Physics Letters B 665 (2008) 260–263

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Hawking radiation as tunneling derived from black hole thermodynamics through the quantum horizon

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ARTICLE INFO

Article history:
Received 30 March 2008
Received in revised form 5 June 2008
Accepted 11 June 2008
Available online 14 June 2008
Editor: M. Cvetič

PACS:
04.70.Dy
04.60.-m

Keywords:
Black hole
Tunneling
Quantum horizon
Radiation temperature

ABSTRACT

We show that the first law of the black hole thermodynamics can lead to the tunneling probability through the quantum horizon by calculating the change of entropy with the quantum gravity correction and the change of surface gravity is presented clearly in the calculation. The method is also applicable to the general situation which is independent on the form of black hole entropy and this verifies the connection of black hole tunneling with thermodynamics further. In the end we discuss the crucial role of the relation between the radiation temperature and surface gravity in this derivation.

1. Introduction

About 30 years ago, Hawking discovered [1] that when considering quantum effect black holes could radiate particles as if they were hot bodies with the temperature $\kappa/2\pi$ where κ was the surface gravity of the black hole and explained [2] the particles of radiation as stemming from vacuum fluctuations tunneling through the horizon of the black hole with Hartle together. But the semiclassical derivation of Hawking based on the Bogoliubov transformation didn't have the directly connection with the view of tunneling. Parikh and Wilczek [3] calculated directly the particle flux from the tunneling picture and made the tunneling physical explanation holds firm basis. In their consideration the energy conservation played a fundamental role and the outgoing particle itself created the barrier [4]. After this, there have been some works which have extended the Parikh–Wilczek tunneling framework to different cases [5,6] and the question of information loss has been discussed in this framework [7,8]. Recently the general approach has been suggested [9] for the tunneling of matter from the horizon by using the first law of thermodynamics or the conservation of energy. On the other side the tunneling probability has also been calculated [10] directly through the change of the entropy that is proportional to area by the first law of thermodynamics, which verifies the connection of black hole radiation with thermodynamics [11] further.

We have noticed that when the quantum gravity effect is considered the tunneling formula can also be obtained by Parikh–Wilczek method and the Hawking temperature relation [12–14]. In this Letter we will proceed this kind of consideration by using the same method as in Ref. [10] but for the entropy which is modified by the logarithmic term caused by quantum gravity effect as in Ref. [12]. In the new method we show clearly the necessary change of the surface gravity when considering the quantum gravity effect and the crucial role which the Hawking temperature relation plays. We note that the method could be extended to general situation where the tunneling probability is obtained by calculating the change of entropy, independent on the form of the entropy, from the first law of black hole thermodynamics. The generalization verifies the connection of black hole tunneling with thermodynamics further.

In this Letter we take the unit convention $k = \hbar = c = G = 1$.

2. The first law of black hole thermodynamics and entropy

The first law of black hole thermodynamics [15] states:

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0370-2693
doi:10.1016/j.physletb.2008.06.013

If one throws a small amount of mass into a static non-charged and non-rotated black hole, it will settle down to a new static black hole.[1] This change can be described as $dM = \frac{\kappa}{8\pi} dA$, which is analogue to the usual first law of thermodynamics $dM = T\,dS$. The case is the same for radiating a small amount of mass from black hole [11].

According to Hawking, the temperature of black hole is taken as $T = \frac{\kappa}{2\pi}$, so the entropy can be obtained as $S = \frac{1}{4}A$. It has been shown [10] that the tunneling formulas for static, spherically symmetric black hole radiation are obtained by the first law of thermodynamics and the area-entropy relation, even if the radiation temperature is different from the Hawking temperature. From the first law of black hole thermodynamics, we can see that if the black hole temperature is changed, the area-entropy relation will also be changed. Note that in Ref. [10] the author calculated the tunneling probability by using the entropy being proportional to horizon area and so the temperature was also proportional to the surface gravity. But when considering the entropy which is modified by the logarithmic term due to quantum gravity effect [12], it looks as if the black hole temperature were not proportional to the black hole surface gravity. Then in such situation, could the tunneling probability be obtained by calculating the change of entropy with log-area term modification when considering the quantum gravity effect in the same way as in Ref. [10]? The answer is positive! Before discussing this problem, we will first present the method proposed by Pilling.

