Physics Letters B 749 (2015) 547–550

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Pion valence-quark parton distribution function

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ARTICLE INFO

Article history:
Received 30 November 2014
Received in revised form 27 July 2015
Accepted 13 August 2015
Available online 21 August 2015
Editor: A. Ringwald

Keywords:
Dynamical chiral symmetry breaking
Dyson–Schwinger equations
π-meson
Parton distribution functions

ABSTRACT

Within the Dyson–Schwinger equation formulation of QCD, a rainbow ladder truncation is used to calculate the pion valence-quark distribution function (PDF). The gap equation is renormalized at a typical hadronic scale, of order 0.5 GeV, which is also set as the default initial scale for the pion PDF. We implement a corrected leading-order expression for the PDF which ensures that the valence-quarks carry all of the pion's light-front momentum at the initial scale. The scaling behavior of the pion PDF at a typical partonic scale of order 5.2 GeV is found to be $(1-x)^{\nu}$, with $\nu \simeq 1.6$, as x approaches one.

Given its dual roles as a conventional bound-state in quantum field theory and as the Goldstone mode associated with dynamical chiral symmetry breaking, the pion has been proven critical to explaining phenomena as diverse as the long-range nucleon–nucleon interactions and the flavor asymmetry observed in the quark sea of the nucleon [1]. The study of the pion structure function is of great interest as a fundamental test of our understanding of nonperturbative QCD. Experimental information on the parton distribution function (PDF) in the pion has primarily been inferred from the Drell–Yan reaction in pion–nucleon collisions [2–5].

Lattice QCD calculations [6–8] have traditionally been able to yield only the low-order moments of the PDFs. While there has been a recent suggestion of a very promising way [9] to directly compute the x-dependence in lattice QCD, it will take considerable effort to reliably extract the large-x behavior using this method. The calculation of PDFs within models is challenging and various models have given a diversity of results. Most models, including the QCD parton model [10], pQCD [11] and the Dyson–Schwinger equations (DSE) [12,13] indicate that at high-x the PDF should behave as $(1-x)^{\alpha}$, with $\alpha \simeq 2$. The Nambu–Jona–Lasinio (NJL) models [14] with translationally invariant regularization and Drell–Yan–West relation [15] favors a linear dependence on $1-x$.

The first DSE study of the pion PDF was based [12] upon an analysis that employed phenomenological parametrizations of both the Bethe–Salpeter amplitude and the dressed-quark propagators. A numerical solution of the DSE utilizing the rainbow-ladder (RL) truncation has been used to compute the pion and kaon PDFs following same line [13].

In this work we revisit the pion valence PDF within the DSE approach, with the following improvements: 1) the rainbow-ladder gap equation is renormalized at a typical hadron scale, ζ_H, that also serves as the initial scale for the PDF; 2) a corrected leading-order expression for the PDF is employed within the RL trunction; 3) the extraction of the PDF is based on its moments, a method that has been widely used in parton distribution amplitude calculations [16,18,19]. The large-x behavior is naturally reflected in the high moments. In the method used here, we can calculate any large moment and thus we have a reliable tool with which to analyze the large-x behavior.

In order to help explain the numerical results and place them in some perspective, we introduce several models which produce pointwise PDFs. Our suggestions cover a broad range of possibilities, against which the predictions of the present model may be compared, especially calculations that can be described within the amplitude language, such as the DSE and NJL models with various regularization frameworks.

In Ref. [17] a corrected, leading-order expression was given for the pion's valence-quark PDF. This expression produces the model-independent result that quarks dressed via the RL truncation carry all of the pion's light-front momentum at a characteristic hadronic scale, if the meson amplitude is momentum dependent. We quote the form of the quark distribution function in the RL truncation here:

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http://dx.doi.org/10.1016/j.physletb.2015.08.036

$$q(x) = N_c \mathrm{tr} \int_{dk}^{\Lambda} \delta(n \cdot k - xn \cdot P)\, \partial_k \left[\Gamma(k - \frac{P}{2}; -P) S(k) \right]$$
$$\Gamma(k - \frac{P}{2}; P) S(k - P) \,. \tag{1}$$

In the infinite momentum frame, $q(x)$ is the number density for a single parton of flavor q to carry the momentum fraction $x = n \cdot k / n \cdot P$, which is positive definite over the physical region $0 < x < 1$. Here, n is a light-like four-vector, $n^2 = 0$; P is the pion's four-momentum, $P^2 = -m_\pi^2$, with m_π the pion mass; $\int_{dk}^{\Lambda}$ is a Poincaré-invariant regularization of the four-dimensional momentum integral (over k), with Λ the ultraviolet regularization mass-scale. In addition, S and Γ are the quark propagator and pion Bethe–Salpeter amplitude, respectively. In the present work the ultraviolet behavior of S and Γ is controlled by the one-gluon exchange interaction. In this case the above integral is ultraviolet divergence free and Λ can be set to infinity safely.

As the derivative in Eq. (1) acts on the full expression within the brackets it naturally yields two terms. The term related to the derivative of the quark propagator yields the so-called impulse-approximation. That corresponds to the textbook "handbag" contribution to virtual Compton scattering. The second term, arising from the action of the derivative on the amplitude originates in the initial/final state interactions. This expression is the minimal expression that retains the contribution to the quark distribution function from the gluons which bind dressed-quarks into the meson. This contribution may be thought of as a natural consequence of the nonlocal properties of the pion wave function. That is, it expresses the process where a photon is absorbed by a dressed quark, which then proceeds to become part of the pion bound-state before re-emitting the photon. It is easy to prove that the distribution function is symmetric, $q(x) = q(1 - x)$, under isospin symmetry and the valence quarks carry all of the momentum of the meson.

