Physics Letters B 754 (2016) 39–42

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Hawking radiation, the Stefan–Boltzmann law, and unitarization

Steven B. Giddings

Department of Physics, University of California, Santa Barbara, CA 93106, United States

ARTICLE INFO

Article history:
Received 9 December 2015
Accepted 31 December 2015
Available online 6 January 2016
Editor: B. Grinstein

ABSTRACT

Where does Hawking radiation originate? A common picture is that it arises from excitations very near or at the horizon, and this viewpoint has supported the "firewall" argument and arguments for a key role for the UV-dependent entanglement entropy in describing the quantum mechanics of black holes. However, closer investigation of both the total emission rate and the stress tensor of Hawking radiation supports the statement that its source is a near-horizon quantum region, or "atmosphere," whose radial extent is set by the horizon radius scale. This is potentially important, since Hawking radiation needs to be modified to restore unitarity, and a natural assumption is that the scales relevant to such modifications are comparable to those governing the Hawking radiation. Moreover, related discussion suggests a resolution to questions regarding extra energy flux in "nonviolent" scenarios, that does not spoil black hole thermodynamics as governed by the Bekenstein–Hawking entropy.

Hawking radiation is commonly perceived as originating from the horizon of a black hole. One reason for this is the structure of Hawking's original calculation [1]: highly blueshifted modes just outside the horizon, which are entangled with similar inside excitations, can be described as evolving to become the radiation. This view is buttressed by a nice match to the thermal description of the observations of detectors at constant radius r. These detectors are properly accelerating, and so experience the Unruh effect with a temperature that is related to Hawking's by a blueshift, in accord with the Tolman law; see for example [2].

It is important, however, to check this picture, since the requirement of unitarity of the black hole decay tells us that the Hawking radiation must be modified. If we wish to understand what kind of modification is needed, and where it occurs, we should first fully understand the properties of the Hawking radiation, which is responsible for the problem of information loss to begin with. This is emphasized, for example, by the structure of the "firewall" argument: if one presupposes a near-horizon origin of the Hawking radiation, and that the corresponding near-horizon excitations must therefore be modified in order to restore unitarity, one concludes that the state is very singular, with an enormous energy density also rendering the spacetime geometry singular at the horizon [3–6].

So, in order to better understand both where unitarizing modifications might appear, and also other aspects of the thermodynamics of black holes, we seek other tests for the source of the Hawking radiation.

One way to infer the size of a radiating body is via the Stefan–Boltzmann law, giving the radiated power (in the case of two polarization degrees of freedom, *e.g.* photons)

$$\frac{dE}{dt} = \sigma_S A T^4 \tag{1}$$

in terms of the area A of an emitting black body, and its temperature; here $\sigma_S = \pi^2/60$ is the Stefan–Boltzmann constant. From this, one finds the area of the emitting surface from the power and the temperature, which for Hawking radiation we expect to be the Hawking temperature. A complication, however, is that a black hole emits as a gray body – it is not precisely thermal. But, once gray-body factors are taken into account, numerical calculation [7] shows that the emission rate *exceeds* the rate (1) for particles with spin ≤ 1 if A is taken to be the horizon area – suggesting a larger effective emitting surface. Specifically, considering for example photon emission, ref. [7] (see eq. (29) and below) shows a total rate for a black hole of mass M

$$\frac{dE}{dt} = 3.4 \times 10^{-5} M^{-2}\,, \tag{2}$$

as compared to a rate

$$\frac{dE}{dt} = 2.1 \times 10^{-5} M^{-2} \tag{3}$$

from (1) if $T = 1/(8\pi M)$ is the Hawking temperature and $A = 16\pi M^2$ the horizon area.

E-mail address: giddings@physics.ucsb.edu.

http://dx.doi.org/10.1016/j.physletb.2015.12.076
0370-2693/

The calculation and conclusions can be sharpened by looking at the power spectrum, which can be expressed as

$$\frac{dE}{dtd\omega} = \frac{1}{\pi}\sum_l (2l+1)\omega \frac{\Gamma_{\omega l}}{e^{\beta\omega}-1} = \frac{1}{\pi^2}\frac{\omega^3}{e^{\beta\omega}-1}\sigma(\omega) \tag{4}$$

for two degrees of freedom, where l is the orbital angular momentum, $\Gamma_{\omega l}$ are the gray-body factors, and $\beta = 1/T$. In the second equality, the spectrum has been related to the absorption cross section at frequency ω,

$$\sigma(\omega) = \frac{\pi}{\omega^2}\sum_l (2l+1)\Gamma_{\omega l}\,. \tag{5}$$

For a spherical blackbody of area $A = 4\pi r^2$, $\sigma(\omega) = \pi r^2 = A/4$, and (1) is reproduced. In the case of Hawking radiation, the gray-body factors vary nontrivially with ω, but in the large-ω limit,

$$\sigma(\omega) \to \pi R_a^2 \tag{6}$$

where

$$R_a = 3\sqrt{3}M = \frac{3\sqrt{3}}{2}R \tag{7}$$

and $R = 2M$ is the Schwarzschild radius. This limit is the geometric-optics, massless limit, and so this result can be understood from the effective potential (see *e.g.* [8]) for a classical massless particle. Here absorption is perfect for $l < \omega R_a$, and vanishes for $l > \omega R_a$, and so

$$\Gamma_{\omega l} \approx \theta(\omega R_a - l)\,, \tag{8}$$

giving (6), and yielding a high-energy power spectrum (4) matching that of [7].

Thus the effective emitting area for the Hawking radiation can be read off from this high-energy emission, and is $A = 4\pi R_a^2$; the effective emitting radius R_a is considerably outside the horizon radius, which is indicative of a source well outside the horizon. Note that for lower-energy modes, where quantum effects become more relevant, the gray-body factors are suppressed from unity. Since most of the emission is in such modes, this yields [7] a total power (2) that is suppressed from (1) evaluated with $A = 4\pi R_a^2$.

