Physics Letters B 664 (2008) 70–77

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Single pion production in CC $\nu_\mu N$ scattering within a consistent effective Born approximation

C. Barbero [a], G. López Castro [b,1], A. Mariano [a,*]

[a] Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, cc. 67, 1900 La Plata, Argentina
[b] Instituto de Física, Universidad Nacional Autónoma de México, México D.F., C.P. 04510, Mexico

A R T I C L E I N F O

Article history:
Received 18 February 2008
Received in revised form 15 April 2008
Accepted 1 May 2008
Available online 9 May 2008
Editor: G.F. Giudice

PACS:
25.30.Pt
25.75.Dw
14.20.Gk

A B S T R A C T

The one pion production in charged current $\nu_\mu N$ scattering is analyzed within an extended version of a model used previously to include the $\Delta(1232)$ contribution in elastic and radiative pion–nucleon scattering, and in pion photoproduction. Because the resonant amplitude needs to be invariant under contact transformations, we identify the correct forms of the Δ propagator and the $W^- N\Delta$ vertex that are consistent with this requirement. The only free parameter of the model, the axial form factor at zero momentum transfer, is fixed from data on the differential cross section for $\nu_\mu p \to \mu^- p\pi^+$ scattering. A reasonable agreement with the experimental data of all the $\nu N \to \mu N'\pi$ total cross sections is obtained. We show that the use of the complete Δ propagator instead of the Rarita–Schwinger one improves the theoretical results, leading to differences ranging from 10 to 30%, depending on the specific process.

The interest on weak pion production processes from νN collisions is twofold: (i) they have become a very important element in the analysis of neutrino oscillation experiments, and (ii) they are a powerful tool to study the hadron structure. For example, in the atmospheric neutrino experiment at Kamioka [1], the energy spectrum of weak pion production amounts a 20% of the quasielastic lepton production cross section and it is the major source of uncertainty in the identification of electron and muon events. In addition, neutral current π^0 production might play an important role to distinguish between different oscillation mechanisms of neutrinos produced at accelerators [2–4]. Thus, it is important to have under good control all possible contributions to the $\nu N \to lN'\pi$ cross section. In particular, it is important to have another check of the hadronic matrix element of these processes, which involves the contribution of nucleonic resonances. This is usually done by using effective dynamical models and the values of resonance parameters extracted from data are usually confronted with those coming from constitutive quarks models (QM) and QCD calculations in the lattice. Also, the availability of high intensity ν beams at Fermilab offers a unique opportunity to gain new information on the structure of the nucleon and baryonic resonances. Experiments as MINERνA [5] and FINeSSE [6] will address relevant problems like the extraction of the nucleon and $N \to \Delta$ axial form factors (ANΔ) or the measure of the strange spin of the nucleon.

The weak pion production is closely related to other pion production reactions that involve the Δ resonance. For instance, pion photoproduction $\gamma N \to N'\pi$ and pion electroproduction $eN \to e'N'\pi$ processes have been also extensively studied and have revealed important information on the electromagnetic form factors (FF). From the multipole $\gamma N \to \Delta$ amplitudes $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ it is possible to extract the values for the transition FF, G_M and G_E, at $q^2 = 0$ (q is the photon momentum) [7–11] using dynamical models. As it is well known, the ratio $R_E = -\operatorname{Im} E_{1+}^{3/2}/\operatorname{Im} M_{1+}^{3/2}|_{E_\gamma = M_\Delta}$ is closely related to the size of the nucleon deformation. Also, pion electroproduction brings the possibility to analyze the momentum dependence at $q^2 \neq 0$ of the mentioned FF [12,13].

