



# A geometric formulation of Higgs Effective Field Theory: Measuring the curvature of scalar field space



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## ARTICLE INFO

### Article history:

Received 19 November 2015

Received in revised form 15 January 2016

Accepted 21 January 2016

Available online 27 January 2016

Editor: B. Grinstein

## ABSTRACT

A geometric formulation of Higgs Effective Field Theory (HEFT) is presented. Experimental observables are given in terms of geometric invariants of the scalar sigma model sector such as the curvature of the scalar field manifold  $\mathcal{M}$ . We show how the curvature can be measured experimentally via Higgs cross-sections,  $W_L$  scattering, and the  $S$  parameter. The one-loop action of HEFT is given in terms of geometric invariants of  $\mathcal{M}$ . The distinction between the Standard Model (SM) and HEFT is whether  $\mathcal{M}$  is flat or curved, and the curvature is a signal of the scale of new physics.

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## 1. Introduction

The recent discovery of a neutral scalar particle with a mass of  $\sim 125$  GeV has renewed interest in non-linear effective Lagrangians. An effective Lagrangian for a spontaneously broken gauge theory with three “eaten” Goldstone bosons and one additional neutral scalar particle yields the most general low-energy description of the interactions of the new scalar particle detected at the LHC. This Higgs Effective Field Theory (HEFT) Lagrangian [1, 2] is constructed with the three “eaten” Goldstone bosons and the light neutral Higgs boson transforming as a triplet and a singlet, respectively, under the custodial symmetry. The only implicit assumption in this HEFT framework is that there are no other light states in the few hundred GeV range which couple to SM particles, which is known to be satisfied experimentally. The theory contains as a limiting case the renormalizable Standard Model (SM) Higgs Lagrangian, where the neutral scalar and the Goldstone bosons of the spontaneously broken electroweak gauge symmetry form a complex scalar doublet  $H$  that transforms linearly as  $\mathbf{2}_{1/2}$  under the electroweak gauge symmetry  $SU(2)_L \times U(1)_Y$ . An important special case of HEFT is the Standard Model Effective Field Theory (SMEFT), where the scalar fields of the EFT transform linearly as a complex scalar doublet  $H$ . Schematically,

$$\text{SM} \subset \text{SMEFT} \subset \text{HEFT}. \quad (1)$$

The path integral formulation of quantum field theory gives a prescription for computing the  $S$ -matrix of the theory from the Lagrangian. An important result in quantum field theory is that the  $S$ -matrix is independent of the fields chosen to parametrize the theory: field redefinitions which change the form of the Lagrangian leave the  $S$ -matrix invariant. The well-known analysis of sigma models by Callan, Coleman, Wess and Zumino (CCWZ) [3,4] uses this freedom to make field redefinitions on the scalar fields to put all spontaneously broken theories into a standard form. The CCWZ convention specifies a definite choice for the scalar fields of a given sigma model. Although making such a choice eliminates the ambiguity of the Lagrangian due to field redefinitions, it does so at a cost of obscuring the geometry of the sigma model, which describes the field-independent properties. In this work, we present a geometric formulation of HEFT which emphasizes the field-independent observables of the scalar sigma model sector. This geometric formulation of HEFT makes explicit the connection between the geometry of scalar field space and experimental measurements.

There are many important features of the scalar sector of a spontaneously broken theory that are obscured in the standard presentation of the Higgs sector of the SM. The usual SM Higgs sector with a fundamental Higgs doublet can be written in the universal CCWZ formulation as a non-linear effective Lagrangian of a Goldstone boson triplet and an additional neutral singlet scalar field  $h$ . The non-linear version leads to the same  $S$ -matrix as the linear formulation, including quantum corrections. It is clear that adding the scalar singlet  $h$  with precisely the right couplings is key

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to the equivalence of the non-linear and linear parameterizations of the SM Higgs sector. The non-linear parameterization of the SM Higgs sector in which the field  $h$  is not necessarily part of a doublet can be consistently generalized to a non-renormalizable EFT, which is HEFT. Theories in which  $h$  is the dilaton or a Goldstone boson of an enlarged global symmetry provide examples of HEFT. SMEFT is a special case of HEFT since it requires that the scalar  $h$  and the three Goldstone bosons transform as a complex scalar doublet.

