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Geometric phase for neutrino propagation in magnetic field

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ABSTRACT

The geometric phase for neutrinos propagating in an adiabatically varying magnetic field in matter is calculated. It is shown that for neutrino propagation in sufficiently large magnetic field the neutrino eigenstates develop a significant geometric phase. The geometric phase varies from 2π for magnetic fields $\sim$ fraction of a micro gauss to π for fields $\sim 10^7$ gauss or more. The variation of geometric phase with magnetic field parameters is shown and its phenomenological implications are discussed.

1. Introduction

The existence of non-zero neutrino mass is so far the only experimental proof [1] of physics beyond standard model. This calls for a minimal extension of standard model wherein a Dirac neutrino acquires a magnetic moment given by [2]

$$\mu_\nu = 3.2 \times 10^{-19} \left(\frac{m_\nu}{1\text{ eV}}\right) \mu_B, \tag{1}$$

where m_ν is the neutrino mass and μ_B is the Bohr Magneton, $\mu_B = 5.8 \times 10^{-15}$ MeV/G. Many theoretical considerations [3–5,7] which utilize physics beyond minimally extended standard model put much more stronger upper bound on the neutrino magnetic moments, as large as $\approx 10^{-14}\mu_B$ for a Dirac neutrino and $\approx 10^{-12} - 10^{-10}\mu_B$ for a Majorana neutrino. Thus experimental observation of neutrino magnetic moment greater than Eq. (1) would be a definite evidence of physics beyond minimally extended standard model. Also the results from the GEMMA experiment [6] put an upper bound on the neutrino magnetic moment at $2.9 \times 10^{-11}\mu_B$. The non-zero magnetic moment imparts non-trivial electromagnetic properties to the neutrino which opens up "a window to new physics" [7].

Inspired by the fact that the neutrinos have a non-zero magnetic moment, we envisage a physical situation where its effect could manifest in the presence of magnetic field. Such situations are encountered in astrophysical environments where neutrinos propagate through large distances in magnetic field in vacuum and in matter. As the neutrino propagates in a magnetic field, its magnetic moment will couple to the field leading to neutrino spin rotation, which may result in spin-flip transitions of neutrinos.

For the case of neutrino spin precession in presence of a magnetic field in matter with constant density, the spin conversion probability is given by [7]

$$P_{\nu_L \to \nu_R}(x) = \frac{(2\mu_\nu B_\perp)^2}{V^2 + (2\mu_\nu B_\perp)^2} \sin^2\left(\frac{1}{2}\sqrt{V^2 + (2\mu_\nu B_\perp)^2}\,x\right), \tag{2}$$

where μ_ν is the neutrino magnetic moment, $B_\perp$ is the perpendicular component of the magnetic field and V is the weak interaction potential for neutrinos propagating in matter. For a neutrino of flavor l, matter potential is given by $V_l = V_{CC}\delta_{le} + V_{NC}$, V_{CC} and V_{NC} being charged-current and neutral current potentials respectively.

There may also arise spin-flavor precession of neutrinos due to transition magnetic moments of neutrino, which involves both spin flip and flavor conversion of the neutrino in presence of a magnetic field. If the magnetic field is twisting uniformly in the transverse plane of propagating neutrino, then the spin-flavor conversion probability in matter with constant density is given by [9–11]

$$P_{\nu_L \to \nu_R}(x) = \frac{(2\mu_\nu B_\perp)^2}{(V_m - \dot{\phi})^2 + (2\mu_\nu B_\perp)^2} \times \sin^2\left(\frac{1}{2}\sqrt{(V_m - \dot{\phi})^2 + (2\mu_\nu B_\perp)^2}\,x\right), \tag{3}$$

where $V_m = \sqrt{2}G_F n^{eff} - \frac{\Delta m^2}{2E_\nu}\cos 2\theta$; G_F is the Fermi's constant, Δm^2 is the mass squared difference $\Delta m^2 = m^2(\nu_L) - m^2(\nu_R)$, θ is the neutrino mixing angle, E_ν is the neutrino energy, n^{eff} is the effective concentration of particles interacting with neutrinos,

$$n^{eff} = \begin{cases} (n_e - n_n), & \text{for } \nu_{eL} - \bar{\nu}_{\mu,\tau R} \\ (n_e - n_n/2), & \text{for } \nu_{eL} - \nu_{eR} \end{cases}, \tag{4}$$

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n_e and n_n are the concentrations of electrons and neutrons respectively, and $\dot{\phi}$ is the precession frequency of the magnetic field.

