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Cardy–Verlinde formula in Taub–NUT/Bolt–(A)dS space

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ABSTRACT

We consider a finite action for a higher dimensional Taub–NUT/Bolt–(A)dS space via the so-called counter term subtraction method. In the limit of high temperature, we show that the Cardy–Verlinde formula holds for the Taub–Bolt–AdS metric and for the specific dimensional Taub–NUT–(A)dS metric, except for the Taub–Bolt–dS metric.

1. Introduction

The AdS/CFT duality was first conjectured by [1] in his search for relationship between gauge theories and strings. The AdS/CFT correspondence [2–6] asserts there is an equivalence between a gravitational theory in the bulk and a conformal field theory in the boundary. According to AdS/CFT, a $(d+1)$-dimensional S-(A)dS action A is given by

$$A = A_B + A_{\partial B} + A_{\text{ct}}, \tag{1}$$

where the bulk action A_B, action boundary $A_{\partial B}$, and counterterm action A_{ct} are given as

$$\begin{aligned}
A_B &= \frac{1}{16\pi G_{d+1}} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g}(\mathcal{R} - 2\Lambda), \\
A_{\partial B} &= -\frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-\gamma}\,\Theta, \\
A_{\text{ct}} &= -\frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-\gamma}\Bigg\{-\frac{d-1}{l} \\
&\quad - \frac{lR}{2(d-2)}\mathcal{F}(d-3) - \frac{l^3}{2(d-2)^2(d-4)} \\
&\quad \times \left(R_{ab}R^{ab} - \frac{d}{4(d-1)}R^2\right)\mathcal{F}(d-5) \\
&\quad + \frac{l^5}{(d-2)^3(d-4)(d-6)} \\
&\quad \times \bigg(\frac{3d+2}{4(d-1)}RR_{ab}R^{ab} - \frac{d(d+2)}{16(d-1)^2}R^3 \\
&\quad + \frac{d-2}{2(d-1)}R^{ab}\nabla_a\nabla_b R - R^{ab}\Box R_{ab} \\
&\quad + \frac{1}{2(d-1)}R\Box R\bigg)\mathcal{F}(d-7) + \cdots\Bigg\},
\end{aligned} \tag{2}$$

where a negative cosmological constant Λ is $\Lambda = -d(d-1)/2l^2$, Θ is the trace of extrinsic curvature. Here, $\mathcal{F}(d)$ is a step function, 1 when $d \geqslant 0$, 0 otherwise. The boundary action $A_{\partial B}$ is added to the action A to obtain equations of motion well behaved at the boundary. Then the boundary energy–momentum tensor is expressed in [7]

$$\frac{2}{\sqrt{-\gamma}}\frac{\delta A_{\partial B}}{\delta\gamma^{ab}} = \Theta_{ab} - \gamma_{ab}\Theta. \tag{3}$$

The counterterm action A_{ct} is added to the action A to remove the divergence appearing as the boundary goes to infinity [8]. For low dimensional S-AdS, a few terms in the counterterm action A_{ct} were explicitly evaluated in [8,9]. Using the universality of the structure of divergences, the counterterm action A_{ct} for arbitrary dimension is suggested in [10]. This action A (1) leads to the entropy S via the Gibbs–Duhem relation

$$S = \frac{E}{T} - A, \tag{4}$$

where T denotes the temperature and E is the total energy.

The entropy of the $(1+1)$-dimensional CFT is expressed in terms of the Virasoro operator L_0 and the central charge c, the so-called the Cardy formula [11]. Using conformal invariance, the generalized Cardy formula in arbitrary dimension is shown to be given universal form as [12] (for the review articles of the issue, see, e.g., [13–15])

$$S_{\text{CFT}} = \frac{2\pi R}{\sqrt{ab}}\sqrt{E_c(2E - E_c)}, \tag{5}$$

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where a and b are certain constants. R denotes the radius of the universe at a given time and E_c is the Casimir energy defined by

