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Thermodynamics of $(2+1)$-flavor strongly interacting matter at nonzero isospin

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ABSTRACT

We investigate the phase structure of strongly interacting matter at non-vanishing isospin before the onset of pion condensation in the framework of the unquenched Polyakov–Quark-Meson model with $2+1$ quark flavors. We show results for the order parameters and all relevant thermodynamic quantities. In particular, we obtain a moderate change of the pressure with isospin at vanishing baryon chemical potential, whereas the chiral condensate decreases more appreciably. We compare the effective model to recent lattice data for the decrease of the pseudo-critical temperature with the isospin chemical potential. We also demonstrate the major role played by the value of the pion mass in the curvature of the transition line, and the need for lattice results with a physical pion mass. Limitations of the model at nonzero chemical potential are also discussed.

1. Introduction

The thermodynamics of strongly interacting matter under extreme conditions plays a major role in the understanding of the physical scenario shortly after the Big Bang [1,2], in the outcome of high-energy heavy ion collisions [3], in the mechanism of supernovae explosions [4], and in the structure of compact stars [5,6]. The possibility to probe such large temperatures and densities in current experiments at LHC-CERN and RHIC-BNL, and especially in future experiments at FAIR-GSI, calls for a detailed study of the transition to the chirally symmetric quark–gluon plasma phase and the properties of this new extraordinary state of matter [7–9].

In all systems mentioned above matter does not consist of equal amounts of protons and neutrons, i.e. one has a non-vanishing isospin density. Using Au or Pb beams in heavy ion collisions, the proton to neutron ratio is $\sim 2/3$. In astrophysical environments the initial proton fraction in supernovae is 0.4, reduces to 0.2 and finally reaches values of less than 0.1 in cold neutron stars. In the universe a large asymmetry in the lepton sector is allowed ($-0.38 < \mu_\nu/T < 0.02$) [10], which can shift the equilibrium conditions at the cosmological QCD transition [11]. Hence, in the description of the thermodynamics in all these scenarios of nature isospin should not be overlooked.

In this work we investigate the phase structure of strongly interacting matter at non-vanishing isospin before the onset of pion condensation in the framework of the Polyakov–Quark-Meson model with $2+1$ quark flavors. First we compute the chiral condensate, the pressure and the trace anomaly at vanishing quark densities, reproducing well-established lattice data. Then we compute all relevant thermodynamic quantities and the phase diagram for a nonzero isospin chemical potential. For vanishing baryon chemical potential, our results can be compared directly to lattice data since this case is free of the Sign Problem [12,13].

Effective chiral models, combined with some version of the Polyakov loop potential, usually have their parameters adjusted to provide a good description of lattice data at zero density. Testing effective models built in this fashion against lattice data at nonzero isospin is crucial to understand whether their extended versions provide qualitative and quantitative accurate descriptions of the phase structure of strong interactions.

To date, most of the calculations at non-vanishing isospin were done with only two quark flavors, neglecting strange quarks as relevant degrees of freedom at the energy scale of the chiral and deconfinement transition [14–31]. Furthermore, lattice calculations at nonzero isospin were performed so far only with unphysical heavy quark masses [14–16]. The impact of the quark mass on the deconfining critical temperature at nonzero isospin was investigated in Refs. [17,18], using a framework that combines chiral perturbation theory to describe the low-energy sector with

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the phenomenological fuzzy bag model for high energies, showing that quark masses play a relevant role. Previously, there have also been investigations with the hadron resonance gas model [32], the $O(2N)$-symmetric ϕ^4-theory [33], the Nambu–Jona-Lasinio model [19–27,34], the Quark-Meson model [28], and its Polyakov-loop extended versions [29–31].

One of the aspects in the extension of the phase diagram to nonzero isospin is the arising of a new phase. Charged pions couple to the isospin chemical potential and at $\mu_I = \mu_u - \mu_d = m_\pi$ there is the onset of pion condensation [35]. The running of the pion mass in the medium shifts the appearance of pion condensation to larger temperatures and densities. Depending on the analyzed region of isospin, temperature and baryon chemical potential, pion condensation must be taken into account [14,15,17,18,21–29,33,34] or not [16,19,20,31,32].

In contrast to what is considered in this work, the two-flavor renormalization group (RG) improved Quark-Meson model was so far only applied to analyze the pion condensate phase [28] and investigations with the PNJL model only considered two quark flavors and applied the simpler pure gauge Polyakov-loop potential [29–31].

We restrict our analysis to moderate isospin chemical potential values, before the onset of pion condensation. To describe strongly interacting matter we adopt the framework of the Polyakov–Quark-Meson model [36–52] that we enhance by applying the unquenched Polyakov-loop potential [50,51]. We build its extension to nonzero isospin in Section 2. In Section 3 we discuss our results on the evolution of the order parameters and thermodynamics with increasing isospin. We compare the decrease of the pseudo-critical temperature with isospin chemical potential to recent lattice data to test the model in its extension to nonzero isospin density. Moreover, we point out the impact of the pion mass on the curvature of the transition line.

