Physics Letters B 749 (2015) 592–596

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Probing the quark–gluon interaction with hadrons

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ARTICLE INFO

*Article history:*
Received 23 June 2015
Received in revised form 31 August 2015
Accepted 31 August 2015
Available online 2 September 2015
Editor: W. Haxton

*Keywords:*
Quark–gluon vertex
Baryons
Mesons
QCD's Green's functions

ABSTRACT

We present a unified picture of mesons and baryons in the Dyson–Schwinger/Bethe–Salpeter approach, wherein the quark–gluon and quark–(anti)quark interactions follow from a systematic truncation of the QCD effective action and include all its tensor structures. The masses of some of the ground-state mesons and baryons are found to be in reasonable agreement with the expectations of a 'quark-core calculation', suggesting a partial insensitivity to the details of the quark–gluon interaction. However, discrepancies remain in the meson sector, and for excited baryons, that suggest higher order corrections are relevant and should be investigated following the methods outlined herein.



## 1. Introduction

Hadrons provide a rich experimental environment for the study of the strong interaction, from details of the resonance spectrum to form factors and transition decays via electromagnetic probes. These reflect the underlying substructure of bound states by resolving, in a non-trivial way, the quarks and gluons of which they are composed. A theoretical understanding of hadrons in terms of these underlying degrees of freedom, interacting as dictated by quantum chromodynamics (QCD), is an on-going effort. Many approaches tackle it in different ways, simplifying certain aspects of the theory. Probing sensibly our theoretical constructs with experimental input thus provides understanding of the theory itself.

In continuum approaches to QCD, it is not possible in general to include all possible correlation functions in a calculation, as there are infinitely many of them. Although this can be viewed as a limitation of continuum approaches, only a finite number of these correlation functions have a significant role in the observable properties of hadrons. Therefore, by including a greater number of relevant correlation functions into the system, continuum methods provide an ideal framework to unravel the underlying mechanisms that generate observable effects from the elementary and non-observable degrees of freedom of QCD. This is in contrast to lattice QCD calculations, which can be viewed as theoretical experiments in the sense that, although they contain *a priori* all the dynamics of QCD, it is challenging to single out individual contributions to a particular measurement. This makes these two approaches complementary.

Amongst the different continuum approaches, the combination of Dyson–Schwinger (DSE) and Bethe–Salpeter equations (BSE) has proven to be extremely useful in the calculation of hadronic properties from QCD [1–3]. Typically, solutions of DSEs constitute the building blocks (propagators and vertices) of bound-state calculations using BSEs, which provide the bridge between QCD and observables. As described in more detail below, the interaction terms that are kept in the DSE determine the interaction kernels among constituents in the BSEs, thereby defining a particular truncation of the DSE/BSE system. One works towards a model-independent truncation by including a larger set of interaction terms; although this programme is obviously not achievable in its totality, it is expected that there will be some degree of convergence at the level of this vertex expansion.

Here we focus upon the inclusion of the quark–gluon interaction, the reliable construction of which is a challenging task. However, one is guided by various symmetries – notably that of chiral symmetry – that provide for stringent constraints. To implement these symmetries at the level of the quark and gluon interaction, simplifications are clearly necessary which typically fall into three categories: (i) the quark–gluon vertex is truncated to its tree-level component times a momentum-dependent effective coupling with the quark DSE and hadron BSEs solved self-consistently [4,5]; (ii) a more sophisticated model for the quark–gluon vertex is used, with the contribution from its different tensor structures modelled,

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http://dx.doi.org/10.1016/j.physletb.2015.08.067

hence abandoning self-consistency but gaining instead flexible insight into the relative importance of each of these structures [6–9]; (iii) some non-perturbative effects of the quark and gluon interaction are taken into account by solving the quark–gluon vertex DSE self-consistently, but potentially introducing some truncation artifacts [10–12]. We follow here the latter approach, since as we demonstrate in this letter it enables the controlled inclusion of interaction mechanisms based on a loop expansion of the effective action.

The effective action is a generating functional for proper vertex functions, and may be considered as a means to define the quantum field theory given an action, since all necessary Green's functions can be derived from it. Related are the n-particle irreducible (nPI) effective actions that form a family of different, but equivalent, representations of the same generating functional (see, *e.g.* [13]). They are defined as functionals of all $m \leq n$ Green's functions of the theory (fields, propagators, vertices, etc.). Although its exact form is not known in general, its loop expansion in $\hbar$ is well-defined [13] and can in practice be performed. Moreover, each term in the expansion already captures both perturbative and non-perturbative physics.

One reason that makes nPI techniques a powerful tool is that they provide a natural link between bound-state equations described by BSEs and the propagators and vertices provided by DSEs [14–18]. A truncation of the loop expansion at a certain order translates into a unique prescription for the truncation of the DSEs and the BSEs that maintains symmetries. For the study of two- and three-body states, it suffices to use either the 2PI effective action, which is defined in terms of fully-dressed propagators but bare vertices [19], or the 3PI effective action, which is defined also in terms of the fully-dressed vertices. In this work we restrict ourselves to the 2PI case and defer the use of the 3PI effective action to a future and more comprehensive study.

