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Gauge invariance and holographic renormalization

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ABSTRACT

We study the gauge invariance of physical observables in holographic theories under the local diffeomorphism. We find that gauge invariance is intimately related to the holographic renormalization: the local counter terms defined in the boundary cancel most of gauge dependences of the on-shell action as well as the divergences. There is a mismatch in the degrees of freedom between the bulk theory and the boundary one. We resolve this problem by noticing that there is a residual gauge symmetry (RGS). By extending the RGS such that it satisfies infalling boundary condition at the horizon, we can understand the problem in the context of general holographic embedding of a global symmetry at the boundary into the local gauge symmetry in the bulk.

1. Introduction

According to AdS/CFT correspondence, any global symmetry at the boundary theory is lifted to a local symmetry in the bulk [1,2]. The gauge symmetry is essential to reduce the degree of freedom which is enlarged by going into one higher dimension. The physical goal in holography is the boundary quantities which do not know the presence of higher dimension or gauge degrees of freedom, while we use the tools in the bulk theory. Therefore the gauge invariance of a physical quantity is a critical issue for the validity of the AdS/CFT. Also tracing the gauge invariance gives much intuition on the way how holography actually works, especially how global symmetry is encoded in the local gauge symmetry.

One can find gauge invariant combinations of the fields, and express the physical quantities in terms of such master variables, however, it is not always easy to find such gauge invariant combination. Even in the case they are available, it is not very convenient to use such fields, especially if many fields are coupled, because the physical quantities are defined in terms of the field variables which are formally gauge dependent. For example [2], energy momentum tensor and chemical potential are defined in terms of metric/gauge field which is not gauge invariant. Similarly, heat currents can be related to the metric perturbation defined only in a specific gauge where time period has definite relation with temperature.

In recent works [3,4], based on [5,6], we developed a systematic method to numerically calculate the Green's functions and all AC transports quantities simultaneously for the case where many fields are coupled and there are *constraints* due to gauge symmetry. Although we have tested the validity of the procedure by showing the agreement of zero frequency limits of AC conductivities with the known analytic DC conductivities [7–9] we still think that we need to prove the gauge invariance of our procedure as a matter of principle. We found that the bulk gauge invariance is intimately related to the holographic renormalization. Although the local counter terms were introduced to kill the divergences, they also kill most of gauge dependence.

Furthermore, there is a residual gauge symmetry (RGS) even after we fix the axial gauge $g_{rx} = 0$. While equations of motion can be written in terms of the gauge invariant master fields $\mathcal{P}_h, \mathcal{P}_\chi$ (3.8), it turns out that the quadratic on-shell action, the generating function for two point retarded Green's functions, cannot be written as such. However, we prove that the Green's functions are still invariant under such a symmetry.

There is a mismatch in the degrees of freedom in the bulk and those at the boundary: there are only two independent bulk solutions satisfying the in-falling boundary conditions while we need three solutions at the boundary since there are three independent source fields. The RGS is the one that resolves the problem: since it cannot satisfy a proper boundary condition, it is not a proper gauge

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symmetry but a 'solution generating symmetry'. It generate the desired solution at the boundary and therefore we should accept its bulk counter part as a new physical degree of freedom as well although it cannot satisfy the infalling boundary condition (BC). By extending the RGS such that it satisfies infalling boundary condition at the horizon, we can make the bulk solution more natural in the sense that it satisfies the infalling BC. With such solution we can also understand the problem in the context of general structure of holography, namely the correspondence between a global symmetry at the boundary and the local gauge symmetry in the bulk.

2. Action and background solution

Let us first briefly review the system we will discuss, which has been analysed in detail in [3,7,10]. The holographically renormalized action ($S_{\rm ren}$) is given by

$$S_{\rm ren} = S_{\rm EM} + S_\psi + S_c\,, \tag{2.1}$$

where

$$S_{\rm EM} = \int_M {\rm d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M}{\rm d}^3x\sqrt{-\gamma}K\,, \tag{2.2}$$

is the usual action for charged black hole in AdS space ($\Lambda < 0$) with the Gibbons–Hawking term and

$$S_\psi = \int_M {\rm d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]\,, \tag{2.3}$$

is the action for two free massless scalars added for a momentum relaxation effect. S_c is the counter term

$$S_c = \eta_c\int_{\partial M}{\rm d}x^3\sqrt{-\gamma}\left(-4 - R[\gamma] + \frac{1}{2}\sum_{I=1}^{2}\gamma^{\mu\nu}\partial_\mu\psi_I\partial_\nu\psi_I\right)\,, \tag{2.4}$$

which is included to cancel the divergence in $S_{\rm EM} + S_\psi$. Here we introduced η_c to keep track of the effect of the counter term. At the end of the computation we will set $\eta_c = 1$.