2. Theoretical framework

We perform our investigation within the framework of a low energy effective model that includes important aspects of QCD: chiral symmetry breaking and (partial) confinement (in the gluonic sector). These properties are contained in the effective Lagrangian of the theory

$$\begin{aligned}\mathcal{L} = {} & \bar{q}(i\not{D} - g\phi_5 + \gamma_0\mu_f)q + \mathrm{Tr}(\partial_\mu\phi^\dagger\partial^\mu\phi) \\ & - m^2\,\mathrm{Tr}(\phi^\dagger\phi) - \lambda_1\big[\mathrm{Tr}(\phi^\dagger\phi)\big]^2 - \lambda_2\,\mathrm{Tr}(\phi^\dagger\phi)^2 \\ & + c(\det\phi + \det\phi^\dagger) + \mathrm{Tr}\big[H(\phi+\phi^\dagger)\big] \\ & - \mathcal{U}(\Phi,\bar{\Phi}),\end{aligned} \tag{1}$$

where ϕ and ϕ_5 are 3×3 matrices that combine scalar and pseudoscalar meson fields. All the contributions to the Lagrangian are discussed in the following as well as in detail e.g. in Refs. [42,44,46,53–55]. We enhance it by the application of the unquenched Polyakov-loop potential $\mathcal{U}_{\mathrm{glue}}$ [50,51]. To describe the creation of constituent quark masses by spontaneous breaking of chiral symmetry we use a $(2+1)$-flavor Quark-Meson model [53,55]. To include isospin effects, we generalize the self-interaction potential of the meson fields to distinguish between the up and down quark condensates [56,57]

$$\begin{aligned}U(\sigma_u,\sigma_d,\sigma_s) = {} & \frac{\lambda_1}{4}\left[\left(\frac{\sigma_u^2+\sigma_d^2}{2}\right)^2 + \sigma_s^4 + (\sigma_u^2+\sigma_d^2)\sigma_s^2\right] \\ & + \frac{\lambda_2}{4}\left(\frac{\sigma_u^4+\sigma_d^4}{4} + \sigma_s^4\right) - \frac{c}{2\sqrt{2}}\sigma_u\sigma_d\sigma_s \\ & + \frac{m^2}{2}\left(\frac{\sigma_u^2+\sigma_d^2}{2} + \sigma_s^2\right) - \frac{h_{ud}}{2}(\sigma_u+\sigma_d) - h_s\sigma_s.\end{aligned} \tag{2}$$

Table 1
Parameters of the Polyakov loop potential of Ref. [60] converted in Ref. [48] to the definition of the coefficients in Eqs. (3) and (4).

a_0	a_1	a_2	a_3	b_3	b_4
1.53	0.96	−2.3	−2.85	13.34	14.88

This potential contains spontaneous and explicit breaking of chiral symmetry. In accord with the Vafa–Witten theorem [58] isospin symmetry of the vacuum is not broken in QCD and therefore we consider only one single explicit symmetry breaking term for the up-and-down quark sector h_{ud}.

An effective description of confinement can be implemented by the Polyakov loop as an order parameter of center symmetry [59]. Further details can be found in Ref. [48]. For the Polyakov-loop potential, one should choose a functional form that is invariant under center symmetry transformations [37,38,40,60,61]. Different parametrizations of the potential are available, one of which is the polynomial form of Ref. [40]

$$\frac{\mathcal{U}(\Phi,\bar{\Phi};t,\mu_f)}{T^4} = -\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2, \tag{3}$$

with the temperature-dependent coefficient b_2 defined as

$$b_2(t) = a_0 + \frac{a_1}{1+t} + \frac{a_2}{(1+t)^2} + \frac{a_3}{(1+t)^3}. \tag{4}$$

Here $t = (T - T_0)/T_0$ is the reduced temperature with the critical temperature of the Polyakov-loop potential given by T_0. The parameters of the potential are chosen such that it reproduces the temperature dependence of the Polyakov-loop expectation value and the thermodynamics of pure gauge theory as obtained in lattice calculations [40,60,61]. We use the parameter set of Ref. [60], which we summarize in Table 1.

To convert this Yang–Mills potential for the Polyakov loop to the glue potential of full QCD we use the relation

$$t_{YM}(t_{\mathrm{glue}}) \approx 0.57\, t_{\mathrm{glue}} \tag{5}$$

that connects the temperature scales of both theories [50,51]. This rescaling accounts for the back-reaction of quarks on the gluon sector at zero quark density [62,63]. Furthermore, we include the running of the critical temperature of the Polyakov-loop potential with the quark densities. Therefore, we generalize the description presented in Refs. [41,49] to include different chemical potentials for each quark flavor,

$$T_0(\mu_f) = T_\tau\, e^{-1/(\alpha_0 b(\mu_f))}, \tag{6}$$

with

$$b(\mu_f) = \bar{b}_0 - \frac{16}{\pi}\sum_{N_f}\frac{\mu_f^2}{T_\tau^2}\frac{\bar{T}_0^2}{\bar{T}_0^2 + m_f^2}. \tag{7}$$

Here, T_τ is the UV scale that is fixed to the mass of the τ-lepton, $m_\tau = 1777$ GeV which gives a coupling $\alpha_0 \simeq 0.303$ consistent with observations [64]. The parameter $\bar{b}_0$ can be adjusted to consider a dependence on the number of quark flavors. We choose $\bar{b}_0 = 11N_c/6\pi$ such that $\bar{T}_0 = 270$ MeV. Here m_f stands for the current quark masses and we adopt $m_s = 95$ MeV [64].

The thermal fluctuation contribution from quarks and antiquarks, which comes from the thermal fermionic determinant,