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# Nuclear limits on gravitational waves from elliptically deformed pulsars

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ABSTRACT

Gravitational radiation is a fundamental prediction of General Relativity. Elliptically deformed pulsars are among the possible sources emitting gravitational waves (GWs) with a strain-amplitude dependent upon the star's quadrupole moment, rotational frequency, and distance from the detector. We show that the gravitational wave strain amplitude $h_0$ depends strongly on the equation of state of neutron-rich stellar matter. Applying an equation of state with symmetry energy constrained by recent nuclear laboratory data, we set an upper limit on the strain-amplitude of GWs produced by elliptically deformed pulsars. Depending on details of the EOS, for several millisecond pulsars at distances 0.18 kpc to 0.35 kpc from Earth, the *maximal* $h_0$ is found to be in the range of $\sim [0.4\text{–}1.5] \times 10^{-24}$. This prediction serves as the first *direct* nuclear constraint on the gravitational radiation. Its implications are discussed.



## 1. Introduction

Gravitational waves are tiny disturbances in space–time and are a fundamental, although not yet directly confirmed, prediction of General Relativity. They can be triggered in cataclysmic events involving (compact) stars and/or black holes. They could even have been produced during the very early Universe, well before any stars had been formed, merely as a consequence of the dynamics and expansion of the Universe. Because gravity interact extremely weakly with matter, gravitational waves would carry a genuine picture of their sources and thus provide undisturbed information that no other messenger can deliver [1]. Gravitational wave astrophysics would open an entirely new non-electromagnetic window making it possible to probe physics that is hidden or dark to current electromagnetic observations [2].

(Rapidly) rotating neutron stars could be one of the major candidates for sources of continuous gravitational waves in the frequency bandwidth of the LIGO [3] and VIRGO (e.g., Ref. [4]) laser interferometric detectors. It is well known that a rotating object self-bound by gravity and which is perfectly symmetric about the axis of rotation does not emit gravitational waves. In order to generate gravitational radiation over extended period of time, a rotating neutron star must have some kind of long-living axial asymmetry [5]. Several mechanisms leading to such an asymmetry have been studied in literature: (1) Since the neutron star crust is solid, its shape might not be necessarily symmetric, as it would be for a fluid, with asymmetries supported by anisotropic stress built up during the crystallization period of the crust [6]. (2) Additionally, due to its violent formation (supernova) or due to its environment (accretion disc), the rotational axis may not coincide with a principal axis of the moment of inertia of the neutron star which make the star precess [7]. Even if the star remains perfectly symmetric about the rotational axis, since it precesses, it emits gravitational waves [7,8]. (3) Also, the extreme magnetic fields presented in a neutron star cause magnetic pressure (Lorenz forces exerted on conducting matter) which can distort the star if the magnetic axis is not aligned with the axis of rotation [9], which is widely supposed to occur in order to explain the pulsar phenomenon. Several other mechanisms exist that can produce gravitational waves from neutron stars. For instance, accretion of matter on a neutron star can drive it into a non-axisymmetric configuration and power steady radiation with a considerable amplitude [10]. This mechanism applies to a certain class of neutron stars, including accreting stars in binary systems that have been spun up to the first instability point of the so-called Chadrasekhar–Friedman–Schutz (CFS) instability [11]. Also, Andersson [12] suggested a similar instability in $r$-modes of (rapidly) rotating relativistic stars. It has been shown that the effectiveness of these instabilities depends on the viscosity of stellar matter which in turn is determined by the star's temperature.

Gravitational wave strain amplitude depends on the degree to which the neutron star is deformed from axial symmetry which, in turn, is dependent upon the equation of state (EOS) of neutron-rich stellar matter. At present time the EOS of matter under extreme conditions (densities, pressures and isospin asymmetries) is still rather uncertain and theoretically controversial. One of the main source of uncertainties in the EOS of neutron-rich matter is the poorly known density dependence of the nuclear symmetry en-

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ergy, $E_{\rm sym}(\rho)$, e.g., [13]. On the other hand, heavy-ion reactions with radioactive beams could provide unique means to constrain the uncertain density behavior of the nuclear symmetry energy and thus the EOS of neutron-rich nuclear matter, e.g., [14–17,26]. Applying several nucleonic EOSs, in this Letter we calculate the gravitational wave strain amplitude for selected neutron star configurations. Particular attention is paid to predictions with an EOS with symmetry energy constrained by very recent nuclear laboratory data. These results set an upper limit on the strain amplitude of gravitational radiation expected from rotating neutron stars.

The pulsar population is such that most have spin frequencies that fall below the sensitivity band of current detectors. In the future, the low-frequency sensitivity of VIRGO [45] and Advanced LIGO [46] should allow studies of a significantly larger sample of pulsars. Moreover, LISA (the Laser Interferometric Space Antenna) is currently being jointly designed by NASA in the United States and ESA (the European Space Agency), and will be launched into orbit by 2013 providing an unprecedented instrument for gravitational waves search and detection [2].