We describe pion as bound state using the Bethe–Salpeter equation. This takes the abbreviated form:

$$\Gamma_\pi(k; P) = \int_{dq}^{\Lambda} K(q, k; P) \chi_\pi(q; P) \tag{2}$$

where q and k are the relative momenta between the quark–antiquark pair, P is the pion's four momentum and

$$\chi_\pi(q; P) = S(q_+) \Gamma_\pi(q; P) S(q_-) \tag{3}$$

is the pion's Poincaré-covariant Bethe–Salpeter wave-function, with Γ_π the Bethe–Salpeter amplitude. Using isospin symmetry we label the dressed quark propagators $S(q_\pm)$, where $q_\pm = q \pm \frac{P}{2}$, without loss of generality. Explicitly, these take the form:

$$S^{-1}(k) = i\gamma \cdot k A(k^2) + B(k^2) \tag{4}$$

where the scalar functions A, B depend on both momentum and the choice of renormalization point.

In this work, we perform the ladder truncation for the quark–antiquark scattering kernel, $K(q, k; P)$. This approximation has been widely used to compute the spectrum of meson bound states and related properties. In this framework the quark–gluon vertex is bare and a judicious choice of effective gluon propagator provides a connection between the infrared and ultraviolet scales. We use the interaction provided in Ref. [20], which contains two different parts. Its ultraviolet composition preserves the one-loop renormalization group behavior of QCD so that, as we shall see, the leading Bethe–Salpeter amplitude takes the well known, model independent ultraviolet behavior. The parameters of the infrared interaction, $D\omega$ and ω, manifest the strength and width of the interaction, respectively. It is chosen deliberately to be consistent with that determined in modern studies of the gauge sector of QCD.

The rainbow ladder truncation of the DSEs preserves the chiral symmetry of QCD. The renormalization constants for the wave function and mass function $Z_{2,4}(\zeta, \Lambda)$ must be included to regularize the logarithmic ultraviolet divergences. In the present calculation we follow the current quark mass independent renormalization approach introduced in Ref. [21]. In practice, the renormalization is defined by the conditions $A(k = \zeta) = 1$ and $\frac{\partial B(k=\zeta)}{\partial m_\zeta} = 1$ in the chiral limit. It should be noted that the renormalization point can be chosen in either the ultraviolet or infrared region and the quark mass function is independent of this choice. The renormalization constants $Z_{2,4}(\zeta, \Lambda)$ decrease as the scale decreases, reflecting the increase in the coupling strength in the infrared region. Here we choose $\zeta = 0.5$ GeV.

Before discussing any numerical results, it is interesting to recall some general features of the shape of the PDF and the Bethe–Salpeter amplitude for a meson. It has been shown that the significant features of $q(x)$, in Eq. (1), can be illustrated algebraically with some simple models. To take a close look at the relation between $q(x)$ and the pion Bethe–Salpeter amplitude, we consider an algebraic model where the quark propagator and the meson amplitude take of constituent mass form [16]

$$S^{-1}(k) = i\gamma \cdot k + M \tag{5}$$

and

$$\Gamma_\pi(k; P) = i\gamma_5 \frac{12}{5} \frac{M}{f_\pi} \int_{-1}^{1} dz \rho(z) \frac{M^2}{k^2 + zk \cdot P + M^2} \,, \tag{6}$$

where M is a dressed-quark mass, f_π is the pion decay constant and we focus on the case of a massless pion. The factor 12/5 is the normalization constant needed to ensure the charge conservation. k is the relative momentum of the quarks in the pion and we choose the amplitude to behave like $1/k^2$ asymptotically, as this is the leading order result if one takes a one-gluon exchange interaction between the quark and antiquark. We introduce the Nakanishi-like representation [22] with weight function $\rho(z)$, which takes a form different from that considered in Ref. [16]. We will see that different choices of ρ lead to different behaviors of $q(x)$. The present algebraic model makes it possible for us to determine the x-dependence of the PDF.

In Ref. [16] it has been shown that $\rho(z) = \frac{1}{2}(\delta(1 - z) + \delta(1 + z))$ describes a bound state with point-particle-like characteristics. It should be noted that it also gives a constant PDF, if one calculates the PDF exactly, even though the Bethe–Salpeter amplitude is momentum dependent. We infer that such behavior corresponds to the NJL prediction if one performs a Pauli–Villas regularization.

The QCD conformal limit can be reproduced with the weight function $\rho(z) = \frac{3}{4}(1 - z^2)$. Ref. [17] deduced a PDF which can be approximately expressed by $30x^2(1 - x)^2$. Following this line, we extend the model of spectral density as $\rho(z) = \frac{3}{4}(1 - z^2)\big(1 + 6a_2 C_2^{3/2}(z)\big)$, where a second Gegenbauer polynomial has been introduced and a_2 is a parameter. The corresponding PDA has the form $\varphi(x) = 6x(1 - x)\big(1 + a_2 C_2^{3/2}(2x - 1)\big)$. Obviously this form reproduces the Chernyak–Zhitnitsky (CZ) form, with $a_2 = 2/3$ [23]. Following the method in Ref. [17], the PDF related to the CZ PDA can be computed consistently. The result is depicted in Fig. 1. Near $x = 1$ this model has the power-law behavior, $\beta(1 - x)^2$, predicted by the QCD parton model. Here a_2 only affects the coefficient β, not the power. However, the PDF shows oscillatory behavior that