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Anisotropic fluid dynamics in the early stage of relativistic heavy-ion collisions ☆

Wojciech Florkowski [a,b,*]

[a] The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland
[b] Institute of Physics, Jan Kochanowski University, ul. Świętokrzyska 15, 25-406 Kielce, Poland

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ABSTRACT

A formalism for anisotropic fluid dynamics is proposed. It is designed to describe boost-invariant systems with anisotropic pressure. Such systems are expected to be produced at the early stages of relativistic heavy-ion collisions, when the timescales are too short to achieve equal thermalization of transverse and longitudinal degrees of freedom. The approach is based on the energy–momentum and entropy conservation laws, and may be regarded as a minimal extension of the boost-invariant standard relativistic hydrodynamics of the perfect fluid. We show how the formalism may be used to describe the isotropization of the system (the transition from the initial state with no longitudinal pressure to the final state with equal longitudinal and transverse pressure).

1. At present, the most successful description of early parton dynamics is achieved with the help of the relativistic hydrodynamics [1–3]. With the equation of state incorporating the phase transition and with the appropriate modeling of the freeze-out process, the hydrodynamic approach leads to very successful description of the hadron transverse-momentum spectra and the elliptic flow coefficient v_2 [4–6]. We also note that with a suitable modification of the initial conditions, the hydrodynamic approach describes consistently the HBT radii [7].

In spite of those clear successes, the use of the hydrodynamics is faced with the problem of so-called early thermalization—in order to have a successful description of the data, the hydrodynamic evolution (implicitly assuming the three-dimensional local equilibrium) should start at a very early time, well below 1 fm/c. Such short values can be hardly explained within the microscopic calculation.

Recently, a possible solution to the problem of early thermalization has been proposed [8]. With the assumptions that only transverse degrees of freedom are thermalized and the longitudinal dynamics is essentially the free-streaming (as proposed originally in Ref. [9]), one can obtain the parton transverse-momentum spectra and v_2 which agree well with the data [8,10]. In this approach, called below the *transverse hydrodynamics* (for general formulation see [11]), the longitudinal pressure vanishes while the transverse pressure is large and leads to the formation of a substantial transverse flow, which is the main effect responsible for the good agreement with the data.

One naturally expects that after some time the purely transverse hydrodynamic evolution is transformed into the standard hydrodynamic evolution with isotropic pressure. The typical way to describe such transformation would be to use the kinetic theory [12] or dissipative hydrodynamics [13,14] (see Refs. [15–17] for the presentation of the current status of the dissipative hydrodynamics). Another mechanism to describe a similar transition, from the initial quasithermal two-dimensional parton distribution to the final three-dimensionally isotropic parton distribution, was studied in Ref. [18]. This approach conserves the entropy and is based on the coupled Vlasov and Yang–Mills equations for the quark-gluon plasma. The full isotropization of the system is in this case an effect of bending of parton trajectories in strong color fields.

The aim of this Letter is to propose the extension of the standard boost-invariant hydrodynamics, which would be suitable for the description of systems with anisotropic pressure. In the special cases, our approach is reduced to the transverse hydrodynamics or standard hydrodynamics. It may be also used to describe *effectively* the process of full isotropization of the pressure.

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* Correspondence address: The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland.
E-mail address: wojciech.florkowski@ifj.edu.pl.

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The proposed formalism is based on the energy, momentum, and entropy conservation laws. The use of the entropy conservation is suggested by various modeling of the RHIC data. For example, the PHOBOS data [20] shows that the numbers of produced hadrons per wounded nucleon in central $d+\mathrm{Au}$ and $\mathrm{Au}+\mathrm{Au}$ collisions are very much similar (differences smaller than 30%). This indicates that equilibration/isotropization effects do not produce a large amount of the entropy. In addition, the recently observed scalings of the hadron production with the number of so-called wounded constituents [21] leave little room for extra particle production during the evolution of the system.

From the microscopic point of view, one may consider two extreme cases where the dynamics conserves the entropy: either the collisions between the particles may be neglected or the scattering rate is so high that the system stays in local thermal equilibrium. Since in the latter case the pressure is isotropic, our formalism may be adequate only for the effective description of the collisionless systems. We may consider here the complicated mean-field dynamics (see Ref. [18] discussed above) or the approximate macroscopic description of systems during a limited period of time when the effects of the collisions may be neglected (see Ref. [19] for the discussion of a related problem).

2. Our starting point is the following form of the energy–momentum tensor[1]

$$T^{\mu\nu} = (\varepsilon + P_T)U^\mu U^\nu - P_T g^{\mu\nu} - (P_T - P_L)V^\mu V^\nu, \tag{1}$$

where ε is the energy density, P_T and P_L are the transverse and longitudinal pressure, and U^μ is the four-velocity of the fluid satisfying the normalization condition $U^\mu U_\mu = 1$. In the isotropic case, the pressures P_T and P_L are equal, $P_T = P_L = P$, and the energy–momentum tensor takes the standard form. For the anisotropic fluid, the last term in (1) is different from zero. The fourvector V^μ defines the direction of the longitudinal pressure. It is space-like, orthogonal to U^μ, $U^\mu V_\mu = 0$, and normalized by the condition $V^\mu V_\mu = -1$. In the local rest frame of the fluid element we have $U^\mu = (1,0,0,0)$ and $V^\mu = (0,0,0,1)$, hence the energy–momentum tensor (1) takes the expected form