The weak and electromagnetic currents of some hadrons are closely related to each other via isospin symmetry. Pursuing with the program of describing resonant parameters having at the same time a consistent determination of the pion weak production cross section, it would be important to extend the models previously used in the electromagnetic case to fix the $N \to \Delta$ weak FF (WNΔ). Along the last three decades there have been developed dynamical models studying these FF for the free space cross section. In particular, we mention the contribution of Fogli and Nardulli [14], where the amplitude was calculated at the tree-level by introducing *some* non-resonant terms

* Corresponding author.
E-mail address: mariano@fisica.unlp.edu.ar (A. Mariano).
[1] On sabbatical leave from Departamento de Física, Cinvestav, Apartado Postal 14-740, 07000 México D.F., Mexico.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.05.011

and the Δ resonance through the Rarita–Schwinger (RS) fields. Recently, there has been a renewed interest [15–18] on these topics due to the new neutrino scattering experiments mentioned above, aiming to compare form factors at $q^2=0$ values (now q is the momentum transferred by the W bosons) with those obtained from quite recent QCD calculations on the lattice [19] and including nuclear medium effects [20]. Among the main features of these contributions we find: (i) the use of the QM to fix the axial parameters [15]; (ii) they treat the production ($WN\to\Delta$) and decay ($\Delta\to\pi N$) vertexes in an independent way [16]; (iii) they consider the effects of final state interactions (FSI) and meson exchange contributions [17]; (iv) they extend the model of Fogli and Nardulli by including other non-resonant terms and additional FF in the WNΔ vertex [18].

Most of the previous works mentioned above treat inconsistently the vertexes and the propagator of Δ resonance. As mentioned in [21], the Lagrangian densities $\hat{\mathcal{L}}_\Delta(A)$ (kinetic term) and $\hat{\mathcal{L}}_{\pi N\Delta}(A)$ (interaction term) are invariant under the contact transformation on the ψ_Δ^μ field

$$\psi_\Delta^\mu\to\psi_\Delta^\mu+a\gamma^\mu\gamma_\alpha\psi_\Delta^\alpha,\qquad A\to A'=\frac{A-2a}{1+4a},$$

where A and a ($a\neq-1/4$) are arbitrary parameters. This invariance assures that spurious spin-1/2 components are removed from the field describing an on-shell Δ particle [22]. The Feynman rules involving the propagator and vertexes of the Δ depend on A, but the physical amplitudes are independent of this parameter, as it should be. As shown in Ref. [21] the A-dependent Feynman rules involving the Δ can be replaced by a set of A-independent vertexes and propagators called *reduced* Feynman rules. This scheme was recently extended to include the $\hat{\mathcal{L}}_{\gamma N\Delta}(A)$ Lagrangian looking for the A-independent $\gamma N\to\Delta$ vertex, consistent with the reduced Δ propagator [21]. Since the vector part of the weak $N\to\Delta$ (VNΔ) vertex is related to the electromagnetic vertex $\gamma N\Delta$ through the CVC hypothesis, it is clear that we also need to use a weak production vertex *consistent* with the reduced Δ propagator. Nevertheless, this consistency condition has not been preserved in some of the previous works and we judge important to know how this fact affects the results on the one pion weak production cross section. On the other hand, it has been also shown that FSI between both the final pion and nucleon play an important role in 'dressing' the $\gamma N\to\Delta$ vertex [8,11,12] in such way that the bare $G^0_{M,E}$ values are roughly 40% below the dressed ones. With a lower percentage, this fact has been corroborated in the WNΔ case [17]. Nevertheless, it is common to observe that some works do not consider the influence of FSI [14,18].

In the present Letter we propose an effective Lagrangian model for the calculation of one pion production cross section and the extraction of the ANΔ FF, $F^A_\Delta(q^2)$. Our formalism is fully consistent and extends to the weak pion production case the model used to treat elastic and radiative π^+p scattering [23] and pion photoproduction [11]. The N, Δ, π, ρ and ω degrees of freedom and their interactions are introduced by preserving electromagnetic gauge invariance and invariance under contact transformations, when the finite width of the Δ resonance is considered. FSI effects on $F^A_\Delta(0)$ are included considering that it is an effective parameter (bare + rescattering contributions) fixed from a fitting procedure to reproduce the experimental data, as we have done previously for the photoproduction case [11].

