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Dynamical domain wall and localization

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ABSTRACT

Based on the previous works (Toyozato et al., 2013 [24]; Higuchi and Nojiri, 2014 [25]), we investigate the localization of the fields on the dynamical domain wall, where the four-dimensional FRW universe is realized on the domain wall in the five-dimensional space–time. Especially we show that the chiral spinor can localize on the domain wall, which has not been succeeded in the past works as the seminal work in George et al. (2009) [23].

1. Introduction

The scenarios that our universe could be a brane or domain wall embedded in a higher-dimensional space–time are not new [1,2] but after we have found the strong conjecture that there could be a classical solution called D-brane in string theories [3,4], the scenarios of the brane [5–9] or the domain wall [13–23] with reality have been well investigated. Especially inflationary brane world models by using the trace anomaly have been proposed [10–12]. We may regard brane with a limit where the thickness of the domain wall vanishes.

Recently some of the authors proposed a domain wall model with two scalar fields, Refs. [24,25]; we have shown to be able to construct a model which generates space–time, where the scale factor of the domain wall universe, which could be the general FRW universe, and the warp factor are arbitrarily given. The formulation is an extension of the formalism of the reconstruction of the domain wall [26]. We should note that in [16], a formulation has been proposed where only the warp factor of the domain wall is arbitrary.

In this paper, we consider the localization of several fields in the model [24,25]. The localization of the graviton has been already shown in [25] and we have found that there appear extra terms which are related with the extra dimension. These terms may affect the fluctuations of the universe. We now investigate the localization of the spinor field and the vector field in this paper. If the domain is static, there appear chiral fermions localized on the domain wall [20]. There were attempts to localize the chiral fermion on the dynamical domain wall corresponding to the FRW universe [23] but it was not succeeded. In [23], it was assumed that the warp factor does not depend on the time, but in this paper, we use the time-dependent warp factor and show the localization of the chiral fermion. In this paper, A, B denote indices on a curved space–time manifold, while $\hat{A}$, $\hat{B}$ denote indices on its tangent space–time.

In the next section, we review the formulation in [24,25] including the localization of the graviton. In section 3, we investigate the localization of the chiral fermion on the domain wall. In section 4, the localization and non-normalizability of vector field are shown. The final section, section 5, is devoted to the discussions.

2. Domain wall model with two scalar fields

We now briefly review the formulation of the domain wall model with two scalar fields based on [24,25] and we show the localization of the graviton. The formulation is given by extending the formulation in [27] and similar procedure was invented for the reconstruction of the FRW universe by single scalar model [28].

In the domain wall solution, which can be regarded as a general FRW universe in the five-dimensional space–time, the metric is given by

$$ds^2 = dw^2 + L^2 e^{u(w,t)} ds_{\mathrm{FRW}}^2 \,. \tag{1}$$

Here ds_{FRW}^2 is the metric of the general FRW universe,

$$ds_{\mathrm{FRW}}^2 = -dt^2 + a(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right\} . \tag{2}$$

The FRW space–time is embedded by the arbitrary warp factor $L^2 e^{u(w,t)}$.

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In [24,25], we considered the following action with two scalar fields ϕ and χ:

$$S_{\phi\chi} = \int d^5x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \frac{1}{2}A(\phi,\chi)\partial_M\phi\partial^M\phi - B(\phi,\chi)\partial_M\phi\partial^M\chi - \frac{1}{2}C(\phi,\chi)\partial_M\chi\partial^M\chi - V(\phi,\chi)\right\}. \tag{3}$$

It has been shown that we can construct a model to realize the arbitrary metric (1) by using the model (3). In the model (3), the energy–momentum tensor for the scalar fields ϕ and χ is given by

$$\begin{aligned}T^{\phi\chi}_{MN} = {} & g_{MN}\left\{-\frac{1}{2}A(\phi,\chi)\partial_L\phi\partial^L\phi - B(\phi,\chi)\partial_L\phi\partial^L\chi - \frac{1}{2}C(\phi,\chi)\partial_L\chi\partial^L\chi - V(\phi,\chi)\right\} \\ & + A(\phi,\chi)\partial_M\phi\partial_N\phi + B(\phi,\chi)\left(\partial_M\phi\partial_N\chi + \partial_N\phi\partial_M\chi\right) \\ & + C(\phi,\chi)\partial_M\chi\partial_N\chi\,. \end{aligned}\tag{4}$$

On the other hand, by the variation of ϕ and χ, we obtain the field equations as follows:

$$0 = \frac{1}{2}A_\phi\partial_M\phi\partial^M\phi + A\nabla^M\partial_M\phi + A_\chi\partial_M\phi\partial^M\chi + \left(B_\chi - \frac{1}{2}C_\phi\right)\partial_M\chi\partial^M\chi + B\nabla^M\partial_M\chi - V_\phi\,, \tag{5}$$

$$0 = \left(-\frac{1}{2}A_\chi + B_\phi\right)\partial_M\phi\partial^M\phi + B\nabla^M\partial_M\phi + \frac{1}{2}C_\chi\partial_M\chi\partial^M\chi + C\nabla^M\partial_M\chi + C_\phi\partial_M\phi\partial^M\chi - V_\chi\,. \tag{6}$$