In connection with the three categories outlined above, it is worth mentioning here that the somewhat hybrid possibility of supplementing some of QCDs degrees of freedom in favor of effective ones, such as pions, has also been explored [20–25]. These can be viewed as approximate representations of the four-quark vertex in the 4PI formalism that introduces at the first step decay channels and a mixing with tetraquark states.

In the present work, we incorporate the results of a recent study of the quark–gluon vertex from a truncated DSE [12] in the calculation of meson and baryon masses. That truncation can be interpreted in the context of the 2PI effective action at 3-loop. Although on the technical side this is no novelty for meson calculations [11,26], it is the first time that corrections incorporating the gluon self-interaction have been included in the covariant three-body baryon calculation. While there exist other recent investigations of the quark–gluon vertex [27–29], these have not yet been confronted with the challenge of reproducing hadron phenomena for reasons we discuss below.

Finally, we wish to stress that this work represents only the first step in an on-going effort to incorporate realistic QCD's Green's functions into the self-consistent study of bound states. Not surprisingly, the low-order of the truncation used performs only as well as simple phenomenological models such as rainbow-ladder. However, but most importantly, it serves as a proof of principle that such an endeavor is feasible, as further increasing the order of the truncation does not increase the technical complexity dramatically.

## 2. Framework

The starting point for the study of hadronic observables in the present framework is thus the effective action

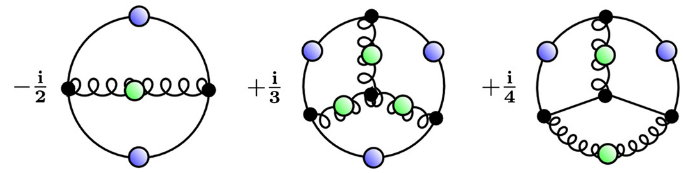


**Fig. 1.** The 2-particle irreducible term $\Gamma_2[\Psi, G]$ in the definition of the effective action, up to three loops.

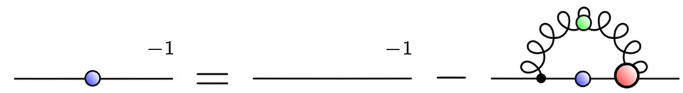


**Fig. 2.** The Dyson–Schwinger equation for the quark propagator.

$$\Gamma[\Psi, G] = S[\Psi] + i\,\mathrm{Tr}\log G - i\,\mathrm{Tr}\,G_0^{-1}G + \Gamma_2[\Psi, G]\,, \qquad (1)$$

where $S$ is the classical action, and $\Psi$ and $G$ collectively represent the fields and full propagators of QCD, respectively. The term $\Gamma_2[\Psi, G]$ contains two-particle irreducible diagrams only and $G_0$ denotes the classical propagators. To proceed, we perform a loop expansion of $\Gamma_2[\Psi, G]$ to three-loop order, as shown in Fig. 1. Moreover, we keep only the non-Abelian term that connects the gauge to the matter sector and neglect the Abelian correction (third diagram), as it is expected to be subleading in the large-$N_c$ limit; whether this is indeed the case for the description of hadron phenomena must certainly be tested and will be the subject of future work.

The next basic element in meson and baryon calculations is the fully-dressed quark propagator. It is given by the quark DSE, see Fig. 2

$$S^{-1}(p;\mu) = Z_2 S_0^{-1}(p) + \Sigma(p;\mu)\,, \qquad (2)$$

with quark self-energy

$$\Sigma(p;\mu) = g^2 Z_{1f} C_F \int_k \gamma^\mu S(q)\Gamma^\nu(q,p)D_{\mu\nu}(k)\,. \qquad (3)$$

Here $q = k + p$, the integral measure is $\int_k = \int d^4k/(2\pi)^4$ and $Z_{1f}$, $Z_2$ are renormalization constants for the quark–gluon vertex and quark propagator respectively. It is clearly dependent upon both the gluon propagator $D_{\mu\nu}(k)$ and the quark–gluon vertex $\Gamma^\nu(q,p)$. The (Landau gauge) propagators are

$$S^{-1}(p) = Z_f^{-1}(p^2)\left(i\not{p} + M(p^2)\right)\,, \qquad (4)$$

$$D^{\mu\nu}(k) = T^{\mu\nu}_{(k)} Z(k^2)/k^2\,, \qquad (5)$$

with quark wavefunction $Z_f^{-1}(p^2)$, dynamical mass $M(p^2)$ and gluon dressing $Z(k^2)$. The transverse projector is $T^{\mu\nu}_{(k)} = \delta^{\mu\nu} - k^\mu k^\nu/k^2$. From the 2PI effective action, the quark DSE is determined via a functional derivative with respect to the quark propagator

$$\Sigma = -i\frac{\delta\Gamma_2}{\delta G}\,. \qquad (6)$$

The expansion of the effective action in Fig. 1 thus defines a truncation of the quark DSE, which is equivalent to the truncation of the quark–gluon vertex DSE shown in Fig. 3. Specifically, the truncated vertex DSE can be given as a summation of vertex corrections

$$\Gamma^\mu(l,k) = Z_{1f}\gamma^\mu + \Lambda^\mu_{\mathrm{NA}} + \ldots\,, \qquad (7)$$