Since the statement that the source of the Hawking radiation is well outside the horizon runs contrary to various perceptions, we should try to test it by other means. A more refined picture of the Hawking radiation comes from examining its stress tensor. This is particularly tractable in the case of a two-dimensional metric, taken to be of the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} = f(r)(-dt^2 + dx^2) = -f(r)dx^+dx^- \tag{9}$$

where

$$dx = \frac{dr}{f(r)}\,, \tag{10}$$

and $x^\pm = t \pm x$. The conformal coordinate x is sometimes referred to as a tortoise coordinate. For the two-dimensional black hole of [9], studied in the soluble collapse models of [10],

$$f(r) = 1 - e^{-2(r-R)}\,. \tag{11}$$

However, the metric (9) may also be thought of the metric induced on a cosmic string that threads a higher-dimensional black hole, allowing us to probe that case as well.

The expectation value of the stress tensor for Hawking radiation can be computed via the conformal anomaly [11,10]:

$$\begin{aligned}
\langle T_{--}\rangle &= \frac{1}{24\pi}\left[\frac{\partial_-^2 f}{f} - \frac{3}{2}\frac{(\partial_- f)^2}{f^2}\right] + t_-(x^-)\\
\langle T_{++}\rangle &= \frac{1}{24\pi}\left[\frac{\partial_+^2 f}{f} - \frac{3}{2}\frac{(\partial_+ f)^2}{f^2}\right] + t_+(x^+)\\
\langle T_{+-}\rangle &= -\frac{1}{24\pi}\left(\frac{\partial_+\partial_- f}{f} - \frac{\partial_+ f\,\partial_- f}{f^2}\right)
\end{aligned} \tag{12}$$

where $t_-(x^-)$ and $t_+(x^+)$ are arbitrary functions characterizing the particular state. It is readily verified that (12) is conserved. Indeed, the conformal anomaly determines $\langle T_{+-}\rangle$, and then conservation fixes $\langle T_{--}\rangle$ and $\langle T_{++}\rangle$, up to the functions $t_\pm$.

Eq. (12) may be written in terms of r-derivatives of f, denoted by primes, using (10). This gives

$$\begin{aligned}
\langle T_{--}\rangle &= \frac{1}{96\pi}\left[ff'' - \frac{1}{2}(f')^2\right] + t_-\\
\langle T_{++}\rangle &= \frac{1}{96\pi}\left[ff'' - \frac{1}{2}(f')^2\right] + t_+\\
\langle T_{+-}\rangle &= \frac{1}{96\pi} ff''\,.
\end{aligned} \tag{13}$$

For the Hartle–Hawking [12] or Unruh [13] states, regularity of $\langle T_{\mu\nu}\rangle$ at the future horizon, checked in terms of the *Kruskal* components of $\langle T_{\mu\nu}\rangle$, then implies

$$t_- = \frac{1}{192\pi}[f'(R)]^2\,. \tag{14}$$

Since the other terms in $\langle T_{--}\rangle$ vanish asymptotically at $r \to \infty$, t_- is the asymptotic Hawking flux. For the Hartle–Hawking vacuum, this flux is balanced by incoming flux, $t_+ = t_-$, and $\langle T_{\mu\nu}\rangle$ is also regular on the past horizon. For the Unruh vacuum, $t_+ = 0$, so there is no incoming asymptotic flux, but there is a negative energy flux into the horizon. Note that $\langle T_{--}\rangle$ also vanishes to *next* order in $r - R$, as can be readily verified by taking its r-derivative, from (13); that is, $\langle T_{--}\rangle$ vanishes as $f^2(r)$ at $r = R$.

We now see properties that support the preceding claim. The outward Hawking flux $\langle T_{--}\rangle$ can be converted into that in an orthonormal frame (*cf.* (9)) by multiplying by $1/f$, but the resulting proper $\langle T_{\hat{-}\hat{-}}\rangle$ still vanishes at the horizon; the proper outward flux builds up from there, over a range of $r \sim R$, to its asymptotic value. That is, the outgoing Hawking flux, as measured by its stress tensor, originates not at the horizon, but in a larger quantum region or atmosphere. For the Hartle–Hawking vacuum, $\langle T_{\hat{0}\hat{1}}\rangle$ identically vanishes due to cancellation between ingoing and outgoing flux. For the Unruh vacuum, $\langle T_{\hat{0}\hat{1}}\rangle$ is nonvanishing at the horizon due to the negative *influx* [14][1] of energy described by $\langle T_{++}\rangle$. This energy flux at a near-horizon coordinate r does satisfy a two-dimensional version of the Stefan–Boltzmann law of the form

$$\frac{dE}{dt} = -\langle T_{\hat{0}\hat{1}}\rangle = \sigma_2 T^2(r)\,, \tag{15}$$

where $T(r)$ is the locally blueshifted temperature, which is seen by the locally accelerated observers at constant r, and σ_2 is a

[1] Indeed, following the first appearance of this paper, the author became aware of [14] which gave closely related arguments, for an origin of Hawking particles in the vicinity of a black hole rather than from the collapsing body that formed it. Unruh's arguments were based on 1) the fact that energy appears outside the black hole and is compensated by the negative influx; 2) the failure of infalling observers to detect particles near the horizon (see also [15]); and 3) the existence of stimulated emission due to an emitter falling into a black hole. Refs. [16,17] have also investigated the role of the negative energy density at the horizon, and pointed out vanishing of an effective "Tolman" temperature there, and ref. [18] makes possibly related comments about negative influx.