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Testing black hole superradiance with pulsar companions

João G. Rosa [1]

Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal

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ABSTRACT

We show that the magnetic dipole and gravitational radiation emitted by a pulsar can undergo superradiant scattering off a spinning black hole companion. We find that the relative amount of superradiant modes in the radiation depends on the pulsar's angular position relative to the black hole's equatorial plane. In particular, when the pulsar and black hole spins are aligned, superradiant modes are dominant at large angles, leading to an amplification of the pulsar's luminosity, whereas for small angles the radiation is dominantly composed of non-superradiant modes and the signal is attenuated. This results in a characteristic orbital modulation of the pulsar's luminosity, up to the percent level within our approximations, which may potentially yield a signature of superradiant scattering in astrophysical black holes and hence an important test of general relativity.

It has been known since the 1970s that low-frequency waves can be amplified upon scattering off a rotating black hole (BH) [1–5]. Superradiant scattering occurs for waves of frequency $\omega < m\Omega$, where Ω denotes the angular velocity of the BH horizon and m the azimuthal number that characterizes the wave's angular momentum. Such waves carry negative energy and spin into the BH from the point of view of a distant observer, which is possible inside the ergoregion that surrounds a rotating BH. Such an observer thus sees the BH lose a fraction of its mass and angular momentum in the scattering process (see [6] for a recent review of this topic).

Superradiance is thus inherently associated to the space–time distortion produced by rotating BHs. Finding observational evidence for this process would then provide an extremely important test of Einstein's general relativity and a unique insight into the properties of the most compact objects in the universe.

Pulsar-BH binaries have long been regarded as the 'holy grail' for testing the fundamental theory of gravity, combining one of the most accurate 'clocks' in the universe with the strong space–time warping produced by a BH. Finding these systems is, in fact, one of the key goals of the Square Kilometer Array [7], as well as for current gravitational wave observatories such as Advanced LIGO/VIRGO [8], where detection of up to a few binary mergers per year may be expected [9].

In this Letter, we show that the radiation emitted by a pulsar can undergo superradiant scattering off a rotating BH companion, constituting the first example of an astrophysical system where observational evidence for BH superradiance can be found.

We begin by considering the angular velocity of a Kerr BH of mass M and spin $J = aMc$, which is given by:

$$\Omega = \frac{ac}{r_+^2 + a^2} \simeq 10\ \text{kHz} \left(\frac{10M_\odot}{M}\right)\left(\frac{\tilde{a}}{1+\sqrt{1-\tilde{a}^2}}\right) , \tag{1}$$

where $r_+ = (GM/c^2)(1+\sqrt{1-\tilde{a}^2})$ denotes the radial position of the outer horizon and $\tilde{a} = Jc/GM^2$ is the dimensionless spin parameter. This implies that, for stellar mass BHs in the range $10M_\odot$–$30M_\odot$, superradiant amplification can only be significant for waves below the 1 kHz–10 kHz frequency range, particularly since it is suppressed for large multipole numbers l and m [6].

Such low frequency radiation is naturally produced by pulsars. These rapidly spinning neutron stars harbor extremely large magnetic fields formed in the collapse of the parent star. For ordinary pulsars, these can reach 10^8–10^9 T, and even larger values for magnetars. This makes pulsars powerful emitters of magnetic dipole radiation at the rotation frequency $\omega_p \sim 1$ Hz–10 kHz, the upper bound corresponding to the rapidly spinning millisecond pulsars (see e.g. [10]).

In addition, spinning neutron stars may also be an important source of gravitational waves due to deviations from axial symmetry. Several sources of asymmetry have been discussed in the literature, such as anisotropic stresses supported by the neutron star's solid crust, a misalignment between the rotation axis and the principal axis of inertia as a result of its violent formation,

E-mail address: joao.rosa@ua.pt.

[1] Also at Departamento de Física e Astronomia, Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal.

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or a misalignment between the rotation and magnetic axes (see e.g. [11]). Due to its quadrupole nature, gravitational radiation is generically emitted at both the pulsar's angular frequency and twice its value, in amounts depending on the structure of the deformation.