In the literature, SMEFT and HEFT are referred to as the linear Lagrangian and the non-linear or chiral Lagrangian, respectively. This nomenclature is somewhat misleading, given that SMEFT is a special case of HEFT. Throughout this paper, we use the term HEFT to refer to the most general EFT containing the physical Higgs boson. We focus on theories with custodial  $SU(2)$ , which have the symmetry breaking pattern  $O(4) \rightarrow O(3)$ . We use linear and non-linear to refer to whether the scalar fields transform linearly or non-linearly under the  $O(4)$  symmetry.

HEFT describes the dynamics of the physical Higgs scalar  $h$ , and the Goldstone bosons  $\varphi$  from  $\mathcal{G} \rightarrow \mathcal{H}$  symmetry breaking, which together form coordinates on a scalar manifold  $\mathcal{M}$ . Different parameterizations of the scalar sector correspond to different coordinate choices on  $\mathcal{M}$ . The  $S$ -matrix is unchanged by such scalar field redefinitions, i.e. by coordinate transformations in scalar field space, and depends only on the geometry of  $\mathcal{M}$ . From this perspective, the key question is not whether  $SU(2)_L \times U(1)_Y$  gauge symmetry is realized linearly or non-linearly at the level of the Lagrangian,<sup>1</sup> since one can convert from the linear form to the non-linear form by a field redefinition, but whether the scalar manifold  $\mathcal{M}$  is *curved* or *flat*. The renormalizable SM has a flat scalar manifold. A geometric formulation of HEFT makes clear that all physical observables are independent of the parameterization of the scalar fields. SMEFT is a special case of HEFT where there is an  $O(4)$  invariant point on the scalar manifold. The SMEFT Lagrangian is given by expanding the HEFT Lagrangian about this special point (which is at  $H = 0$ ) in a power series in the scalar fields.

In this paper, we compute the Riemann curvature tensor of  $\mathcal{M}$ , and show how the curvature can be measured experimentally in terms of the couplings of the physical Higgs boson to the massive electroweak gauge bosons  $W^\pm$  and  $Z$ . We also explore other couplings of the Higgs boson to the massless SM gauge boson, the photon, and to fermions from a geometric point of view. We emphasize that an important goal of precision Higgs boson physics will be to constrain the curvature of the scalar manifold  $\mathcal{M}$ , which is a measure of the scale of new physics.

The geometric formulation of non-linear sigma models is well-known, and has been used extensively for supersymmetric sigma models, and, to a lesser extent, for chiral perturbation theory [5–9]. In this paper, we apply it to HEFT with a single light neutral scalar  $h$ . A general formulation of spontaneously broken  $\mathcal{G}/\mathcal{H}$  theories with an arbitrary number of additional scalars is given in a subsequent work [10].

A coordinate-invariant formulation of HEFT also clears up a number of subtleties which arise in the one-loop corrections to the theory. Calculations of radiative corrections in sigma models by Appelquist and Bernard [11,12], and more recently for HEFT by Gavela et al. [13], require intermediate steps in which chiral non-invariant terms proportional to the equations of motion (EOM) are redefined away. The appearance of these terms for curved scalar manifolds does not have physical implications, and they are avoided when employing a covariant formalism for perturbation

theory [5,6]. In our geometric formulation of HEFT, quantum corrections to the theory are given in terms of the curvature of the scalar manifold  $\mathcal{M}$ , and non-covariant terms which vanish on-shell can be understood and systematically dealt with.

The organization of this paper is as follows. Section 2 defines the curvature of the scalar field manifold for HEFT, and gives ways to measure the curvature experimentally. Section 3 presents the path integral formalism for calculating radiative corrections in an arbitrary sigma model. A geometric formulation of all quantities is given. Section 4 specializes to renormalization in the case of interest, namely HEFT. Section 5 gives the conclusions.