The spin-precession $\nu_{eL} \to \nu_{eR}$ in the magnetic field of the sun was considered by Cisneros [12] as a possible solution to account for the deficit of the active solar ν_e flux. Okun et al. [13] included matter effects and showed that the it results in the suppression of $\nu_{eL} \to \nu_{eR}$ transitions. It was soon realised [9,10,14] that the spin-flavor precession of the neutrinos can lead to resonant conversion $\nu_e \to \bar{\nu}_{\mu,\tau}$. However, it became eventually clear that the resonant spin flavor precession is not the correct solution of the solar neutrino problem [15].

The geometric phase, on the other hand, is a general property of quantum systems which arises if the Hamiltonian governing the system contains two or more time-dependent parameters. If the system is initially prepared in an eigenstate of the Hamiltonian then as the system undergoes an adiabatic evolution along a closed curve in the parameter space, the eigenstate develops a geometric phase in addition to the usual dynamical phase [16]. The geometric phase can be generalised for systems undergoing non-adiabatic, non-cyclic and non-unitary evolutions [17–19] and has observable consequences in a wide variety of physical systems ranging from classical motion of Foucault pendulum to nuclear magnetic resonance studies [20].

The most common example where geometric phase arises is the spin-precession of a particle with an intrinsic magnetic moment in a magnetic field. The possibility of geometric phase associated with neutrino spin precession was explored in [8,9] in the context of solar neutrino problem. In [9] it was shown that the geometric phase may induce resonant spin conversion of neutrinos inside the sun. Also, there has been some interesting earlier work on geometric phase in neutrino oscillations where electromagnetic interactions of neutrinos are not included. Naumov [21] calculated Berry phase for a three-flavor Dirac neutrino system as the neutrino propagation occurs in a medium whose density and element composition varies cyclically with distance. He et al. [22] generalised [21] and studied Berry phase in neutrino oscillations for both Dirac and Majorana neutrinos including active and active-sterile neutrino mixing and non-standard interactions. The above papers [21,22] claim that for non-trivial Berry phase to arise, the neutrino interactions with background matter must depend on two independent densities, and CP violating phase must be non-zero. Blasone et al. [23] studied Berry phase for the case of two and three flavor neutrino oscillations in vacuum. They showed that the cyclic time evolution of a neutrino flavor state produces an overall phase factor which, following Aharonov–Anandan prescription [17], can be written as sum of two parts, a purely dynamical part and a part which depends only on the mixing angle and hence geometric in nature. Wang et al. [24] generalised the results of [23] for the case of non-cyclic time evolution. Mehta [25] examined geometric phase for two neutrino flavor oscillations with CP conservation and showed that non-zero geometric phase appears not just at the amplitude level [23,24] but even at the level of probability and hence is directly observable. Syska et al. [26] studied the geometric phase in π^+ decay and pointed out that geometric phase value of π in neutrino oscillations is consistent with the mixing matrix parameters.

Since it is known that the geometric phase does not have significant effect in causing the spin or spin-flavor transitions of neutrinos in the sun, we approach the problem from a more general perspective. We assume that the neutrinos are propagating in an adiabatically varying arbitrary magnetic field in matter with constant density and, following Berry [16], we explicitly calculate the geometric phase that arises in such situation. We then write transitions probability $P(\nu_L \to \nu_R)$ in terms of geometric phase and analyse the values of geometric phase which lead to resonance.