$$E_c = dE - (d-1)TS. \tag{6}$$

Employing AdS/CFT dual picture, $\sqrt{ab}$ is fixed to $(d-1)$ exactly, in particular, for a d-dimensional CFT on $\mathbf{R}\times S^{d-1}$ [12]. Then, the entropy is given as

$$S_{\rm CFT} = \frac{2\pi R}{d-1}\sqrt{E_c(2E-E_c)}, \tag{7}$$

which is shown to hold for Schwarzschild (A)dS (S-(A)dS) [12,16], charged (A)dS [17,18], Kerr–(A)dS [18,19], and Taub–Bolt-AdS_4 [20]. There are many other relevant papers on the subject [21–25]. Thus, one may naively expect that the entropy of all CFTs that have an AdS-dual description is given as the form (7). However, AdS black holes do not always satisfy the Cardy–Verlinde formula (see, e.g., [17,26]). Therefore, one intriguing question is whether this formula is valid for higher dimensional Taub–NUT–(A)dS at high temperature. In this Letter, we will endeavor to do this.

2. Taub–NUT/Bolt–AdS black hole

When the total number of dimension of the spacetime is even, $(d+1) = 2u+2$, for some integer u, the Euclidean section of the arbitrary $(d+1)$-dimensional-Taub–NUT-AdS metric, for a $U(1)$ fibration over a series of the space $\mathcal{M}^2$ as the base space $\bigotimes_{i=1}^{u}\mathcal{M}^2$, is given by [27–33] (for the generalized versions of the issue, see, e.g., [34,35])

$$ds^2_{\rm AdS} = f(r)\left[dt_E + 2N\sum_{i=1}^{u}\cos(\theta_i)\,d\phi_i\right]^2 + \frac{dr^2}{f(r)} + \left(r^2-N^2\right)\sum_{i=1}^{u}\left[d\theta_i^2 + \sin^2(\theta_i)\,d\phi_i^2\right], \tag{8}$$

where N represents a NUT charge for the Euclidean section, and the metric function $f(r)$ has the general form

$$f(r) = \frac{r}{(r^2-N^2)^u}\int^r\left[\frac{(p^2-N^2)^u}{p^2} + \frac{(2u+1)(p^2-N^2)^{u+1}}{l^2p^2}\right]dp - \frac{2mr}{(r^2-N^2)^u}, \tag{9}$$

with a cosmological parameter l and a geometric mass m.

Requiring $f(r)|_{r=N}$ the NUT solution occurs. Then for AdS spacetime the inverse of the temperature β arises from imposed condition in order to ensure regularity in the Euclidean time t_E and radial coordinate r [27–33]

$$\beta = \frac{4\pi}{f'(r)}\bigg|_{r=N} = \frac{2(d+1)\pi N}{q}, \tag{10}$$

where β is the period of t_E. Here q is a positive integer, which originates from removing Misner string singularities. Using counter term subtraction method the regularized action is given as [27–33]

$$I_{\rm NUT} = \frac{(4\pi)^{\frac{d}{2}}N^{d-2}((d-1)N^2-l^2)}{32\pi^2l^2}\Gamma\left(\frac{2-d}{2}\right)\Gamma\left(\frac{d+1}{2}\right)\beta. \tag{11}$$

Employing the thermal relation $E = \partial_\beta I$ the total energy can also be written by

$$E = \frac{(4\pi)^{\frac{d}{2}}(d-1)N^{d-2}((d+1)N^2-l^2)}{32\pi^2ql^2}\Gamma\left(\frac{2-d}{2}\right)\Gamma\left(\frac{d+1}{2}\right), \tag{12}$$

and the entropy is given as [27–33]

$$S_{\rm NUT,AdS} = \frac{(4\pi)^{\frac{d}{2}}N^{d-2}(d(d-1)N^2-(d-2)l^2)}{32\pi^2l^2} \times\Gamma\left(\frac{2-d}{2}\right)\Gamma\left(\frac{d+1}{2}\right)\beta, \tag{13}$$

by the Gibbs–Duhem relation $S = \beta M - I$ where M denotes the conserved mass

$$M = \frac{(d-1)(4\pi)^{\frac{d}{2}}}{16\pi^{\frac{3}{2}}}m. \tag{14}$$

Substituting (10), (12), and (13) into (6), one gets the Casimir energy [12]

$$E_c = \frac{(4\pi)^{\frac{d}{2}}(d-1)N^{d-2}(dN^2-l^2)}{16\pi^2ql^2}\Gamma\left(\frac{2-d}{2}\right)\Gamma\left(\frac{d+1}{2}\right). \tag{15}$$