## 2. Formalism

In what follows we review briefly the formalism used to calculate the gravitational wave strain amplitude. A spinning neutron star is expected to emit GWs if it is not perfectly symmetric about its rotational axis. As already mentioned, non-axial asymmetries can be achieved through several mechanisms such as elastic deformations of the solid crust or core or distortion of the whole star by extremely strong misaligned magnetic fields. Such processes generally result in a triaxial neutron star configuration [3] which, in the quadrupole approximation and with rotation and angular momentum axes aligned, would cause gravitational waves at *twice* the star's rotational frequency [3]. These waves have characteristic strain amplitude at the Earth's vicinity (assuming an optimal orientation of the rotation axis with respect to the observer) of [18]

$$h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz}\nu^2}{r}, \tag{1}$$

where $\nu$ is the neutron star rotational frequency, $I_{zz}$ its principal moment of inertia, $\epsilon = (I_{xx} - I_{yy})/I_{zz}$ its equatorial ellipticity, and $r$ its distance to Earth. The ellipticity is related to the neutron star maximum quadrupole moment (with $m = 2$) via [19]

$$\epsilon = \sqrt{\frac{8\pi}{15}}\frac{\Phi_{22}}{I_{zz}}, \tag{2}$$

where for *slowly* rotating (and static) neutron stars $\Phi_{22}$ can be written as [19]

$$\Phi_{22,\max} = 2.4\times 10^{38}\ \mathrm{g\,cm^2}\left(\frac{\sigma}{10^{-2}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^{6.26}\left(\frac{1.4M_\odot}{M}\right)^{1.2}. \tag{3}$$

In the above expression $\sigma$ is the breaking strain of the neutron star crust which is rather uncertain at present time and lies in the range $\sigma = [10^{-5}\text{–}10^{-2}]$ [18]. To maximize $h_0$ in this work we take $\sigma = 10^{-2}$ which might be too optimistic. From Eqs. (1) and (2) it is clear that $h_0$ does not depend on the moment of inertia $I_{zz}$, and that the total dependence upon the EOS is carried by the quadrupole moment $\Phi_{22}$. Thus Eq. (1) can be rewritten as

$$h_0 = \chi\frac{\Phi_{22}\nu^2}{r}, \tag{4}$$

with $\chi = \sqrt{2045\pi^5/15}G/c^4$. In a recent work we have calculated the neutron star moment of inertia of both static and (rapidly) rotating neutron stars. For slowly rotating neutron stars Lattimer and Schutz [20] derived the following empirical relation

$$I \approx (0.237 \pm 0.008)MR^2\left[1 + 4.2\frac{M\ \mathrm{km}}{M_\odot R} + 90\left(\frac{M\ \mathrm{km}}{M_\odot R}\right)^4\right]. \tag{5}$$

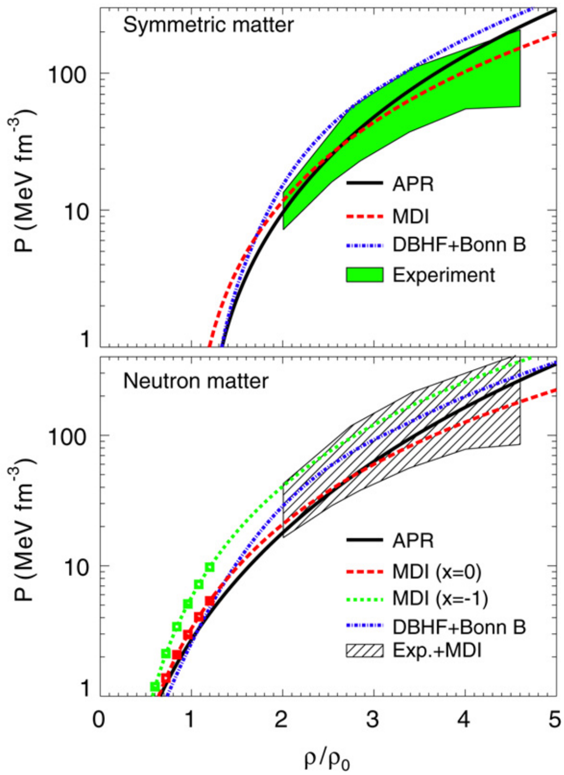


**Fig. 1.** (Color online.) Pressure as a function of density for symmetric (upper panel) and pure neutron (lower panel) matter. The green area in the upper panel is the experimental constraint on symmetric matter extracted by Danielewicz, Lacey and Lynch [22] from analyzing the collective flow in relativistic heavy-ion collisions. The corresponding constraint on the pressure of pure neutron matter, obtained by combining the flow data and an extrapolation of the symmetry energy functionals constrained below $1.2\rho_0$ ($\rho_0 = 0.16\ \mathrm{fm}^{-3}$) by the isospin diffusion data, is the shaded black area in the lower panel. Results are taken partially from Ref. [22].

This expression is shown to hold for a wide class of equations of state which do not exhibit considerable softening and for neutron star models with masses above $1M_\odot$ [20]. Using Eq. (5) to calculate the neutron star moment of inertia and Eq. (3) the corresponding quadrupole moment, the ellipticity $\epsilon$ can be readily computed (via Eq. (2)). Since the global properties of spinning neutron stars (in particular the moment of inertia) remain approximately constant for rotating configurations at frequencies up to $\sim 300$ Hz [21], the above formalism can be readily employed to estimate the gravitational wave strain amplitude, provided one knows the exact rotational frequency and distance to Earth, and that the frequency is relatively low (below $\sim 300$ Hz). These estimates are then to be compared with the current upper limits for the sensitivity of the laser interferometric observatories (e.g., LIGO).

## 3. Results and discussion

We calculate the gravitational wave strain amplitude $h_0$ for several selected pulsars employing several nucleonic equations of state. We assume a simple model of stellar matter of nucleons and light leptons (electrons and muons) in beta-equilibrium. For many astrophysical studies (as those in this Letter), it is more convenient to express the EOS in terms of the pressure as a function of density and isospin asymmetry. In Fig. 1 we show pressure as a function of density for two extreme cases: symmetric (upper panel) and pure neutron matter (lower panel). We pay particular attention to the EOS calculated with the MDI [23] (momentum dependent) interaction because its symmetry energy has been constrained in the subsaturation density region by the available nuclear laboratory data. The EOS of symmetric nuclear matter with the MDI inter-