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A note on the field-theoretical description of superfluids

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ABSTRACT

Recently, a Lagrangian description of superfluids attracted some interest from the fluid/gravity-correspondence viewpoint. In this respect, the work of Dubovksy et al. has proposed a new field theoretical description of fluids, which has several interesting aspects. On another side, we have recently provided a supersymmetric extension of the original works. In the analysis of the Lagrangian structures a new invariant appeared which, although related to known invariants, provides, in our opinion, a better parametrization of the fluid dynamics in order to describe the fluid/superfluid phases.

Motivated by fluid/gravity correspondence [2–5] and the recent developments in the field theory description of fluids by [6–10], in [1] we have given a field-theoretical description of fluid dynamics suitably extended to a supersymmetric framework. The supersymmetric formulation was achieved by suitable extension in superspace of the description of a relativistic non-dissipative fluid in terms of an effective Lagrangian whose fields are identified with the comoving-coordinates $\phi^I(x)$ ($I = 1,\dots,d$ labeling the spatial indices), as first considered in [6].[1]

In Refs. [7–9] the authors also introduced a further field ψ representing a U(1)-phase related to the presence of a charge. Assuming that the Lagrangian is invariant under a set of spatial symmetries and under an internal gauge symmetry acting on ψ named *chemical shift symmetry*, they were able to determine two fundamental invariants, which from the thermodynamical point of view turn out to be the entropy density s and the chemical potential μ.

The extension to a supersymmetric framework was naturally obtained in Ref. [1] by promoting the fields to superfields so that each field has a fermionic partner. In particular the supersymmetrization requires the introduction, besides the fermionic coordinates $\theta^\alpha(x)$[2] partners of the $\phi^I(x)$, an additional fermionic field $\tau(x)$ partner of the local coordinate $\psi(x)$ also transforming under the chemical-shift symmetry. In that way, the basic ingredients of the Lagrangian description are a set of superfields, invariant under the same set of symmetries of the bosonic theories, their bosonic part coinciding with the invariants discussed in the quoted papers, namely the entropy and the chemical potential.

While in Ref. [1] we were mainly focusing on the supersymmetric extension of the bosonic entropy current, we also discussed the possibility of considering further new Poincaré invariants Z^I out of the fields $\partial_\mu\phi^I$ and $\partial_\mu\psi$, namely $Z^I \equiv \partial_\mu\phi^I\partial^\mu\psi$, which could be useful, together with the other invariants, to describe in a more natural way the dynamics of a fluid in some particular conditions, such as, for example, the *superfluid phase transition*. While the Z^I are not invariant under the chemical shift symmetry, a particular combination of the Z^I with the other Poincaré invariants is actually invariant under chemical-shift symmetry and its introduction also provides a natural understanding of the kinetic term of ψ. It actually coincides with $y^2 = u^\mu u^\nu\partial_\mu\psi\partial_\nu\psi$, u^μ being the fluid four-velocity.

In this Letter we want to substantiate the observation given in Ref. [1] by explaining why the introduction of the invariants Z^I can describe a superfluid as a spontaneous broken phase of a field theory invariant under chemical shift symmetry.

In the following, we first give a short review of the Lagrangian formalism for fluid mechanics as presented in [8,9], paying particular attention to the chemical shift symmetry and to the derivation of the energy–momentum tensor. Then we discuss the properties of the new Poincaré invariants Z^I, presented at the bosonic level in [1], further extending them to a supersymmetric setting. In the application to the superfluid we show how they provide a better

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[1] See also [11] for a complete review.
[2] Their precise definition will be specified in the final part of this note.

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parametrization of the superfluid phase separation.[3] The new invariants would be also useful for the quantum extension of the theoretical description of fluids along the lines of [14]. Finally we give the extension of the supersymmetric approach to the fluid dynamics given in [1] using the new variables Z^I.

The field-theory Lagrangian approach to fluid dynamics was developed in Refs. [6–9]. It is based on the use of the comoving coordinates of the fluid as fundamental fields. We will adopt the same notations as [8].

Working, for the sake of generality, in $d+1$ space–time dimensions, one introduces d scalar fields $\phi^I(x^I, t)$, $I = 1, \ldots, d$, as Lagrangian comoving coordinates of a fluid element at a point x^I and time t, such that a background is described by $\phi^I = x^I$ and requires, in the absence of gravitation, the following symmetries:

$$\delta\phi^I = a^I \quad \left(a^I = \text{const.}\right), \tag{1}$$

$$\phi^I \to O^I{}_J \phi^J \quad \left(O^I{}_J \in \mathrm{SO}(d)\right), \tag{2}$$

$$\phi^I \to \xi^I(\phi), \quad \det\left(\partial\xi^I/\partial\phi^J\right) = 1. \tag{3}$$