## 2. Curvature of scalar field space

The Higgs sector of the SM has a complex scalar doublet  $H$  which can be defined in terms of 4 real scalar fields  $\phi_H^i$ ,  $i = 1, 2, 3, 4$ , by

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_H^2 + i\phi_H^1 \\ \phi_H^4 - i\phi_H^3 \end{bmatrix}. \quad (2)$$

The scalar potential  $V(H)$  depends only on the magnitude of the scalar 4-vector

$$(\phi_H^1)^2 + (\phi_H^2)^2 + (\phi_H^3)^2 + (\phi_H^4)^2 \equiv 2H^\dagger H, \quad (3)$$

and it has a minimum at the vacuum expectation value  $v = 246$  GeV,

$$\langle (\phi_H^1)^2 + (\phi_H^2)^2 + (\phi_H^3)^2 + (\phi_H^4)^2 \rangle = v^2, \quad (4)$$

which spontaneously breaks  $\mathcal{G} = O(4)$  symmetry down to  $\mathcal{H} = O(3)$ . It is convenient to define the  $2 \times 2$  scalar field matrix

$$\Sigma \equiv (\tilde{H}, H), \quad (5)$$

where  $\tilde{H} \equiv (i\sigma_2)H^*$ . Under the  $O(4) \sim SU(2)_L \times SU(2)_R$  symmetry,  $\Sigma$  transforms as

$$\Sigma \rightarrow L \Sigma R^\dagger, \quad \langle \Sigma \rangle = \frac{v}{\sqrt{2}} \mathbb{1}, \quad (6)$$

where  $L$  and  $R$  are  $2 \times 2$  unitary matrices, and  $\mathbb{1}$  is the  $2 \times 2$  identity matrix. One sees that the unbroken  $\mathcal{H}$  symmetry is custodial  $O(3) \sim SU(2)_V$ , which ensures the gauge boson mass relation  $M_W = M_Z \cos \theta_W$  at tree level. The vacuum manifold  $\mathcal{G}/\mathcal{H}$  is isomorphic to the three-sphere  $S^3$  and is parametrized by three Goldstone boson coordinates  $\varphi^a$ ,  $a = 1, 2, 3$  where Latin letters from the beginning of the alphabet will be used to distinguish  $O(3)$  indices from  $O(4)$  indices,  $i = 1, 2, 3, 4$ . The radial direction perpendicular to  $S^3$  corresponds to the Higgs boson direction  $h$ , which transforms as a singlet under the unbroken symmetry group. We can make this geometric relationship explicit using spherical polar coordinates in scalar field space:

$$\begin{bmatrix} \phi_H^1 \\ \phi_H^2 \\ \phi_H^3 \\ \phi_H^4 \end{bmatrix} \equiv (v + h) \begin{bmatrix} u^1(\varphi) \\ u^2(\varphi) \\ u^3(\varphi) \\ u^4(\varphi) \end{bmatrix}, \quad \begin{aligned} u(\varphi) \cdot u(\varphi) &= 1, \\ u^i(0) &= \delta^{i4}, \end{aligned} \quad (7)$$

where  $u^i(\varphi)$  is a unit 4-vector depending only on the angular coordinates  $\varphi$  on  $S^3$ , and  $h$  is the radial coordinate.

The above discussion focuses on the global symmetries of the  $O(4)$  sigma model. Now, we account for the partial gauging of the  $O(4)$  symmetry. The electroweak gauge symmetry group  $\mathcal{G}_{\text{gauge}}$  in the Higgs sector is a subgroup of the global symmetry group,  $\mathcal{G}_{\text{gauge}} \subset \mathcal{G}$ . Specifically, using the definition of  $\Sigma$  in Eq. (6),  $O(4) \sim SU(2)_L \times SU(2)_R \supset SU(2)_L \times U(1)_Y$ , where  $U(1)_Y$  is the Abelian

<sup>1</sup> Note that this commonly used terminology of linear and non-linear electroweak symmetry breaking derives from the non-linear chiral Lagrangian for QCD.

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