The action (2.1) yields general equations of motion[1]

$$R_{MN} = \frac{1}{2}g_{MN}\left(R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right) + \frac{1}{2}\sum_I\partial_M\psi_I\partial_N\psi_I + \frac{1}{2}F_M{}^PF_{NP}\,, \tag{2.5}$$

$$\nabla_M F^{MN} = 0\,, \qquad \nabla^2\psi_I = 0\,, \tag{2.6}$$

which admit the following solutions

$$ds^2 = G_{MN}{\rm d}x^M{\rm d}x^N = -f(r){\rm d}t^2 + \frac{{\rm d}r^2}{f(r)} + r^2\delta_{ij}{\rm d}x^i{\rm d}x^j\,, \tag{2.7}$$

$$f(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_0^2}{r^2}\,, \quad m_0 = r_0^3\left(1 + \frac{\mu^2}{4r_0^2} - \frac{\beta^2}{2r_0^2}\right)\,, \tag{2.8}$$

$$A = \mu\left(1 - \frac{r_0}{r}\right){\rm d}t\,, \tag{2.9}$$

$$\psi_I = \beta_{Ii}x^i = \beta\delta_{Ii}x^i\,. \tag{2.10}$$

These are reduced to AdS–Reissner–Nordstrom (AdS–RN) black brane solutions when $\beta = 0$. Here we have taken special β_{Ii}, which satisfies $\frac{1}{2}\sum_{I=1}^{2}\vec\beta_I\cdot\vec\beta_I = \beta^2$ for general cases.

The solutions (2.7)–(2.10) are characterized by three parameters: r_0, μ, and β. r_0 is the black brane horizon position ($f(r_0) = 0$) and can be replaced by temperature T for the dual field theory:

$$T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi}\left(3r_0 - \frac{\mu^2 + 2\beta^2}{4r_0}\right)\,. \tag{2.11}$$

Non-vanishing components of energy–momentum tensor and charge density read

$$\langle T^{tt}\rangle = 2m_0\,, \quad \langle T^{xx}\rangle = \langle T^{yy}\rangle = m_0\,, \quad \langle J^t\rangle = \mu r_0\,. \tag{2.12}$$

$\langle T^{tt}\rangle = 2\langle T^{xx}\rangle$ implies that charge carriers are still of massless character. From here we set $r_0 = 1$ not to clutter.

3. Gauge fixing and residual gauge transformation

To study electric, thermoelectric, and thermal conductivities we introduce small fluctuations around the background (2.7)–(2.10)

$$\delta A_x(t,r) = \int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)\,, \tag{3.1}$$

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}e^{-i\omega t}r^2h_{tx}(\omega,r)\,, \tag{3.2}$$

$$\delta g_{rx}(t,r) = \int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}e^{-i\omega t}r^2h_{rx}(\omega,r)\,, \tag{3.3}$$

$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}e^{-i\omega t}\chi(\omega,r)\,. \tag{3.4}$$

The fluctuations are chosen to be independent of x and y. This is allowed since all the background fields appearing in the equations of motion turn out to be independent of x and y. The gauge field fluctuation ($\delta A_x(t,r)$) sources metric ($\delta g_{tx}(t,r)$, $\delta g_{rx}(t,r)$) and scalar field ($\delta\psi_1(t,r)$) fluctuation and vice versa and all the other fluctuations are decoupled. We will work in momentum space and $h_{tx}(\omega,r)$ and $h_{rx}(\omega,r)$ is defined so that it goes to constant as r goes to infinity.

By linearizing the full equation of motion, we get four equations. However one of them can be obtained by the others. Thus we may consider following three equations:

$$(\chi' - \beta h_{rx}) - \frac{i\mu\omega a_x}{\beta r^2 f(r)} - \frac{ir^2\omega(h_{tx}' + i\omega h_{rx})}{\beta f(r)} = 0\,, \tag{3.5}$$

$$a_x''(r) + \frac{a_x'(r)f'(r)}{f(r)} + \frac{\omega^2 a_x(r)}{f(r)^2} + \frac{\mu(h_{tx}' + i\omega h_{rx})}{f(r)} = 0\,, \tag{3.6}$$

$$f(r)f'(r)(\chi'(r) - \beta h_{rx}) + f(r)^2(\chi' - \beta h_{rx})' + \frac{2f(r)^2(\chi' - \beta h_{rx})}{r} + \omega^2\chi(r) - i\beta\omega h_{tx}(r) = 0\,. \tag{3.7}$$

If we differentiate the third equation with respect to r, all equations can be written in terms of three variables, $\mathcal{P}_\chi$, $\mathcal{P}_h$, and a_x, where

$$\mathcal{P}_\chi \equiv \chi' - \beta h_{rx}\,, \qquad \mathcal{P}_h \equiv h_{tx}' + i\omega h_{rx}\,. \tag{3.8}$$

Therefore, h_{rx} is a non-dynamical degree of freedom. Indeed, $\mathcal{P}_\chi$, $\mathcal{P}_h$, and a_x are invariant under a diffeomorphism generated by

[1] Index convention: $M, N, \cdots = 0, 1, 2, r$, and $\mu, \nu, \cdots = 0, 1, 2$, and $i, j, \cdots = 1, 2$.