It is thus clear that pulsars can emit both electromagnetic (EM) and gravitational waves (GWs) with frequency below the threshold for superradiant scattering, potentially providing two different channels for astrophysical observations. The question that remains to be addressed is whether this radiation has the correct multipolar character to extract energy and spin from a BH companion.

Let us first review the basic features of EM and GW scattering in the Kerr spacetime. A convenient framework to study these processes is the Newman–Penrose (NP) formalism [12], where one projects the Maxwell and Weyl tensors describing the waves along a null tetrad. For the Kerr spacetime, a useful choice is the Kinnersley tetrad [13], where two of the 4-vectors coincide with the principal null directions of the Weyl tensor. This allows one to construct complex scalar functions, Υ_s, known as the NP scalars and which encode the different spin components of the EM ($s=\pm1$) and GW ($s=\pm2$) radiation.

These quantities satisfy independent scalar wave equations that can be solved using separation of variables [3]. In particular, they admit a generic mode expansion in Boyer–Lindquist coordinates (t,r,θ,ϕ) [14] of the form:

$$\Upsilon_s=\sum_{l,m,\omega}e^{-i\omega t}e^{im\phi}{}_sS_{lm}(\theta)_sR_{lm}(r)+(\omega\rightarrow-\omega) \tag{2}$$

For simplicity, we will first focus on modes of the form $e^{-i\omega t}$ and discuss the $e^{i\omega t}$ modes below. The angular functions ${}_sS_{lm}(\theta)e^{im\phi}$ correspond to spin-weighted spheroidal harmonics, which reduce to the well-known spin-weighted spherical harmonics ${}_sY_{lm}(\theta,\phi)$ [15] for $a\omega\ll1$. The problem then reduces to a single radial wave equation, known as the Teukolsky equation [3]:

$$\begin{aligned}\Delta\frac{d^2{}_sR_{lm}}{dr^2}&+2(s+1)(r-M)\frac{d_sR_{lm}}{dr}\\&+\left(\frac{K^2-2is(r-M)K}{\Delta}+4is\omega r-{}_sA_{lm}\right){}_sR_{lm}=0\,,\end{aligned} \tag{3}$$

where, in units such that $G=c=\hbar=1$, $K(r)=(r^2+a^2)\omega-ma$, $\Delta=r^2-2Mr+a^2$ and the angular eigenvalue can be written as a series expansion ${}_sA_{lm}=l(l+1)-s(s+1)+\sum_{k=1}^{+\infty}c_k(a\omega)^k$ [16].

Although no exact solution of this equation is known, approximate solutions can be found for low frequencies, $\omega\ll r_+^{-1}$, in two overlapping regions. These "near" and "far" solutions, valid for $r-r_+\ll\omega^{-1}$, and $r-r_+\gg r_+$, respectively, can then be matched to produce a complete solution. We then obtain a relation between the incoming and outgoing radiation, and consequently the energy flowing into the BH horizon. This is encoded in the gain/loss factor [2]:

$$\begin{aligned}{}_sZ_{lm}&\equiv\frac{dE_{out}/dt}{dE_{in}/dt}-1=-\frac{dE_{BH}/dt}{dE_{in}/dt}\\&\simeq-\varpi\frac{2}{\epsilon}(4\omega M\epsilon)^{2l+1}C_{ls}^2\prod_{k=1}^{l}\left(k^2+\frac{4\varpi^2}{\epsilon^2}\right)\,,\end{aligned} \tag{4}$$

where $\varpi=(\omega-m\Omega)r_+$, $\epsilon=(r_+-r_-)/2M$ and $C_{ls}=(l+s)!(l-s)!/(2l)!(2l+1)!$. We thus conclude that superradiant scattering, ${}_sZ_{lm}>0$, occurs when $\omega<m\Omega$ for co-rotating wave modes, $m>0$. Also, amplification is larger for the lowest multipoles and, since $l\geq|s|$, it is more pronounced in the EM case (at low frequencies).