The following current respects the symmetries (1)–(3):

$$J^\mu = \frac{1}{d!}\epsilon^{\mu,\nu_1,\ldots,\nu_d}\epsilon_{I_1,\ldots,I_d}\partial_{\nu_1}\phi^{I_1}\ldots\partial_{\nu_d}\phi^{I_d}, \tag{4}$$

and enjoys the important property that its projection along the comoving coordinates does not change:

$$J^\mu\partial_\mu\phi^I = 0. \tag{5}$$

This is equivalent to saying that the spatial d-form current $J^{(d)} = -\star^{d+1} J^{(1)}$, where

$$\begin{aligned} J^{(1)} &= \frac{1}{d!}\epsilon_{\mu\nu_1\ldots\nu_d}\epsilon_{I_1\ldots I_d}\partial^{\nu_1}\phi^{I_1}\ldots\partial^{\nu_d}\phi^{I_d}\, dx^\mu \\ &= (-1)^d \star^{d+1}\left(\frac{1}{d!}\epsilon_{I_1\ldots I_d}\, d\phi^{I_1}\wedge\cdots\wedge d\phi^{I_d}\right), \end{aligned} \tag{6}$$

is closed identically, that is it is an exact form. Hence it is natural to define the fluid four-velocity as aligned with J^μ:

$$J^\mu = bu^\mu \to b = \sqrt{-J^\mu J_\mu} = \sqrt{\det\left(B^{IJ}\right)}, \tag{7}$$

where $B^{IJ} \equiv \partial_\mu\phi^I\partial^\mu\phi^J$. From a physical point of view, the property of J^μ to be identically closed identifies it with the entropy current of the perfect fluid in the absence of dissipative effects, so that $b = s$, s being the entropy density.

If there is a conserved charge (particle number, electric charge etc.), one introduces a new field $\psi(x^I, t)$ which is a phase, that is it transforms under U(1) as follows

$$\psi \to \psi + c \quad (c = \text{const.}). \tag{8}$$

Moreover, if the charge flows with the fluid, charge conservation is obeyed separately by each volume element. This means that the charge conservation is not affected by an arbitrary comoving position-dependent transformation

$$\psi \to \psi + f\left(\phi^I\right), \tag{9}$$

f being an arbitrary function. This extra symmetry requirement on the Lagrangian is dubbed *chemical-shift symmetry*. Using the entropy current J^μ one finds that, by virtue of Eq. (5), the quantity $J^\mu\partial_\mu\psi$ is invariant under (9).

[3] See [12,13] for a review on superfluids in the context of holography and fluid/gravity correspondence.

From these premises the authors of [8] constructed the low energy Lagrangian respecting the above symmetries. To lowest order in a derivative expansion the Lagrangian will depend on the first derivatives of the fields through invariants respecting the symmetries (1)–(3), (8):

$$\mathcal{L} = \mathcal{L}\left(\partial\phi^I, \partial\psi\right). \tag{10}$$

In principle, there are two such invariants constructed out of J^μ and $\partial_\mu\psi$, namely b and $J^\mu\partial_\mu\psi$, so that the (Poincaré invariant) action functional can be written as follows:

$$S = \int d^{d+1}x\, F(b, y), \tag{11}$$

where y is

$$y = u^\mu\partial_\mu\psi = \frac{J^\mu\partial_\mu\psi}{b}. \tag{12}$$

By coupling (11) to a gravity background, one can obtain the energy–momentum tensor by taking, as usual, the variation of S with respect to a background (inverse) metric $g^{\mu\nu}$:

$$T_{\mu\nu} = (yF_y - bF_b)u_\mu u_\nu + \eta_{\mu\nu}(F - bF_b). \tag{13}$$

On the other hand, from classical fluid dynamics, we also have

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu + \eta_{\mu\nu}p, \tag{14}$$

from which we identify the pressure and density

$$\rho = yF_y - F \equiv yn - F, \qquad p = F - bF_b. \tag{15}$$

Comparing the two expressions of the energy–momentum tensor one can derive the relations between the thermodynamical functions and the field-theoretical quantities (see [8] for a complete review). In particular, it turns out that the quantity y defined in Eq. (12) coincides with the chemical potential μ. We conclude that the Lagrangian density of a perfect fluid is a function of s and μ

$$F = F(s, \mu). \tag{16}$$

The results presented here can be straightforwardly generalized to the supersymmetric case, where the comoving coordinates ϕ^I and phase ψ are extended to superfields. This was given in [1].

The Lagrangian (16) used above depends on the two Poincaré-invariant variables b and y, but at first sight it does not seem to allow for the presence of a kinetic term for the dynamical field ψ, namely $X = \partial_\mu\psi\partial^\mu\psi$. This term respects the translational invariance (8), and was actually considered in [9], but fails to satisfy the *chemical-shift symmetry* (9). However, as we are going to see, the kinetic term for ψ is in fact included in y^2.

To show this, let us observe that the Poincaré invariants one can build from the fields $\partial_\mu\phi^I$ and $\partial_\mu\psi$ are given by B^{IJ}, y, X together with the variables $Z^I \equiv \partial_\mu\psi\partial^\mu\phi^I$. Under chemical-shift symmetry they transform as

$$\begin{aligned} &\delta B^{IJ} = 0, \\ &\delta X = 2\partial_I f Z^I, \\ &\delta y = 0, \\ &\delta Z^I = \partial_J f B^{IJ}. \end{aligned} \tag{17}$$

We note that B^{IJ} and y are invariant under the chemical-shift symmetry, while the other two quantities X and Z^I are not. However we can construct out of them a new invariant $I(B, X, Z)$:

$$I(B, X, Z) = X - Z^I Z^J\left(B^{-1}\right)_{IJ}, \tag{18}$$