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Mimic the optical conductivity in disordered solids via gauge/gravity duality

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ABSTRACT

We study the optical conductivity in a $(2+1)$-dimensional non-relativistic field theory holographically dual to a $(3+1)$-dimensional charged Lifshitz black brane within the Einstein–Maxwell-dilaton theory. Surprisingly, we find that the optical AC conductivity satisfies the nontrivial (non-)power law scaling in the high frequency regime rather than approaching to a constant when the dynamical critical exponent $z>1$, which is qualitatively similar to those in various disordered solids in condensed matter systems. Besides, this (non-)power law scaling behavior shows some universality, which is robust against the temperatures. We argue that the peculiar scaling behavior of AC conductivity may stem from the couplings of the dilaton field with the gauge fields and also the logarithmic behavior near the boundary in the Lifshitz spacetime.

1. Introduction

The AdS/CFT correspondence [1–3] or generally the gauge/gravity duality, has provided us very effective tools to study the properties of the strongly coupled quantum field theories which live on the boundary of certain gravitational backgrounds. One of the most important characteristics of the gauge/gravity duality is that it is a kind of strong–weak duality. In view of this duality, various important phenomena of the strongly coupled field theories can be studied by performing calculations on their dual weakly coupled gravity side. Recently, motivated from the study in condensed matter physics, many attempts have been made in constructing bulk gravitational solutions to model numerous types of strongly coupled phenomena in condensed matter systems, especially close to the phase transition or quantum critical points, including the superconductor (superfluid) phase transition [4], Fermi and non-Fermi liquids [5], superconductor–insulator transitions [6], etc., for recent review, see [7,8]. There are also some quantum phase transition systems in condensed matter physics which contain the Lifshitz-fixed points have received much attention. On one hand, these developments successfully extended the gauge/gravity duality into the more general form, namely, non-relativistic version [9–16]. On the other hand, they allow us to study strongly coupled systems toward realistic laboratory conditions by holographic principle, which may also be used as a test for the gauge/gravity duality itself.

This Letter focuses on dealing with Lifshitz field theory in the framework of the non-relativistic gauge/gravity duality. Following our previous work [17], we continue studying the holographic optical conductivity in the quantum field theory which is dual to the Lifshitz black brane with two independent $U(1)$ gauge fields [18]. To compare with the phenomena in condensed matter physics, we work in a $(3+1)$-dimensional Lifshitz spacetime, *i.e.*, the dual field theory is $(2+1)$-dimensional, and we focus on the case of $1\leqslant z\leqslant 2$, where z is the dynamical critical exponent. When $z=1$, the Lifshtiz black brane will return to the usual Reissner–Nordström (RN)–AdS black brane, therefore, the optical conductivity we obtain is similar to those studied in previous AdS/condensed matter literatures, such as [7]. However, when $z>1$ the optical conductivity, especially its AC part, shows interesting behavior which is less discussed before as far as we know. More explicitly, we find that the optical conductivity will possess a non-trivial scaling with respect to the frequency in large frequency regime when $z>1$, such as $\omega^{s(z)}$, where $s(z)>0$ is a function of z. This feature is very interesting, since in the previous literatures people argued that the large frequency behavior of the electric conductivity in $(2+1)$-dimensional field theory will

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approach to a constant due to the dimensionless of the conductivity [19]. While in this Letter we do find a counterexample. We argue from the viewpoint of non-relativistic gauge/gravity duality that the particular scaling behavior of the optical conductivity with respect to large frequency is caused by the couplings between the dilaton and the electromagnetic fields in the Einstein–Maxwell-dilaton (EMD) theory. This kind of non-minimal coupling will probably introduce extra-dimensional scale into the boundary field system, which results in the peculiar evolution of the optical conductivity associated with the frequency. For earlier studies on holographic properties of charged dilatonic black brane in EMD theory with non-minimal coupling in asymptotically AdS spacetime, see for example [20,21].

More interestingly, we surprisingly find from the condensed matter literatures that there indeed exists a similar universal behavior of the optical conductivity in $(2+1)$-dimensional condensed matter systems, such as [22], in which the authors studied the optical conductivity in various disordered solids both experimentally and theoretically. We observe that the optical conductivity studied in the present holographic model has very like behaviors to those in disordered solids in the extreme disorder limit for both high and low frequencies, at least qualitatively. In order to figure out this interesting phenomenon in more detail, we extend our previous study on the optical conductivity [17] into various temperatures and find that in the low frequency regime, the optical conductivity will decrease as the temperature decreases, which is consistent with the experiments in [22] qualitatively. In particular, at zero temperature the conductivity will vanish, which suggests that the conducting electrons will be frozen at zero temperature.[1] In addition, we show that all the optical conductivities will have the same scaling with respect to the frequency whatever the temperature is for the fixed z in the high frequency regime. This robust phenomenon for the high frequency behavior of the conductivity is also in accordance with the experiments [22]. Furthermore, we also find a linear relation between the logarithmic of the optical conductivity versus the reciprocal of the temperature in some regime of the temperatures, which is qualitatively similar to those in disordered solids [28], as well. All of these consistencies of the holographic optical conductivity with those in various disordered solids allow us to guess that there might be some deep relationship between them, although the underlying precise details are not very clear at present. Strictly speaking, there are no apparent disorder parameters in our model, namely, there is neither spatial inhomogeneity in the background spacetime, nor interaction terms randomly distributed on the spatial coordinates like those studied in [29–32]. However, we want to point out that the fluctuation of the second $U(1)$ gauge field in our construction could be interpreted as the impurity field [33], which interacts with the first $U(1)$ gauge field indirectly through the dilaton field. The homogeneous optical conductivity may relate the extreme disorder limit of disordered solids in which local randomly varying mobilities of charge carriers cover many orders of typical length scale of the condensed matter system [22].