From now on, for convenience we use l/z instead of the universe radius R in (7) since the AdS metric is always asymptotically taken to be [36]

$$ds^2 = \frac{l^2}{z^2}dz^2 + \frac{l^2}{z^2}g_{ab}(x,z)\,dx^a\,dx^b, \tag{16}$$

where the $r=\infty$ is put to $z=0$, and the roman indexes a and b refer to boundary coordinates. When $1/\sqrt{ab}$ in the formula (7) is taken to be $2/(d+1)(d-1)(d-2)$, the CFT entropy is given as

$$S_{\rm CFT} = \frac{4\pi l\sqrt{|E_c(2E-E_c)|}}{(d+1)(d-1)(d-2)} = \frac{(4\pi)^{\frac{d}{2}}|dN^2-l^2|(-1)^{[\frac{d}{2}]}\Gamma(\frac{d+1}{2})\Gamma(\frac{2-d}{2})}{4\pi(d+1)(d-2)q}, \tag{17}$$

where $[x]$ is the Gauss number (greatest integer less than or equal to x). Here it seems that the difference from the standard Cardy–Verlinde formula (7) is due to the distinctive nature of NUT solution in AdS space like asymptotically locally AdS (ALAdS) metric. In the limit of high temperature, $N\to 0$, leading term in the entropy of CFT can be expressed as

$$S_{\rm CFT} = \frac{(4\pi)^{\frac{d}{2}}(d+1)(d-2)N^{d-1}}{16\pi q}(-1)^{[\frac{d}{2}]}\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{2-d}{2}\right) = (-1)^{[\frac{d}{2}]}S_{\rm NUT,AdS}. \tag{18}$$

This result shows that the entropy of the Taub–NUT–AdS space suffices to be the generalized Cardy–Verlinde formula (5) for all even u $(d+1 = 2u+2)$. This is reasonable because the Taub–NUT–AdS metric has the thermodynamically stable range depending on the magnitude of the NUT charge i.e. any NUT solution in AdS space for all odd u is thermodynamically unstable in the lime $N\to 0$.

Requiring $f(r)|_{r=r_{\rm B}>N}$ and $f'(r)|_{r=r_{\rm B}} = \frac{1}{(u+1)N}$, the Bolt solution occurs. In Taub–Bolt–AdS metric, the inverse of the temperature, the total energy, and the entropy are respectively

$$\beta = \frac{4\pi}{f'(r)}\bigg|_{r=r_{\rm B}} = \frac{4\pi l^2r_{\rm B}}{l^2+(2u+1)(r_{\rm B}^2-N^2)}, \tag{19}$$

$$E = \frac{(4\pi)^u u}{8\pi}\left(\sum_{k=0}^{u}\binom{u}{k}\frac{(-1)^kN^{2k}r_{\rm B}^{2u-2k-1}}{2u-2k-1} + \sum_{k=0}^{u+1}\binom{u+1}{k}\frac{(-1)^kN^{2k}r_{\rm B}^{2u-2k-1}}{2u-2k-1}\right), \tag{20}$$

$$S_{\rm Bolt,AdS} = \frac{(4\pi)^u\beta}{16\pi l^2}\left[\frac{(2u-1)(2u+1)(-1)^uN^{2u+2}}{r_{\rm B}} + \sum_{k=0}^{u}\binom{u}{k}(-1)^kN^{2k}r_{\rm B}^{2u-2k}\right.$$