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On the exact solution of the accelerating string in AdS_5 space ☆

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ABSTRACT

In this Letter, the exact accelerating string solution of a heavy quark–antiquark pair is found in AdS_5 space. On the accelerating string, there is a particular scale which separates the radiation and the heavy quark. This scale is explicitly shown to be an event horizon in the proper frame of the heavy quark. Furthermore, we find a new correspondence, which relates the horizon in AdS_5 space on the gravity theory side to the Unruh temperature in Minkowski space on the field theory side of the AdS/CFT correspondence. $p_\perp$-broadening and p_L-broadening of the heavy quark due to radiation are computed using the AdS/CFT correspondence.

1. Introduction

In Ref. [1], in order to describe bare quark energy loss in a finite size plasma, a brief description of the solution for an accelerating string in AdS_5 space was given, corresponding to a heavy quark–antiquark pair accelerated in opposite directions. In this Letter, we will develop and describe in detail the theory of the accelerating string in AdS_5 spacetime.[1] The objective of this Letter is to investigate the uniformly accelerating heavy quark–antiquark pair with a connecting string in AdS_5 spacetime. We find that there exists an event horizon on the string which separates the heavy quark and radiation during the acceleration. In other words, the upper part of the string is moving with the heavy quark, however, the lower part the string corresponds to radiated energy. Moreover, the event horizon is then shown to correspond to the well-known Unruh temperature [4] in a classical gravity calculation. In the end, the energy loss and $p_\perp$-broadening due to acceleration radiation are studied.

2. The accelerating string solution

We set up our accelerating string calculation as follows: a quark–antiquark pair is imbedded in a brane located at $u = u_m$, and a net electric field E_f is imposed in the brane which accelerates the quark and antiquark at a constant acceleration in their own proper frame (an additional small electric field E_{f2} which balances the attracting force between the quark and antiquark is also understood).

The metric of the resulting vacuum AdS_5 space can be written as

$$ds^2 = R^2\left[\frac{du^2}{u^2} - u^2dt^2 + u^2\left(dx^2+dy^2+dz^2\right)\right] = \frac{R^2}{w^2}\left(dw^2 - dt^2 + dx^2 + dy^2 + dz^2\right), \quad (1)$$

where R is the curvature radius of the AdS_5 space and $w = \frac{1}{u}$. The dynamics of a classical string is characterized by the Nambu–Goto action,

$$S = -T_0 \int d\tau\, d\sigma\, \sqrt{-\det g_{ab}}, \quad (2)$$

where (τ, σ) are the string world-sheet coordinates, and $-\det g_{ab} = -g$ is the determinant of the induced metric. T_0 is the string tension. We define $X^\mu(\tau,\sigma)$ as a map from the string world-sheet to the five-dimensional spacetime, and introduce the following notation for derivatives: $\dot{X}^\mu = \partial_\tau X^\mu$ and $X'^\mu = \partial_\sigma X^\mu$. When one chooses a static gauge by setting $(\tau,\sigma) = (t,u)$, and defines $X^\mu = (t, u, x(t,u), 0, 0)$, it is straightforward to find that

$$-\det g_{ab} = \left(\dot{X}^\mu X'_\mu\right)^2 - \left(\dot{X}^\mu \dot{X}_\mu\right)\left(X'^\mu X'_\mu\right) = R^4\left(1 - \dot{x}^2 + u^4 x'^2\right). \quad (3)$$

Therefore, the equation of motion of the classical string reads

$$\frac{\partial}{\partial u}\left(\frac{u^4 x'}{\sqrt{-g}}\right) - \frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0. \quad (4)$$

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[1] There is a similar numerical study of the accelerating string in Ref. [2] in the black three-brane metric of AdS_5 space. However, our focus in this Letter is to study the exact accelerating string solution in vacuum AdS_5 spacetime. There is also a recent interesting study of the accelerating in Ref. [3] which considers a general time-dependent acceleration. The simplicity of our discussion comes about because we only consider constant acceleration in the vacuum for which we are able to find an exact analytic solution.

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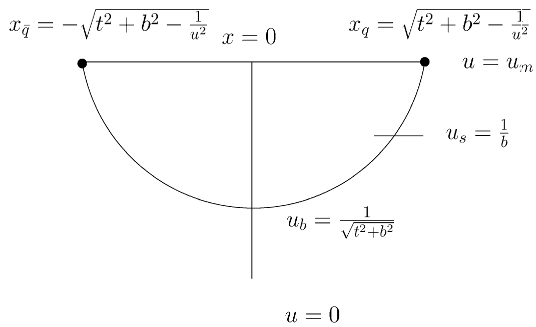

Fig. 1. Illustrating the accelerating string.

In general, this equation is a non-linear differential equation which involves two variables and two derivatives. Thus it is notoriously hard to solve directly when $x(t,u)$ is a non-trivial function of (t,u). Fortunately, we have been able to find the exact solution which corresponds to the accelerating string. The solution reads,

$$x = \pm\sqrt{t^2+b^2-\frac{1}{u^2}}, \tag{5}$$

where the + part represents the right moving part of the string and the − part yields the left moving part of the string, together with the smooth connection in the middle. The quark and anti-quark pair are accelerating and moving away from each other. The constant b can be fixed by the boundary condition. It is very easy to check that Eq. (5) satisfies the equation of motion by noting that $\sqrt{-g/R^4} = \frac{b}{\sqrt{t^2+b^2-1/u^2}}$.