The rest parts of this Letter is organized as follows: The configuration of the asymptotic Lifshitz brane is briefly introduced in Section 2; We show the numerical results of the optical conductivity in Section 3; The conclusions and discussions are drawn in Section 4.

[1] This does not conflict with the fact that the extremal Lifshitz black brane has a nonvanishing entropy, since according to the black hole/CFT correspondence, the entropy of extremal black hole is contributed from the ground state degeneracy of the near horizon microstates. It would also be interesting to study the microscopic entropy of this Lifshitz black brane by extending the methods in the RN/CFT correspondence [23–27]. However, this is beyond the scope of the present Letter.

2. The configuration of the Lifshitz black brane

The bulk gravitational theory we consider is the $(3+1)$-dimensional Einstein–Maxwell-dilaton (EMD) theory with the action

$$I=\frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R-2\Lambda-\frac{1}{4}e^{\lambda_1\phi}F_1^2 -\frac{1}{4}e^{\lambda_2\phi}F_2^2-\frac{1}{2}(\partial\phi)^2\right), \tag{1}$$

where Λ is the cosmological constant, $F_{a\mu\nu}=\partial_\mu A_{a\nu}-\partial_\nu A_{a\mu}$ $(a=1,2)$ are the $U(1)$ gauge field strengths associated with two independent gauge fields $A_{1\mu}$ and $A_{2\mu}$, ϕ is the dilaton field, while λ_1 and λ_2 are the coupling constants between the gauge fields and the dilaton. The dynamical equations in the bulk are

$$\begin{aligned}
&\Box\phi=\frac{1}{4}\sum_{a=1}^{2}\lambda_a e^{\lambda_a\phi}F_a^2,\\
&\nabla_\mu\left(e^{\lambda_a\phi}F_a^{\mu\nu}\right)=0,\\
&R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{2}\sum_{a=1}^{2}e^{\lambda_a\phi}\left(F_{a\mu\lambda}F_{a\nu}^{\lambda}-\frac{1}{4}g_{\mu\nu}F_a^2\right)\\
&\qquad+\frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right).
\end{aligned}\tag{2}$$

One of the solutions for the above EMD theory is a kind of charged Lifshitz black brane derived in [18]

$$\begin{aligned}
&ds^2=-\frac{r^{2z}}{l^{2z}}f(r)\,dt^2+\frac{l^2}{r^2f(r)}dr^2+\frac{r^2}{l^2}\left(dx^2+dy^2\right),\\
&f(r)=1-\frac{m}{r^{z+2}}+\frac{\mu^{-\sqrt{z-1}}l^{2z}q^2}{4zr^{2(z+1)}},\\
&A'_{1t}=l^{-z}\sqrt{2(z+2)(z-1)}\,\mu^{\sqrt{\frac{1}{(z-1)}}}r^{z+1},\qquad A'_{2t}=\frac{q\mu^{-\sqrt{z-1}}}{r^{z+1}},\\
&e^\phi=\mu r^{\sqrt{4(z-1)}},\qquad \lambda_1=-\sqrt{\frac{4}{z-1}},\qquad \lambda_2=\sqrt{z-1},\\
&\Lambda=-\frac{(z+2)(z+1)}{2l^2},
\end{aligned}\tag{3}$$

where l is the curvature radius of the Lifshitz spacetime, μ is the scalar field amplitude, m and q are respectively related to the mass and charge of the black brane. The Hawking temperature of the Lifshitz black brane and the entropy density of boundary non-relativistic field are respectively

$$T=\frac{1}{4\pi}\left(\frac{r_h}{l}\right)^{z+1}\left(\frac{2(z+1)}{r_h}-\frac{mz}{r_h^{z+3}}\right)\quad\text{and}\quad s=\frac{r_h^2}{4G_4l^2}, \tag{4}$$

where we have denoted the location of the outer event horizon to be $r=r_h$, i.e. $f(r_h)=0$.

3. The optical conductivity

In this section, we will numerically compute the optical conductivity of the non-relativistic quantum field theory dual to the above charged Lifshitz black brane.[2] We will show that the holographic optical conductivity is qualitatively similar to those in

[2] The optical conductivity studied in the present Letter is calculated from the current–current 2-point correlator of the bulk perturbed $U(1)$ gauge field A_2 in the linear perturbation limit, i.e., the bulk effective action is expanded up to the second order which corresponds to the tree level approximation from the boundary quantum field theory side. Note that the optical conductivity of the disordered