of each phase in mixed phase, energy per baryon, should be function of particle number contained in each phase or binding energy of mixed matter is a non-linear function of fraction in particle number. This means the description of energy of such system needs an internal phenomenological parameter, the density-dependent fraction

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http://dx.doi.org/10.1016/j.physletb.2014.01.004

in particle number, besides particle number density. To maintain the self-consistency of the system, the standard treatment of this problem is to impose a supplement energy term (or so-called "zero point energy") [4]. In our case, the zero point energy means a Gibbs free enthalpy difference, or equivalently say imbalance of two phases. During transitions, the additional variable, density-dependent fraction in particle number, is generally thought to be a parameter describing non-equilibrium status [5,6]. In this Letter, we will exhibit the related self-consistency of thermodynamics and get the chemical potential difference of the two phases.

Understandings of microphysics of this problem are as follows. We take an example of deconfinement phase transition. When hadrons are converted into quarks, baryon number of hadronic phase decreases but that of quark phase increases, their binding energies both change because binding energy of mixed matter is a non-linear function of the fraction in baryon number. Some energy is released as heat if they reduce. In the case of the phase transition under constant-pressure, the binding energy of each phase is independent of the particle number, the conversion couldn't cause any change in each binding energy, and dissipation is impossible. The crucial difference between the cases is that, each of subsystems (hadronic phase and quark phase) in mixture is of structure for the first case, while the latter only includes two uniform clusters. This can be easier to be understand if the subsystems with structure are regarded as two "giant nuclei". When a real nucleus emits or absorbs a neutron, liberation of nuclear energy is possible under some condition. Likewise, the increase or decrease of baryon number of the "giant nuclei" leads to a rearrangement of particles in the interior of them. One of possible consequences is reducing their binding energies. The excess of the energies is certainly released as heat. If the system of mixed hadron–quark matter is being compressed, the above dissipation processes may occur for converting hadrons into quarks. Not only the energy of the system but also the Gibbs free enthalpy should be lowered by the processes. The decrease of Gibbs free enthalpy is equivalent to imbalance of two phases. This is quite different from constant-pressure phase transition in which no Gibbs free enthalpy changes.

The plan of this Letter is as follows. In Section 2 we briefly review the phase transition with two conserved charges. We introduce the fraction in baryon number instead of the fraction in volume to reexpress the energy per baryon and energy density of mixed phase. This is a useful preparation for a discussion of dissipation processes. In Section 3 we demonstrate the possible existence of non-equilibrium phase transition from thermodynamical analysis and microphysics as well as our general formulism of this problem. In Section 4 we have an application of the general theory by considering the mixed phase with specific equations of state of hadronic and quark matter that may exist in neutron stars.

2. Review of phase transition with more than one charge

As a useful background to our discussion below, we first recount some properties of the particular phase transition following Glendenning's philosophy [1]. A substance composed of two conserved charges or independent components is a hotbed of such phase transition. It is important to realize that although there exist two charges they are conserved only globally rather than locally, and for this reason phase transitions may involve the mixed phase through which the pressure varies continuously.

In general, Gibbs condition for phase equilibrium is that chemical potential, temperature and pressure in two phases be equal. Since the pressure now depends on two independent chemical potentials, the equilibrium condition of two phases can be expressed as

$$P_Q(\mu_b, \mu_e, T) = P_H(\mu_b, \mu_e, T) \tag{1}$$

where Q, H represent respectively high and low density phases or they can also denote quark and hadronic phases subsequently. Satisfying global charge neutrality, Eq. (1) can be solved for the chemical potentials, $\mu_{b,e}(\chi)$, in mixed phase, where χ is fraction in volume, $\chi = \frac{V_Q}{V_H+V_Q}$. These in turn yield the particle and energy densities

$$\rho = \chi\rho_Q + (1-\chi)\rho_H, \tag{2}$$

$$\epsilon = \chi\epsilon_Q + (1-\chi)\epsilon_H. \tag{3}$$

If we introduce replaced parameter for convenience, the fraction in baryon number $\eta(= A_Q/A)$, there are identities $\chi = \eta\frac{\rho}{\rho_Q}$, $1-\chi = (1-\eta)\frac{\rho}{\rho_H}$. The energy per baryon or so-called binding energy can then also be constructed by combining Eqs. (2) and (3),

$$e = \frac{\epsilon}{\rho} = \eta e_Q + (1-\eta)e_H. \tag{4}$$

The energy density is restated as

$$\epsilon = \eta\rho e_Q + (1-\eta)\rho e_H. \tag{5}$$

These illustrate the non-linear phase structure of the mixed phase. At zero temperature, the energy for the system relies on thermodynamical variable, ρ, and internal parameter, η, which is still ρ-dependent. These properties of the mixed phase will prove to be important in following discussions.

If local charge neutrality is enforced in the description of the first-order phase transition, the system would reduce to a simple substance with only one independent chemical potential, the textbook example. The Gibbs condition has a unique solution which implies a fixed phase transition point. Thus, the mixed phase becomes the usual Maxwell construction and shows linear phase structure.

3. Non-equilibrium phase transition

In this section, we try to discuss the non-equilibrium property of the phase transition having more than one conserved charge and give the description of the imbalance of two phases from different aspects, namely, thermodynamics, microphysics and relaxation dynamics.

Thermodynamic self-consistency. The problem that whether the two phases are balance during the phase transition or not arises from thermodynamics. We begin with the thermodynamic formula for the coexistence of two phases

$$d\epsilon = \frac{P+\epsilon}{\rho}\,d\rho + \sum_k \rho\mu_k\,d\eta_k, \tag{6}$$

where P denotes the pressure of system, μ_k, the chemical potential of species k, with $k = H, Q$ for two chemical component "mixture". If chemical balance is assumed, the formula reduces to

$$d\epsilon = \frac{P+\epsilon}{\rho}\,d\rho, \tag{7}$$

or equivalently

$$P = \rho^2\frac{d}{d\rho}\left(\frac{\epsilon}{\rho}\right). \tag{8}$$

One can easily check that the identities (7) and (8) hold for constancy η only, and if η is density dependent it is no longer true. This is the so-called problem of thermodynamic self-consistency.