3. Thermodynamics and tunneling

In this section we will review the method, presented in Ref. [10], which is used to obtain the tunneling probability directly from black hole thermodynamics. Let us start by writing the metric for a general spherically symmetric system in ADM form [16],

$$ds^2 = -N_t(t,r)^2\,dt^2 + L(t,r)^2\big[dr + N_r(t,r)\,dt\big]^2 + R(t,r)^2\,d\Omega^2. \quad (1)$$

The metric is used for the situation where the geometry is spherically symmetric and has a Killing vector which is timelike outside the horizon. Specially one can consider the case of a massless particle and fix the gauge appropriately ($L = 1$, $R = r$) which is particularly useful to study across horizon phenomena. So the metric becomes

$$ds^2 = -N_t(r)^2\,dt^2 + \big[dr + N_r(r)\,dt\big]^2 + r^2\,d\Omega^2. \quad (2)$$

The metric is well behaved on the horizon and for a four-dimensional spherically Schwarzschild solution, $N_t = 1$, $N_r = \sqrt{\frac{2M}{r}}$ (M is the mass of the black hole), for a four-dimensional Reissner–Nordstrom solution, $N_t = 1, N_r = \sqrt{\frac{2M}{r} - \frac{Q^2}{r^2}}$ (M is the mass and Q is the charge of the black hole). And we also note that for $N_t = \sqrt{\frac{f(r)}{g(r)}}, N_r = f(r)\sqrt{\frac{1-g(r)}{f(r)g(r)}}$, the metric (2) becomes the same as that in Ref. [10].

Now let us consider the Parikh–Wilczek tunneling [3]. Supposed the mass of the black hole is fixed and the mass is allowed to fluctuate, then the shell of energy E travels on the geodesics given by the line element (2). Taking into account self-gravitation effects, the outgoing radial null geodesics near the horizon are given approximately by

$$\dot{r} = N_t(r) - N_r(r) \simeq \big(N_t'(R) - N_r'(R)\big)(r - R) + O\big((r-R)^2\big), \quad (3)$$

where the horizon, $r = R$, is determined from the condition $N_t(R) - N_r(R) = 0$ and the last formula is the expansion of the radial geodesics in power of $r - R$.

[1] The law is also applicable to the general situation such as that of charged and rotating black holes.

According to the definition of a time-like Killing vector the surface gravity of the black hole near the horizon is obtained as

$$\kappa \simeq N_t'(R) - N_r'(R). \quad (4)$$

So the radiation temperature is

$$T = \frac{\kappa}{2\pi} = \frac{N_t'(R) - N_r'(R)}{2\pi}. \quad (5)$$

Now we consider the black hole thermodynamics in the region near the horizon. The change of the Bekenstein–Hawking entropy, if the mass of black hole changes from M_i to M_f, is given as

$$\Delta S = \int_{M_i}^{M_f} \frac{dS}{dM} dM = \int_{M_i}^{M_f} 2\pi R \frac{dR}{dM} dM. \quad (6)$$

Considering the small path near R, we can insert the mathematical identity $\mathrm{Im} \int_{r_i}^{r_f} \frac{1}{r-R} dr = -\pi$ in the formula (6). Thus we obtain

$$\Delta S = -2\,\mathrm{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{R}{r-R} \frac{dR}{dM} dM. \quad (7)$$

Using (5) and the expression of the temperature in thermodynamics $\frac{1}{T} = \frac{\partial S}{\partial E}$, we attain

$$R\frac{dR}{dM} = \frac{1}{N_t'(R) - N_r'(R)}. \quad (8)$$

Then Eqs. (3) and (8) give the final form of the change of entropy (7) as

$$\Delta S = -2\,\mathrm{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{dR}{\dot{r}} dM = -2\,\mathrm{Im}\,I, \quad (9)$$

where I is the action for an s-wave outgoing positive particle in WKB approximation.

So the tunneling probability is given as

$$\Gamma \sim e^{\Delta S} = e^{-2\,\mathrm{Im}\,I}. \quad (10)$$

Thus we obtain the tunneling probability from the change of entropy as a direct consequence of the first law of black hole thermodynamics by using the same method as that in Ref. [10]. Let us emphasize that in the original method the author uses the general radiation temperature different from the Hawking temperature in order to discuss the factor of 2 problem. However the new temperature is still proportional to the surface gravity like the Hawking temperature and only the proportional relation is crucial for the discussed problem in this Letter. So we take the Hawking temperature as the black hole temperature without loss of generality.

4. The tunneling through the quantum horizon

We note that for spherically symmetric black holes a generalized treatment [9] has been suggested, in which the tunneling probability is gotten directly from the principle of conservation of energy by calculating the imaginary part of the action in WKB approximation and the method is independent on the form of black hole entropy. For the entropy which is proportional to area [3] or contains the logarithmic modification caused by the presence of quantum gravity [12], we know that the tunneling probability has been obtained by calculating the imaginary part of the action in WKB approximation. Recently Pilling has suggested that the tunneling probability is obtained directly from the first law of thermodynamics by calculating the change of the entropy being proportional to area, even if the radiation temperature is different