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The ambiguity-free four-dimensional Lorentz-breaking Chern–Simons action

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ABSTRACT

The four-dimensional Lorentz-breaking finite and determined Chern–Simons like action is generated as a one-loop perturbative correction via an appropriate Lorentz-breaking coupling of the gauge field with the spinor field. Unlike the known schemes of calculations, within this scheme this term is found to be regularization independent.

The possibility of violation of the Lorentz and CPT symmetries in the nature has been intensively discussed in recent years [1–17]. Several theoretical investigations have pointed out that these symmetries can be broken. In these studies it was mostly suggested that this violation can be implemented in QED via adding the Chern–Simons-like term $\mathcal{L}_{\rm CS} = \frac{1}{2}k_\mu \epsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$, with k_μ being a constant four-vector characterizing the preferred direction of the space–time, to the photon sector, with, at the same time, another CPT-odd term, i.e., $\bar\psi \not{b} \gamma_5 \psi$, is added to the fermion sector, with the b_μ is a constant four-vector introducing CPT symmetry breaking. It is well known that this extension of the QED does not break the gauge symmetry of the action and equations of motion but it modifies the dispersion relations for different polarization of photons and Dirac spinors. The Chern–Simons-like term is known to have some important implications, such as birefringence of light in the vacuum [9]. Many interesting investigations in the context of Lorentz-CPT violation have appeared recently in the literature. For instance, several issues were addressed, such as Cherenkov-type mechanism called "vacuum Cherenkov radiation" to test the Lorentz symmetry [18], changing of gravitational redshifts for differently polarized Maxwell–Chern–Simons photons [19], evidence for the Lorentz-CPT violation from the measurement of CMB polarization [20], supersymmetric extensions [21], breaking of the Lorentz group down to the little group associated with k_μ [22] and magnetic monopoles inducing electric current [23]. Among these developments one of the most interesting and controversial results is the dynamical origin of the Lorentz-breaking parameter k_μ which arises due to integration over the fermion fields in the modified Dirac action involving the b_μ vector. The result is the induction of the Chern–Simons-like term via radiative corrections which may lead to a relation between the parameters k_μ and b_μ.

The induction of the Chern–Simons-like Lorentz-CPT violating term, $\mathcal{L}_{\rm CS}$, is one of the most important results in the study of the Lorentz symmetry violation [3,4,8]. This term naturally emerges as a perturbative correction in the theory suggested in [4] as a possible extension of QED by an axial-vector term

$$\mathcal{L} = \bar\psi(i\not\partial - m)\psi - \bar\psi \not{b}\gamma_5\psi - e\bar\psi \not{A}\psi. \tag{1}$$

After carrying out the integration over fermions, one can obtain the relation between the coefficients k_μ and b_μ in terms of some loop integrals with some of them being divergent. Therefore one has to implement some regularization to calculate these integrals, thus, the constant C relating the coefficients as $k_\mu = C b_\mu$ turns out to be dependent on the regularization scheme used [24]. The ambiguity of the results manifested in the dependence on the regularization scheme has been intensively discussed in the literature, and several studies have shown that C can be found to be finite but undetermined [25–29]. Astrophysical observations impose very stringent experimental bounds on k_μ (see [30] for different estimations of the Lorentz-breaking coefficients). Since the coefficient k_μ of the radiatively induced Chern–Simons term depends on b_μ it is natural to expect that the constant b_μ can also suffer an experimental bound in this framework. However, if ambiguities are present there is no way to know the experimental bounds for the constant b_μ, because C is simply undetermined. In other words, we cannot define the fate of the constant that is responsible for the Lorentz and CPT violation in the fermion sector by straightforward measuring k_μ. In the sequel we are going to extend the well studied Lagrangian (1) in attempt to shed some light on the issue of inducing Chern–Simons term in the ambiguity-free manner.

