Physics Letters B 749 (2015) 425–430

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

How likely are constituent quanta to initiate inflation?

Lasha Berezhiani *, Mark Trodden

Center for Particle Cosmology, Department of Physics & Astronomy, University of Pennsylvania, 209 South 33rd Street, Philadelphia, PA 19104, United States

ARTICLE INFO

Article history:
Received 8 May 2015
Received in revised form 22 July 2015
Accepted 3 August 2015
Available online 6 August 2015
Editor: S. Dodelson

ABSTRACT

We propose an intuitive framework for studying the problem of initial conditions in slow-roll inflation. In particular, we consider a universe at high, but sub-Planckian energy density and analyze the circumstances under which it is plausible for it to become dominated by inflated patches at late times, without appealing to the idea of self-reproduction. Our approach is based on defining a prior probability distribution for the constituent quanta of the pre-inflationary universe. To test the idea that inflation can begin under very generic circumstances, we make specific – yet quite general and well grounded – assumptions on the prior distribution. As a result, we are led to the conclusion that the probability for a given region to ignite inflation at sub-Planckian densities is extremely small. Furthermore, if one chooses to use the enormous volume factor that inflation yields as an appropriate measure, we find that the regions of the universe which started inflating at densities below the self-reproductive threshold nevertheless occupy a negligible physical volume in the present universe as compared to those domains that have never inflated.

1. Introduction

According to the standard lore, inflation is capable of producing the entire observable universe from a small homogeneous domain [1–3]. If one postulates the inflating universe to emerge from the space–time foam at the Planck scale, as happens in a number of scenarios, then it is unclear how to discuss the likelihood of this process. However, in scenarios in which it is reasonable to consider the non-inflating universe at sub-Planckian energies, we may investigate the circumstances under which it is likely for the universe to become dominated by inflating patches.

This question has prompted a great deal of effort devoted to understanding the question of initial conditions [4–22]. While it is possible that homogeneous domains capable of seeding inflation may be rare in the early universe, it is often argued that the exponential increase in volume that occurs during the inflationary phase may nevertheless result in the physical volume of the late time universe being dominated by patches that underwent inflation. Moreover, if self-reproduction – *eternal inflation* – occurs, then most of the universe would be inflating even today [6,23], although there remain open questions about this interesting possibility [24,25].

In this letter, we would like to take a different approach to the problem of quantifying the likelihood of slow-roll inflation without appealing to the tantalizing idea of eternal inflation. Inspired by the recently proposed corpuscular description of the inflationary background [24,26], we propose describing the pre-inflationary universe as a collection of quanta with a certain probability distribution for their wavelengths. As we shall see, this provides considerable constraints on the initial conditions problem of inflation.

The purpose of this work is not to give an account for the origin of the universe; in fact, we are agnostic about this. Instead, we are simply investigating the idea that inflation may begin under very generic circumstances under a set of favorable assumptions about the early universe. In this sense, this work is in the spirit of [10], but approaches the question from the particle physics perspective rather than the general relativistic one. We carry this out by estimating the upper bound on the probability for a given region in the early universe to begin inflating, assuming that certain conditions are met. In particular, our main assumption is that, in a smooth region at sub-Planckian energy density, most of the quanta are expected to have momenta comparable to the energy scale that sets the local energy density. We then use this bound to find out whether most of the physical volume of the late time universe could have originated from inflation, assuming that inflated domains are rewarded by the exponential volume factor relative to the non-inflated ones.

We also emphasize that this work is not an attempt to solve the cosmological measure problem or to promote one proposed measure over any others. We are interested in the question of how

* Corresponding author.
E-mail address: lb1418@nyu.edu (L. Berezhiani).

http://dx.doi.org/10.1016/j.physletb.2015.08.007

likely is it for inflation to begin in a high energy (but in the calculable regime) universe, with our assumptions. We could imagine that, when all the relevant physics is understood, and the measure problem solved, it could be that the conditional probability – given that we observe a universe like ours today, what is the likelihood that it arose from a previously inflating patch – is dominated by these inflated patches. We certainly are not in a position to rule this out. But this is, however, a question we do not know how to answer. We nevertheless think the question we are answering is interesting. Here rather, we merely *assume* the globally defined volume measure, since it a particularly simple one and is frequently invoked in favor of inflation. One could, in principle, carry out a similar analysis to the one here for any other choice of measure should one wish.

2. The puzzle

We will phrase the question we wish to address in the context of the simplest model of inflation, focusing on a single massive scalar field, without self-interactions, minimally coupled to gravity, and ignoring all but gravitational and inflaton degrees of freedom. The action is therefore

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2\right), \qquad (2.1)$$

where we have defined the reduced Planck mass via Newton's constant G_N by $8\pi G_N = M_{\rm pl}^{-2}$, and we require $M_{\rm pl} \gg M \gg m$, with M denoting the scale of inflation.