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_L \end{pmatrix}. \tag{2}$$

For the boost-invariant systems the fluid four-velocity U^μ has the structure

$$U^\mu = \left(u^0 \cosh\eta, u_x, u_y, u^0 \sinh\eta\right), \tag{3}$$

where $u^\mu = (u^0, u_x, u_y, 0)$ is the fluid four-velocity at $z = 0$ (for the vanishing longitudinal coordinate), and $\eta = 1/2 \ln(t+z)/(t-z)$ is the space–time rapidity. With the normalization condition $u_0^2 - u_x^2 - u_y^2 = 1$ we automatically have $U^2 = 1$. The four-vector V^μ satisfying the appropriate normalization and orthogonality conditions has the form

$$V^\mu = (\sinh\eta, 0, 0, \cosh\eta). \tag{4}$$

In this Letter we restrict our considerations to the case of massless particles, where $T^\mu_\mu = 0$ and $\varepsilon = 2P_T + P_L$. As in the standard hydrodynamics, the evolution equations are obtained from the energy–momentum conservation law

$$\partial_\mu T^{\mu\nu} = 0. \tag{5}$$

The projection of Eq. (5) on the four-velocity U_ν yields

$$D\varepsilon + (\varepsilon + P_T)\partial_\mu U^\mu - \Delta U_\nu V^\mu \partial_\mu V^\nu = 0, \tag{6}$$

where we have introduced the short-hand notation for the total time derivative, $D \equiv U^\mu \partial_\mu$ and the difference of the pressures, $\Delta = P_T - P_L$. In addition to the energy–momentum conservation law (5) we demand that there is a conserved entropy current characterizing the system. We write it in the form $\partial_\mu(\sigma U^\mu) = 0$ or equivalently as

$$D\sigma + \sigma \partial_\mu U^\mu = 0, \tag{7}$$

where σ is the entropy density. Moreover, with the definition (4) one finds

$$V^\mu \partial_\mu V^\nu = \partial^\nu \ln\tau, \tag{8}$$

where $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time. Eqs. (7) and (8) allow us to rewrite Eq. (6) in the form

$$D\varepsilon = \frac{(\varepsilon + P_T)}{\sigma} D\sigma + \frac{\Delta}{\tau} D\tau. \tag{9}$$

Eq. (9) indicates that in the general case the energy density may be considered as a function of the two variables, $\varepsilon = \varepsilon(\sigma,\tau)$, hence $\varepsilon(\tau,x,y) = \varepsilon[\sigma(\tau,x,y),\tau]$. We emphasize that this is a novel feature of our approach, which distinguishes it from the standard hydrodynamics where the energy density depends only on the entropy density, $\varepsilon = \varepsilon(\sigma)$ and $\varepsilon(\tau,x,y) = \varepsilon[\sigma(\tau,x,y)]$.

3. The functional dependence $\varepsilon = \varepsilon(\sigma,\tau)$ plays a role of the *generalized equation of state* for our system. Eq. (9) is satisfied if the two conditions hold,

$$\left(\frac{\partial\varepsilon}{\partial\sigma}\right)_\tau = \frac{\varepsilon + P_T}{\sigma}, \qquad \left(\frac{\partial\varepsilon}{\partial\tau}\right)_\sigma = \frac{\Delta}{\tau}. \tag{10}$$

Multiplying Eq. (10) by σ and τ, respectively, and dividing both of them by ε we obtain a simple set of equations

$$\left(\frac{\partial\varepsilon'}{\partial\sigma'}\right)_{\tau'} = \frac{4}{3} + \frac{\Delta'}{3}, \qquad \left(\frac{\partial\varepsilon'}{\partial\tau'}\right)_{\sigma'} = \Delta', \tag{11}$$

where $\varepsilon' = \ln(\varepsilon/\varepsilon_0)$, $\Delta' = \Delta/\varepsilon$, $\sigma' = \ln(\sigma/\sigma_0)$, and $\tau' = \ln(\tau/\tau_0)$ with $\varepsilon_0, \sigma_0, \tau_0$ being arbitrary constants.[2] The thermodynamic consistency requires that $d\varepsilon'$ is the total derivative, hence the mixed second derivatives of the function $\varepsilon'(\sigma',\tau')$ are equal. This condition implies that Δ' must be a function of the single variable, namely $\Delta' = \Delta'(\sigma' + 3\tau')$. By the direct integration of Eq. (11) we find that the energy density function must be of the form

$$\varepsilon = \varepsilon_0 \left(\frac{\sigma}{\sigma_0}\right)^{4/3} R(x), \quad x = \frac{\sigma}{\sigma_0}\left(\frac{\tau}{\tau_0}\right)^3. \tag{12}$$

The function $R(x)$ is related to the function $\Delta'(x)$ by the equation

$$R(x) = \exp\left[\frac{1}{3}\int_0^{\ln x} \Delta'(y)\,dy\right]. \tag{13}$$

In the similar way we may express the transverse and longitudinal pressure, namely

$$P_T = \varepsilon_0 \left(\frac{\sigma}{\sigma_0}\right)^{4/3}\left[\frac{R}{3} + xR'\right],$$

$$P_L = \varepsilon_0 \left(\frac{\sigma}{\sigma_0}\right)^{4/3}\left[\frac{R}{3} - 2xR'\right], \tag{14}$$

[1] Throughout the Letter we use the natural units with $c = \hbar = 1$. The metric tensor $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

[2] Their values may be arranged to impose the appropriate initial conditions.