Following Herzog et al. [2], one can compute the canonical momentum densities associated with the accelerating string,

$$\pi^0_\mu = -T_0\frac{(\dot{X}^\nu X'_\nu)X'_\mu - (X'^\nu X'_\nu)\dot{X}_\mu}{\sqrt{-g}}, \tag{6}$$

$$\pi^1_\mu = -T_0\frac{(\dot{X}^\nu X'_\nu)\dot{X}_\mu - (\dot{X}^\nu \dot{X}_\nu)X'_\mu}{\sqrt{-g}}. \tag{7}$$

The energy density is given by π^0_t,

$$\frac{dE}{du} = -\pi^0_t = \frac{T_0R^4}{\sqrt{-g}}\left(1+u^4x'^2\right). \tag{8}$$

Thus the total energy of the right half of the string at time t is,

$$\int_{u_b}^{u_m}\frac{dE}{du}du = \frac{T_0R^2u_m}{b}\sqrt{t^2+b^2-\frac{1}{u_m^2}}. \tag{9}$$

Moreover, the energy flow is given by π^1_t,

$$\frac{dE}{dt} = \pi^1_t = \frac{T_0R^4}{\sqrt{-g}}u^4x'\dot{x}. \tag{10}$$

Assuming that the quark carries a unite charge and E_f being the external electric field as defined above, thus the force acting on the top of the string is just $qE_f = E_f$, and the energy being put into the system by the external force E_f is $E_f(X(t)-X(0))$. Thus the net energy[2] being put into the right half string from 0 to t is,

$$\int_0^t \frac{dE}{dt}dt\bigg|_{u=u_m} = \frac{T_0R^2u_m}{b}\left(\sqrt{t^2+b^2-\frac{1}{u_m^2}} - \sqrt{b^2-\frac{1}{u_m^2}}\right)$$
$$= \frac{T_0R^2u_m}{b}\big(X(t)-X(0)\big), \tag{11}$$

with the second term in the bracket being the initial energy deposited in the string. Also $b^2 - \frac{1}{u_m^2} \geqslant 0$ is assumed for consistency. Therefore, from energy conservation, the energy increase of the system should be equal to $E_f(X(t)-X(0))$. One can easily fix the constant b by setting $E_f = \frac{T_0R^2u_m}{b}$, then,

$$b = \frac{M}{E_f} = \frac{\sqrt{\lambda}u_m}{2\pi E_f}, \tag{12}$$

where $M = T_0R^2u_m$ is the mass of the heavy quark and $T_0R^2 = \frac{\sqrt{\lambda}}{2\pi}$ according to the Ads/CFT correspondence [5–7]. It is now very easy to see the physical interpretation of the constant b as the reciprocal of the constant acceleration a, i.e., $a = \frac{E_f}{M} = \frac{1}{b}$.

In addition, although $\frac{\partial x}{\partial t} = \frac{t}{\sqrt{t^2+b^2-1/u^2}}$ exceeds 1 when u becomes smaller than $1/b$, one can compute the speed which energy travels by the following,

$$v = \frac{\partial x}{\partial t} + \frac{\partial x}{\partial u}\frac{du}{dt} = \frac{t}{t^2+b^2}\sqrt{t^2+b^2-\frac{1}{u^2}}, \tag{13}$$

and find that $v \leqslant 1$ at all times. In arriving at the above result, one needs to look at the hypersurface where energy is constant (for example, one can focus on a lower segment of string with constant energy. One of the ends of this segment is taken as the bottom of the whole string and the other end can be taken to be somewhere $u < 1/b$),

$$E(u,t) = C \quad \Rightarrow \quad \frac{\partial E}{\partial t} + \frac{\partial E}{\partial u}\frac{du}{dt} = 0. \tag{14}$$

Then, one can obtain $\frac{du}{dt} = -\frac{\partial E}{\partial t}/\frac{\partial E}{\partial u} = -\frac{ut}{t^2+b^2}$ according to the energy flow along the string. Here $\frac{du}{dt}$ simply implies that the separation between the lower segment and the rest part of the string has to move downwards at the rate which we found above. Therefore, although some part of the string may travel with a speed beyond speed of light, the energy in the string can only travel at a speed smaller than speed of light. This indicates that the accelerating string solution is consistent with physical expectations. Finally, the Lorentz boost factor of the string reads,

$$\cosh\eta = \frac{1}{\sqrt{1-v^2}} = \frac{t^2+b^2}{\sqrt{(t^2+b^2)b^2+\frac{t^2}{u^2}}}, \tag{15}$$

and it reduces to t/b in the large t and u limits.

3. The event horizon and the Unruh temperature

In the following, we employ a transformation which transforms our system from AdS_5 to a generalized Rindler spacetime. To a uniformly accelerated observer, Minkowski spacetime becomes the so-called Rinder spacetime. With properly chosen parameters, the heavy quark and the string look static in our generalized Rindler spacetime. In other words, we choose to transform to the proper frame of the accelerating string. This frame is an accelerating frame with a constant acceleration a. The transform reads,

$$x = \sqrt{b^2-r^2}\exp\left(\frac{\alpha}{b}\right)\cosh\frac{\tau}{b},$$

$$t = \sqrt{b^2-r^2}\exp\left(\frac{\alpha}{b}\right)\sinh\frac{\tau}{b},$$

$$w = r\exp\left(\frac{\alpha}{b}\right). \tag{16}$$

[2] The total energy being put into the system should be the sum of work done by E_f and E_{f2}. However, the part from E_{f2} balances the Coulomb potential between quark and antiquark as part of our setup in the beginning of the calculation. Thus only E_f contributes to the non-Coulomb net energy increase.