Physics Letters B 729 (2014) 136–142

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Shock wave evolution and discontinuity propagation for relativistic superfluid hydrodynamics with spontaneous symmetry breaking

Sun Zhang [a,b,c,*]

[a] Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
[b] Key Laboratory of Dark Matter and Space Astronomy, Chinese Academy of Sciences, Nanjing 210008, China
[c] Joint Center for Particle, Nuclear Physics and Cosmology (J-CPNPC), PMO-NJU, Nanjing 210008, China

ARTICLE INFO

*Article history:*
Received 24 October 2013
Received in revised form 17 November 2013
Accepted 7 January 2014
Available online 10 January 2014
Editor: M. Cvetič

*Keywords:*
Relativistic superfluid
Hydrodynamics
Shock wave
Sound speed

ABSTRACT

In this Letter, we have studied the shock wave and discontinuity propagation for relativistic superfluid with spontaneous $U(1)$ symmetry breaking in the framework of hydrodynamics. General features of shock waves are provided, the propagation of discontinuity and the sound modes of shock waves are also presented. The first sound and the second sound are identified as the propagation of discontinuity, and the results are in agreement with earlier theoretical studies. Moreover, a differential equation, called the growth equation, is obtained to describe the decay and growth of the discontinuity propagating along its normal trajectory. The solution is in an integral form and special cases of diverging waves are also discussed.

© 2014 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/3.0/). Funded by SCOAP[3].

## 1. Introduction

In 1941, Landau proposed his famous two-fluid (or two-constituent) theory for superfluid helium [1,2]. The basic idea of the theory is that liquid helium behaves as if it were a mixture of two distinguished liquids when temperature is below the critical point in a local equilibrium state. That is, one of them is a normal viscous fluid (or excitation microscopically) with entropy and thermal conductivity, while the other is a non-viscous fluid, called superfluid (or condensate microscopically) which can move dissipationlessly along a solid surface with zero viscosity. No momentum is assumed to transfer from one to the other, and no viscous communication takes place between these two components. In fact, the two-fluid picture was presented by Tisza firstly [3] inspired by Bose condensate in momentum space, and studied extensively and intensively after Landau's work [4–7]. Phenomenologically and semi-microscopically, the two-fluid theory is now accepted as a fundamental model for the description of superfluid hydrodynamics. In the non-relativistic frame, such a theory was studied and summarized by Khalatnikov [7].

The first approach to the theory of relativistic superfluid was provided by Israel [8] and Dixon [9] based on the idea of two-fluid picture, treating both constituents as perfect fluids. The medium is usually called relativistic in two senses: if it has a relativistic equation of state or when it flows at a relativistic velocity [10]. Both conditions can be satisfied in the massive neutron star [11–15] whose core could be considered as the construction of superfluid nuclear matter [16], superfluid nucleon–hyperon mixture [13] and/or paring quark matter [17].

It is also well known that constituents of superfluid are not perfect fluids strictly and both of them cannot be regarded as completely independent fluids [18,19]. Especially in the relativistic regime the system can be a strongly self-interacting one [19]. That is, the coupling effects would drive the medium deviated far from ideality [10]. At finite temperature, the communication (or entrainment) between the superfluid component (or condensate) and the normal entropy component (or thermal excitations) can play a crucial role in some circumstances [14]. Then it is meaningful to take into account the deviation from the perfect fluids in order to obtain more effective models. Khalatnikov and Lebedev [18,20] and Carter and Khalatnikov [21] suggested two equivalent approaches, the so-called potential and convective variational models respectively [22], to include the interaction between the superfluid and normal fluid.

As pointed out in Ref. [23], a superfluid obeys hydrodynamics microscopically while displays quantum effects macroscopically. From the viewpoint of field theory, a condensate phase is

* Correspondence to: Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China.
*E-mail address:* szhang@pmo.ac.cn.

http://dx.doi.org/10.1016/j.physletb.2014.01.014
0370-2693/© 2014 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/3.0/). Funded by SCOAP[3].

associated with spontaneous breaking of $U(1)$ global symmetry. Son [24] presented another approach to the relativistic superfluid with $U(1)$ symmetry breaking based on the Poisson bracket technique [25,26]. Such a description is also arisen from the two-constituent theory, and is effective to include the long-range Goldstone modes and more clearer to show the relation between superfluid and symmetry breaking. After constructing some basic relations of the fundamental quantities by Poisson brackets, including fluid (thermo-) ones and field ones, the hydrodynamic equations are obtained naturally, which provide the explicit meaning associated with the symmetry breaking. It can be used directly to the relativistic superfluid to study the dissipative effects [27,28] and anomalies [29]. Furthermore, recently this superfluid hydrodynamic framework with spontaneous symmetry breaking has been applied to holographic models by Herzog, Kovtun, Son, Yarom et al. [30–34]. Using AdS/CFT correspondence, they try to construct the condensate phase or superfluid in AdS geometry asymptotically and study the hydrodynamics, including sound modes and possible phase transitions in strongly interacting relativistic superfluid. Alford et al. [22,35] attempt to relate various “macroscopic” two-constituent models to an underlying “microscopic” field theory with a global $U(1)$ symmetry spontaneous breaking. In their framework, using $\phi^4$-model of a complex scalar field, the entrainment coefficient and sound speeds are calculated, and the relationship between different formulations of various two-constituent models is also discussed. The two-constituent or multi-constituent [36] theory is appealing and still in development [37].