In this Letter we propose an extension of the usual theory through introduction of new chiral couplings which can eliminate such ambiguities. So, let us extend the usual Lagrangian (1) as follows:

$$\mathcal{L} = \bar\psi(i\not\partial - m)\psi - \bar\psi \not{b}(1+\gamma_5)\psi - e\bar\psi \not{A}(1-\gamma_5)\psi. \tag{2}$$

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In order to show that the above Lagrangian preserves gauge invariance we rewrite it in terms of new gauge fields, a vector field and an axial field, in the form [31]

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m - \mathcal{V}\!\!\!/ - \mathcal{A}\!\!\!/\gamma_5)\psi, \tag{3}$$

where the vector and axial gauge fields are defined as $\mathcal{V}\!\!\!/ = b\!\!\!/ + eA\!\!\!/$ and $\mathcal{A}\!\!\!/ = b\!\!\!/ - eA\!\!\!/$, respectively. In this sense one can understand b_μ and A_μ as R- and L-handed external fields. Hence, this Lagrangian is invariant under the local vector $U_V(1)$ gauge transformation

$$\psi \to \exp\big[i\alpha(x)\big]\psi,$$
$$\bar{\psi} \to \bar{\psi}\exp\big[-i\alpha(x)\big],$$
$$\mathcal{V}_\mu \to \mathcal{V}_\mu - \partial_\mu\alpha(x). \tag{4}$$

The couplings we propose in the theory imply in a special gauge invariant and Lorentz-CPT violating theory which we call "extended chiral QED", where divergences among loop integrals are canceled. The extension is based on the observation that one can extend and impose some restrictions on the gauge invariant and CPT-Lorentz violating Lagrangian (1) by replacing $b\!\!\!/$ and $A\!\!\!/$ according to the transformations

$$b\!\!\!/\gamma_5 \to b\!\!\!/(1+\gamma_5),$$
$$A\!\!\!/ \to A\!\!\!/(1-\gamma_5). \tag{5}$$

One can also verify that other combinations of signs in the interacting terms above produce either only divergent integrals or mixture of divergent and finite integrals. Note that both b_μ and A_μ manifest themselves as external fields with opposite chiralities coupled to fermion fields. As we will show below, in this model the divergences are canceled and no regularization scheme is required. As we just anticipated, by combining other signs of γ_5 in (2), one could also construct interactions with same chirality, but in this case the divergences would persist.

The one-loop effective action $S_{\text{eff}}[b, A]$ of the gauge field A_μ in this theory can be expressed in the form of the following functional trace

$$S_{\text{eff}}[b, A] = -i\,\text{Tr}\ln\big[p\!\!\!/ - m - b\!\!\!/(1+\gamma_5) - eA\!\!\!/(1-\gamma_5)\big]. \tag{6}$$

Notice that in this expression, the Tr symbol stands for the trace over Dirac matrices, trace over internal space, as well as for the integration in momentum and coordinate spaces. Hence, in the case of Eq. (6) the calculations are complicated since because the electromagnetic field A_μ is coordinate dependent, hence it does not commute with functions of momentum. Therefore it is not easy to separate out the momentum and space dependent quantities and carry out the integrations in respective spaces. To solve this problem, we will use the method of derivative expansion [32] (see also [24]), and proceed as follows.

The functional trace in (6) can be represented as

$$S_{\text{eff}}[b, A] = S_{\text{eff}}[b] + S'_{\text{eff}}[b, A], \tag{7}$$

where the first term is $S_{\text{eff}}[b] = -i\,\text{Tr}\ln[p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)]$, which does not depend on the gauge field, and the only nontrivial dynamics is concentrated in the second term $S'_{\text{eff}}[b, A]$ given by the following power series

$$S'_{\text{eff}}[b, A] = i\,\text{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{1}{p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)}eA\!\!\!/(1-\gamma_5)\right]^n. \tag{8}$$