In this work we analyze as a first step the charged current (CC) modes

$$\nu p\to\mu^-p\pi^+,\qquad \nu n\to\mu^-n\pi^+,\qquad \nu n\to\mu^-p\pi^0, \tag{1}$$

where the tree-level amplitudes are shown schematically in Fig. 1. Clearly, all the Feynman graphs do not necessarily contribute to each of the processes in Eq. (1). We have a non-resonant or background (B) contribution built up of the nucleon Born terms (Fig. 1(a)–(d)), the meson exchange amplitudes (Figs. 1(e) and 1(f)) and the Δ-crossed term (Fig. 1(g)); the genuine resonant contribution (R) coming from the Δ-pole amplitude is shown in Fig. 1(h). The sum of all these terms give rise to the total amplitude which can be separated into a background and a resonant contributions as

$$\mathcal{M}=\mathcal{M}_B+\mathcal{M}_R,$$

where[2]

$$\mathcal{M}_i=-\frac{G_FV_{ud}}{\sqrt{2}}\bar{u}(\mathrm{p}_\mu)\gamma_\lambda(1-\gamma_5)u(\mathrm{p}_\nu)\bar{u}(\mathrm{p}')\mathcal{O}_i^\lambda(\mathrm{p},\mathrm{p}',\mathrm{q})u(\mathrm{p}),\quad i=B,R, \tag{2}$$

with $G_F=1.16637\times10^{-5}$ GeV^{-2}, $|V_{ud}|=0.9740$, and the 4-momenta defined as

$$\mathrm{p}=(E_N,\mathbf{p}),\qquad \mathrm{p}_\nu=(E_\nu,\mathbf{p}_\nu),\qquad \mathrm{p}_\mu=(E_\mu,\mathbf{p}_\mu),\qquad \mathrm{k}=(E_\pi,\mathbf{k}),\qquad \mathrm{p}'=(E_{N'},\mathbf{p}'),$$

with $E_i=\sqrt{|\mathbf{p}_i|^2+m_i^2}$ ($|\mathbf{v}_i|=\frac{|\mathbf{p}_i|}{E_i}$ and we set $m_\nu=0$).

It is well known that the baryonic currents J_i^λ have a vector-axial structure ($J_i^\lambda\equiv V_i^\lambda-A_i^\lambda$). In terms of the vector current, the electromagnetic one is written as $J^\lambda_{\text{elect}}=V^\lambda_{\text{isoscalar}}+V_3^\lambda$ and the weak CC as $V^\lambda_\pm\equiv\mp(V_1^\lambda\pm iV_2^\lambda)$. The conserved vector current (CVC) hypothesis allows to relate the isovector pieces of these vector currents. In this way, the Born and ω-exchange contributions (ρ-exchange graph (f) do not contribute to V since the $\rho-\pi$ current is isoscalar) can be obtained from the usual strong [23] ($\mathcal{L}_{NN\omega}(x)$ is built from $\mathcal{L}_{NN\rho}(x)$ with the replacements $\rho_\mu(x)\to\omega_\mu(x)$ and $\boldsymbol{\tau}\to1$) and electromagnetic interaction Lagrangians [7,11,23]. For the axial currents we adopt the Lagrangians of Refs. [15,17] based on standard effective chiral methods and spin-parity arguments. We get

$$\begin{aligned}
&\mathcal{O}_B^\lambda(\mathrm{p},\mathrm{p}',\mathrm{q})\\
&=-i\sqrt{2}\left[F_1^V(Q^2)\gamma^\lambda-i\frac{F_2^V(Q^2)}{2m_N}\sigma^{\lambda\nu}q_\nu-F^A(Q^2)\gamma^\lambda\gamma_5\right]i\frac{\not{p}'+\not{q}+m_N}{(p'+q)^2-m_N^2}\frac{g_{\pi NN}}{2m_N}\gamma_5(\not{p}-\not{p}'-\not{q})\mathcal{T}_1(m_tm_{t'})\\
&\quad+\frac{g_{\pi NN}}{2m_N}\gamma_5(\not{p}-\not{p}'-\not{q})i\frac{\not{p}-\not{q}+m_N}{(p-q)^2-m_N^2}(-i)\sqrt{2}\left[F_1^V(Q^2)\gamma^\lambda-i\frac{F_2^V(Q^2)}{2m_N}\sigma^{\lambda\nu}q_\nu-F^A(Q^2)\gamma^\lambda\gamma_5\right]\mathcal{T}_2(m_tm_{t'})
\end{aligned}$$

[2] We work in the limit of the local four-fermion interaction.