On the other hand, the shock waves or discontinuity phenomena are widely existent in hydrodynamic systems [38], including superfluid [7]. Significantly, the relativistic shock processes can play important roles in the heavy-ion collisions at high energies and in the early universe [39,40]. The production and evolution of the hot dense matter, quark–gluon plasma are interested theoretically and experimentally. While the experiments of RHIC at Brookhaven and LHC of CERN at Geneva can provide us important events and data, the hydrodynamic framework is shown to be efficient to describe not only the bulk fluid evolution but also the propagation of discontinuity and sonic (or supersonic) waves.

To the best of the author’s knowledge, the first work on the shock wave of relativistic superfluid was given by Carter in Ref. [41]. Inspired by Hadamard’s method, the weak discontinuities are introduced to the space–time derivatives of physical quantities, instead of these quantities themselves, when crossing some characteristic surface. Two propagation modes of the discontinuities are shown as a “heat” (or second sound) mode and an “ordinary” (or first sound) mode respectively. After Carter’s work, some other researches appeared in the same theme [10,42]. Additionally, Vlasov’s works [10] are not based on the Hadamard’s method to some extent, and the shock waves in superconductive cosmic strings are also discussed [43].

In this Letter, we shall study the shock wave and discontinuity propagation for the relativistic superfluid with spontaneous $U(1)$ symmetry breaking. The following sections are organized as follows. The general features of shock waves are presented in Section 2. While the propagation of discontinuity and the sound modes of shock waves are discussed in Section 3, the first sound and the second sound are identified in agreement with the earlier work by Herzog et al. [30]. Furthermore, in Section 4, the growth equation, governing the decay and growth of the discontinuity propagation along its normal trajectory, is also obtained, and special cases of diverging waves are also examined for the solution of the growth equation. Finally, we would like to give a summary in Section 5.

## 2. General features of shock waves for relativistic superfluid

In the framework of relativistic hydrodynamics, shock phenomena have been widely studied especially for its evolution and phase transition in heavy-ion collisions [39,40,44–47]. The hydrodynamic picture on these processes is physically clear and simple, only the conservation laws for energy–momentum and baryon number are required in the beginning. Under the initial conditions the behaviors of the discontinuities crossing some singular surface are governed by the conservation laws and can give a reasonable description for the shock phenomena. Moreover, such a framework can be generalized naturally to more complicated systems, such as multi-component fluid [48], some quark pairing superfluid [49] and so on. These systems can also be studied along the hydrodynamic approach. One of the central issues is on the $U(1)$ symmetry breaking. The related current of $U(1)$ will introduced to the charge conservation equation and the contribution of the condensate also should be included to the energy–momentum tensor. Furthermore, an extra equation would appear to relate the condensate phase and some hydrodynamic driving potential, such as the chemical potential $\mu$ in superfluidity or the scalar potential $V$ in superconductivity. Such a conjugated equation is the Josephson-type equation [50].

To study the general features of shock waves for relativistic superfluids, we briefly rewrite the main results of the superfluid hydrodynamics by Son [24,30]. We take the metric tensor $\eta^{\mu\nu}$ as $\mathrm{diag}(-1,+1,+1,+1)$, then the four-velocity $u^\mu$ is normalized as $\eta_{\mu\nu}u^\mu u^\nu=-1$. The equations for energy–momentum conservation and $U(1)$ charge conservation are

$$\partial_\mu T^{\mu\nu}=0, \tag{1}$$

and

$$\partial_\mu j^\mu=0, \tag{2}$$

where $T^{\mu\nu}$ and $j^\mu$ are clearly expressed by the addition of two components, the normal one and the symmetry breaking one (or superfluid one),

$$T^{\mu\nu}=(\epsilon+P)u^\mu u^\nu+P\eta^{\mu\nu}+f^2\partial^\mu\varphi\partial^\nu\varphi, \tag{3}$$

and

$$j^\mu=nu^\mu+f^2\partial^\mu\varphi, \tag{4}$$

and the Josephson equation is also obtained,

$$u^\mu\partial_\mu\varphi+\mu=0. \tag{5}$$

The energy density $\epsilon$ is defined by $\epsilon=TS+n\mu-P$, a Legendre transformation of $P$, the velocities of the normal fluid component and the superfluid component are identified respectively as

$$u^\mu=(1,\mathbf{v})/\sqrt{1-v^2}, \qquad u_s^\mu=\partial^\mu\varphi/\mu\equiv\xi^\mu/\mu. \tag{6}$$

The normalized $u_\mu$ is the time-like four velocity of the normal component and associated with the entropy flow, while $\mathbf{v}$ is the three dimensional velocity $v_k=\partial x_k/\partial t$. The equation of state, describing the relation of pressure $P$, temperature $T$, chemical potential $\mu$ and phase $\varphi$ could be written in the differential form as

$$dP=s\,dT+n\,d\mu-f^2\xi^\mu\,d\xi_\mu. \tag{7}$$

The conservation of entropy $\partial_\mu(su^\mu)=0$, with no dissipative transports in the system, can be derived from above and is not an independent equation.

Download English Version:

https://daneshyari.com/en/article/1851243

Download Persian Version:

https://daneshyari.com/article/1851243

Daneshyari.com