In order to seed inflation, the scalar field must assume an approximately homogeneous background form $\varphi(t)$ at least over few Hubble distances, where the Hubble scale is related to the energy density stored in the inflaton through the Friedmann equation

$$H^2 = \frac{\rho}{3M_{\rm pl}^2}\,. \qquad (2.2)$$

The question we would like to ask is whether it is natural to have such a homogeneous background value of the inflaton over the Hubble patch.

Our approach is to consider the background as a superposition of modes of given wavelengths. In this language, the requirement that the background be homogeneous over a Hubble patch implies that the constituent quanta have super-horizon wavelengths. As a result, the puzzle can be formulated as follows. In order to start inflation at the energy scale $M \equiv \rho^{1/4} \ll M_{\rm pl}$, the corresponding Hubble patch must be filled with modes of characteristic wavenumber $k < H$. On the other hand, this appears to be puzzling since it is unclear why a given mode would be much softer than the scale M that sets the local energy density. Furthermore, inflation lasts as long as the slow-roll condition is satisfied

$$\frac{m^2}{3H^2} \ll 1\,. \qquad (2.3)$$

Consequently, we must require a hierarchy $m \ll H \ll M$ for accelerated expansion. This means that most regions at such a high energy density may be dominated by sub-horizon modes. In other words, it may be more natural to assume that a given patch of size H^{-1} at energy density $\rho = M^4$ is behaving as a radiation dominated universe rather than an inflating one. Once modes start out with sub-horizon wavelengths, driving decelerated expansion, they will never be able to exit the horizon.

The following remark is in order here. Sometimes it is argued that even if the energy stored in super-horizon modes is subdominant at the beginning, it may become relevant in time. This could happen since the amplitude of these modes is frozen, while sub-horizon modes decay. However, those quanta that are nearly of the horizon scale will soon reenter the horizon and will themselves decay. As for the modes that are much longer than the curvature scale, their homogeneity will be spoiled quickly, unless the background is fine-tuned to be homogeneous over distances longer than their wavelength. This is an important point and its more detailed analysis will be presented elsewhere.

Notice that if M were of order of $M_{\rm pl}$ then H, M and $M_{\rm pl}$ would all be of the same order, and that therefore, in this case, homogeneity over the Hubble patch is not as unnatural as it might be at sub-Planckian energies.

The goal of the remainder of this paper is to quantify the above mentioned unnaturalness.

3. Estimating probabilities

Let us consider the theory (2.1) of gravitons and inflatons only. The pre-inflationary state of the universe can be characterized in many different ways, but we would like to think of it in terms of the constituent quanta. Clearly, in the quantum world, all degrees of freedom need to be quantized. Moreover, the state needs to be consistent with quantum constraints of General Relativity. In order to simplify the analysis, we make a mitigating assumption about gravitons; namely, given that inflatons are in the desired state, we assume the quantum state of gravitons to be fixed by constrains in a way that prefers the beginning of inflation. Independently of what was happening before inflation, which we assume to start at a scale $M \ll M_{\rm pl}$, the pre-inflationary processes should have provided us with a region filled with φ-quanta. Otherwise, the region in question would not have the potential to inflate.

In order for a given region of size ℓ to inflate, the energy density of φ-quanta with wavelength $\lambda > \ell$ must be sufficient to source a curvature $H^2 \geq \ell^{-2}$. This sets a lower bound on the number of long modes in the region under consideration. In particular, if we denote the average wavenumber as $\langle k\rangle$, then the total energy density of N such quanta reduces to $N\langle k\rangle/\ell^3$. As a result, the expression for the curvature sourced by the long modes becomes

$$H^2 = \frac{N\langle k\rangle}{3\ell^3 M_{\rm pl}^2}\,. \qquad (3.1)$$

Therefore, the critical number of φ-quanta necessary to drive inflation in a region of size ℓ is given by

$$N \geq N_c \equiv \frac{3M_{\rm pl}^2\ell}{\langle k\rangle}\,. \qquad (3.2)$$

To estimate the probability of inflation we need to be more specific about the initial state. In order to challenge the idea that inflation can begin efficiently under very generic circumstances, we assume that the most likely momentum of a given particle is comparable to the energy scale that sets the local energy density. Moreover, we assume that the probability for a mode to be much softer than this energy scale is significantly smaller than unity. The validity of this assumption depends on the production mechanism of φ-quanta. For instance, we could think of the modes in question as the byproduct of the processes with sub-Planckian momentum transfer. In this case, we would expect them to have momenta comparable to the energy scale at which they were produced. Therefore, we propose to consider the wavenumber of a mode to be given by a normalized probability distribution $f(k)$, with the following properties

1. $f(0) = 0$. This assumption is not vital for our argument, as we already mentioned above, all we really need to assume is that