To obtain the Chern–Simons-like term we should expand this expression up to the second order in the gauge field

$$S'_{\text{eff}}[b, A] = S^{(2)}_{\text{eff}}[b, A] + \cdots. \tag{9}$$

The dots in (9) stand for the terms of higher orders in the gauge field. Here

$$S^{(2)}_{\text{eff}}[b, A] = -\frac{ie^2}{2}\,\text{Tr}\big[S_b(p)A\!\!\!/(1-\gamma_5)S_b(p)A\!\!\!/(1-\gamma_5)\big], \tag{10}$$

where $S_b(p)$ is the b^μ dependent propagator of the theory defined as

$$S_b(p) = \frac{i}{p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)}. \tag{11}$$

Now, we can apply the key identity of the derivative expansion method [32], that is,

$$A_\mu(x)S_b(p) = S_b(p - i\partial)A_\mu(x), \tag{12}$$

with the propagator $S_b(p - i\partial)$ is expanded up to the first order in derivatives as

$$S_b(p - i\partial) = S_b(p) + S_b(p)\partial\!\!\!/ S_b(p) + \cdots. \tag{13}$$

Substituting this expression into Eq. (10), we obtain

$$S^{(2)}_{\text{eff}}[b, A] = \int d^4x\, \Pi^{\lambda\mu\nu} A_\mu \partial_\nu A_\lambda, \tag{14}$$

with the one-loop self-energy tensor is given by

$$\Pi^{\lambda\mu\nu} = -\frac{ie^2}{2}\int\frac{d^4p}{(2\pi)^4}\,\text{tr}\big[S_b(p)\gamma^\mu(1-\gamma_5) \times S_b(p)\gamma^\lambda S_b(p)\gamma^\nu(1-\gamma_5)\big], \tag{15}$$

where the symbol tr denotes the trace of the product of the gamma matrices. Using a perturbative method for fermion propagator, we can expand $S_b(p)$ in the following series in b^μ

$$S_b(p) = S(p) + S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p) + \cdots, \tag{16}$$

with $S(p)$ being the usual fermion propagator. We find

$$\begin{aligned}\Pi^{\lambda\mu\nu} = -\frac{ie^2}{2}\int\frac{d^4p}{(2\pi)^4}\,\text{tr}\big[&S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p)\gamma^\mu(1-\gamma_5)\\ &\times S(p)\gamma^\lambda S(p)\gamma^\nu(1-\gamma_5) + S(p)\gamma^\mu(1-\gamma_5)\\ &\times S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p)\gamma^\lambda S(p)\gamma^\nu(1-\gamma_5)\\ &+ S(p)\gamma^\mu(1-\gamma_5)S(p)\gamma^\lambda S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)\\ &\times S(p)\gamma^\nu(1-\gamma_5)\big].\end{aligned} \tag{17}$$

Thus, taking into account the fact that $\{\gamma_5, \gamma^\mu\} = 0$ and $(\gamma_5)^2 = 1$ and applying the following relation for trace

$$\text{tr}\big(\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5\big) = 4i\epsilon^{\lambda\mu\nu\rho}, \tag{18}$$

we can write down the simple expression for self-energy tensor $\Pi^{\lambda\mu\nu}$ as

$$\Pi^{\mu\nu\lambda} = 8ie^2m^2b_\rho\int\frac{d^4p}{(2\pi)^4}\frac{N^{\mu\nu\rho\lambda}}{(p^2-m^2)^4}, \tag{19}$$

where

$$N^{\mu\nu\rho\lambda} = 2\epsilon^{\mu\nu\rho\theta}p^\lambda p_\theta + \epsilon^{\mu\nu\rho\lambda}\big(p^2 - m^2\big). \tag{20}$$

The key property of this expression is that, unlike the results obtained earlier [8,24,33–35], this result is manifestly finite and does not require any regularization. The exact, regularization independent, value for $\Pi^{\lambda\mu\nu}$ is

$$\Pi^{\lambda\mu\nu} = \epsilon^{\lambda\mu\nu\rho}b_\rho\frac{e^2}{3\pi^2}. \tag{21}$$

Thus, the effective action (14) acquires the familiar form