Here we use the denotation $A_\phi = \partial A(\phi,\chi)/\partial\phi$, etc. We now choose $\phi = t$ and $\chi = w$. Then we find

$$\begin{aligned} T_0{}^0 &= -\frac{e^{-2u(w,t)}}{2L^2}A - \frac{1}{2}C - V\,, \\ T_i{}^j &= \delta_i{}^j\left(\frac{e^{-2u(w,t)}}{2L^2}A - \frac{1}{2}C - V\right), \\ T_5{}^5 &= \frac{e^{-2u(w,t)}}{2L^2}A + \frac{1}{2}C - V\,, \quad T_0{}^5 = B\,. \end{aligned}\tag{7}$$

By using the Einstein equation, we may solve Eqs. (7) with respect to A, B, C, and V as follows:

$$\begin{aligned} A &= \frac{L^2e^{u(w,t)}}{\kappa^2}\left(G_1{}^1 - G_0{}^0\right) = \frac{L^2e^{u(w,t)}}{\kappa^2}\left(G_2{}^2 - G_0{}^0\right) \\ &= \frac{L^2e^{u(w,t)}}{\kappa^2}\left(G_3{}^3 - G_0{}^0\right) \\ &= \frac{1}{\kappa^2}\left(\frac{2k}{a^2} - \ddot{u} - 2\dot{H} + \frac{(\dot{u})^2}{2} + \dot{u}H\right), \\ B &= \frac{1}{\kappa^2}G_0{}^5 = -\frac{3u'}{2\kappa^2L^2e^u}\left(\dot{u} + 2H\right), \\ C &= \frac{1}{\kappa^2}\left(G_5{}^5 - G_1{}^1\right) = \frac{1}{\kappa^2}\left(G_5{}^5 - G_2{}^2\right) = \frac{1}{\kappa^2}\left(G_5{}^5 - G_3{}^3\right) \\ &= \frac{1}{\kappa^2}\left(-\frac{3}{2}u'' + \frac{2k}{L^2e^ua^2} - \frac{1}{2e^u}\left(\ddot{u} + 2\dot{H} + (\dot{u})^2 + 5\dot{u}H + 6H^2\right)\right), \\ V &= \frac{1}{\kappa^2}\left(G_0{}^0 + G_5{}^5\right) \\ &= \frac{1}{\kappa^2}\left(-\frac{3}{4}\left(u'' + 2\left(u'\right)^2\right) + \frac{3k}{L^2e^ua^2} + \frac{1}{4L^2e^u}\left(3\ddot{u} + 6\dot{H} + 3\left(\dot{u}\right)^2 + 15\dot{u} + 18H^2\right)\right). \end{aligned}\tag{8}$$

Here the Einstein tensor is denoted by G_{MN}. Then by replacing t and w in the r.h.s. of Eqs. (8) by ϕ and χ, respectively, we find the explicit forms of $A(\phi,\chi)$, $B(\phi,\chi)$, $C(\phi,\chi)$, and $V(\phi,\chi)$. Then by using the expressions in the action (3), we obtain a model which realizes the metric (1). We should note that Eqs. (5) and (6) are satisfied automatically because they can be obtained by using the Einstein equation and the 0th and 5th components of the Bianchi identity $\nabla^N\left(R_{MN} - \frac{1}{2}Rg_{MN}\right) = 0$, which corresponds to the conservation of the energy–momentum tensors in (7).

The localization of the graviton can be investigated by considering the perturbation

$$g_{MN} \to g_{MN} + h_{MN}\,. \tag{9}$$

We now impose the gauge condition

$$\nabla^M h_{MN} = g^{MN}h_{MN} = 0\,. \tag{10}$$

In our domain wall model, by writing h_{ij} $(i,j = 1,2,3)$ in a factorized form, $h_{ij}(w,x) = e^{u(w,t)}\hat{h}_{ij}(x)$, we find that the graviton follows the equation,

$$0 = \left(2\frac{\dot{a}\dot{u}}{a} - \dot{u}\partial_0 + 2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\partial_0 - \partial_0^2 + \frac{\triangle}{a^2}\right)\hat{h}_{ij}\,. \tag{11}$$

Here $\triangle$ is the Laplacian in the flat three-dimensional space. Then if u goes to minus infinity rapidly enough for large $|w|$, $h_{ij}(w,x)$ can be normalized in the direction of w and therefore the graviton localizes on the domain wall.

We should note that the graviton $h^{(4)}_{ij}$ in the four-dimensional FRW space–time (2) satisfies the following equation:

$$0 = \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\partial_0 - \partial_0^2 + \frac{\triangle}{a^2}\right)h^{(4)}_{ij}\,, \tag{12}$$

in the standard four-dimensional Einstein gravity. Therefore when $\dot{u}\left(2\frac{\dot{a}}{a} - \partial_0\right)\hat{h}_{ij} = 0$, the expression (11) coincides with the equation for the graviton in (12). Especially when the warp factor does not depend on time, that is, $\dot{u} = 0$, the two expressions coincide with each other. Conversely if $\dot{u} \neq 0$, corrections proportional to $\dot{u}$ are possible when we consider the perturbations.

We have only considered the perturbation of the graviton, which is a tensor field and therefore the graviton does not mix with the scalar modes. This is because we are interested in the localization of the graviton. Of course, it is necessary to consider the perturbations including the scalar modes when we consider more realistic cosmology but the perturbations become complicated and we would like to leave the perturbations including the scalar modes to the future works.

3. Localization of fermion

In this section, we show that chiral fermion can localize on the domain wall. The Dirac equation in five dimensions is given by

$$\Gamma^M\nabla_M\Psi + \tilde{f}\chi(w)\Psi = 0\,. \tag{13}$$

In the last term, $\tilde{f}\chi$ is a function of the scalar field $\chi = w$ in (3) and this term expresses the general Yukawa interaction between