We note that ${}_sZ_{lm}(\omega)={}_sZ_{l,-m}(-\omega)$ for arbitrary frequencies [3], so that superradiance will occur for $m<0$ for the wave modes of the form $e^{i\omega t}$.

Superradiant amplification is, however, much larger for higher frequencies, in particular for near-extremal BHs ($\tilde{a}\simeq1$). In this regime, we need to employ numerical methods to solve the Teukolsky equation. A simple procedure, first used in [4], consists in numerically integrating an ingoing wave solution at the horizon up to a large distance, where one can extract the coefficients of the incoming and outgoing radiation, as we describe in detail in a companion article [17].

For GWs, we obtain a maximum amplification $Z^{max}\simeq1.02$ for the $l=m=2$ mode with $\tilde{a}=0.999$ and $\omega\simeq2\Omega$. In the EM case, the maximum gain is considerably lower, $Z^{max}\simeq0.044$, for $l=m=1$, in agreement with [4]. For non-superradiant modes, $Z<0$ and $|Z|$ increases with the frequency. In particular, the lowest non-superradiant multipoles approach the maximal absorption limit, $Z\simeq-1$, for $\omega<m\Omega$. In addition, for a given frequency, higher multipoles exhibit progressively smaller gain/loss factors, such that only a finite number of modes will effectively be relevant in a scattering problem (see also [18]).

A physical wave, described by real fields, generically contains both $e^{\pm i\omega t}$ frequency modes, as well as different (l,m) multipoles. Whether an overall amplification of the signal occurs upon scattering off a Kerr BH then depends on the relative amplitude of the different incoming modes, which we will now determine for the pulsar-BH binary.

We consider the limit where the orbital distance is large, $d\gg\lambda\gtrsim r_+,R_p$, where λ is the wavelength of the radiation and R_p the neutron star's radius. In this limit, it is a good approximation to consider the electromagnetic and gravitational fields generated by the rotating neutron star in flat space, which then give the incident wave for the scattering problem.

Since in this limit the orbital period will largely exceed the pulsar's rotational period, we may first study the scattering problem for a given pulsar position (d,θ_p,ϕ_p) in the BH frame and then include the orbital variation of these parameters.

It is also a sufficiently good approximation to treat the pulsar as a point-like spinning magnetic dipole and mass quadrupole. Here we will focus on binary systems where the pulsar and BH spins are either aligned or anti-aligned, although the same method can be applied to arbitrary configurations.

For a misalignment angle α between the pulsar's spin and its magnetic dipole moment, we have:

$$\ddot{\mathbf{M}}_p=\frac{1}{2}M_0\omega_p^2\sin\alpha e^{-i\omega_pt}\left[\mathbf{e}_x\pm i\mathbf{e}_y\right]+\text{c.c.}\,, \tag{5}$$

in the Cartesian coordinates associated with the Boyer–Lindquist frame, with the upper (lower) sign corresponding to an aligned (anti-aligned) binary system. We will also consider the case where any deformation rotates with the pulsar, such that its quadrupole moment corresponds to that of a deformed ellipsoid with principal axis of inertia at an angle β from the spin axis. The resulting quadrupole tensor includes two frequency components, as anticipated above, yielding [11]:

$$\begin{aligned}\frac{\ddot{Q}_{ij}}{Q_0}=&\frac{c_\beta}{2}e^{-i\omega_pt}\begin{pmatrix}0&0&\pm i\\0&0&-1\\\pm i&-1&0\end{pmatrix}\\&+s_\beta e^{-2i\omega_pt}\begin{pmatrix}1&\pm i&0\\\pm i&-1&0\\0&0&0\end{pmatrix}+\text{c.c.}\end{aligned} \tag{6}$$

where $c_\beta\equiv\cos\beta$ and $s_\beta\equiv\sin\beta$. The amplitudes M_0 and Q_0 can be obtained from the properties of the neutron star but will not