2. Geometric phase

We assume that the neutrino is propagating along the z-direction, in a magnetic field rotating arbitrarily about the direction of motion of neutrino. The magnetic field vector can be written as

$$\mathbf{B}(t) = B_0(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \tag{5}$$

where B_0, θ, and ϕ are adiabatically varying time dependent parameters.

The evolution equation describing the propagation of the two helicity components of a neutrino $|\nu\rangle = (\nu_L, \nu_R)^T$ in magnetic field Eq. (5) in matter is given by

$$i\frac{d}{dt}|\nu(t)\rangle = H(t)\,|\nu(t)\rangle\,, \tag{6}$$

where the Hamiltonian $H = H_0 + H_{wk} + H_{em}$; H_0 is the vacuum Hamiltonian, H_{wk} is the weak-interaction Hamiltonian for neutrino coupling with matter, and H_{em} is the electromagnetic Hamiltonian for neutrino magnetic moment coupling with external magnetic field. We assume that CP is conserved and for simplicity neglect neutrino mass mixing, so that the flavor eigenstates coincide with the mass eigenstates.

In the $(\nu_L, \nu_R)^T$ basis the total Hamiltonian, neglecting the terms proportional to unit matrix, can be written as [9]:

$$H = \begin{pmatrix} \frac{V}{2} + \mu_\nu B_0\cos\theta & \mu_\nu B_0 e^{-i\phi}\sin\theta \\ \mu_\nu B_0 e^{i\phi}\sin\theta & -\frac{V}{2} - \mu_\nu B_0\cos\theta \end{pmatrix}, \tag{7}$$

where $V = \sqrt{2}G_F n^{eff} - \frac{\Delta m^2}{2E}$, $\Delta m^2 = m^2(\nu_L) - m^2(\nu_R)$, E is the neutrino energy, n^{eff} is the effective concentration of particles interacting with neutrinos given by Eq. (4).

The instantaneous eigenvalues and normalised eigenvectors of H are

$$\lambda_\pm = \pm\sqrt{\left(\frac{V}{2} + \mu_\nu B_0\cos\theta\right)^2 + (\mu_\nu B_0\sin\theta)^2}, \tag{8}$$

$$|+\rangle = \frac{1}{N}\begin{pmatrix} \mu_\nu B_0\sin\theta \\ -e^{i\phi}(\frac{V}{2} + \mu_\nu B_0\cos\theta - \lambda_+) \end{pmatrix}, \tag{9}$$

$$|-\rangle = \frac{1}{N}\begin{pmatrix} e^{-i\phi}(\frac{V}{2} + \mu_\nu B_0\cos\theta - \lambda_+) \\ \mu_\nu B_0\sin\theta \end{pmatrix}, \tag{10}$$

where $|+\rangle$ and $|-\rangle$ corresponds to λ_+ and λ_- respectively, and

$$N = \sqrt{\left(\frac{V}{2} + \mu_\nu B_0\cos\theta - \lambda_+\right)^2 + (\mu_\nu B_0\sin\theta)^2}. \tag{11}$$

If the neutrino is initially in state $|+\rangle$ at $t = 0$, then as it moves along in the magnetic field it picks up a geometrical phase factor given by [16]:

$$\gamma_+ = i\oint_C \mathbf{dr}\cdot\langle +|\nabla +\rangle\,, \tag{12}$$

where the integral is over closed curve C in the parameter space.

For our case we have

$$\nabla|+\rangle = \frac{1}{B_0 N}\begin{pmatrix} \mu_\nu B_0\cos\theta - f\mu_\nu B_0\sin\theta \\ e^{i\phi}[\mu_\nu B_0\sin\theta(1 - \frac{V}{2\lambda_+}) + f(\frac{V}{2} + \mu_\nu B_0\cos\theta - \lambda_+)] \end{pmatrix}\hat{\theta} + \frac{1}{NB_0\sin\theta}\begin{pmatrix} 0 \\ -ie^{i\phi}(\frac{V}{2} + \mu_\nu B_0\cos\theta - \lambda_+) \end{pmatrix}\hat{\phi}, \tag{13}$$

where