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Corrections to the Hawking tunneling radiation in extra dimensions

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ABSTRACT

Although the tunneling approach is fully established for black hole radiation, much work has been done to support the extension of this approach to more general settings. In this paper the Parikh–Kraus–Wilczeck tunneling proposal of black hole tunneling radiation is considered. The thermodynamics of the higher dimensional Schwarzschild black hole is studied based on the generalized uncertainty principle (GUP) and the modified dispersion relation (MDR) analysis, separately. It is shown that entropy and the rate of the higher dimensional Schwarzschild black hole tunneling radiation receive some corrections. The leading-order corrections does not contain the logarithmic term of the entropy, if the dimensions of the space–time is an odd number. Through the comparison, it is found that the results of these two alternative approaches are identical if one uses the suitable expansion coefficients.

1. Introduction

Classically, black holes are perfect absorbers; do not emit anything and their physical temperature is absolute zero. In 1974, Hawking proved that the physical temperature of a black hole is not absolute zero. However, in quantum theory a black hole radiates all species of particles with a perfect black body spectrum, at temperature T proportional to the horizon surface gravity. The entropy is a geometrical object relating to the horizon area through the well-known entropy–area relation [1].

Recently, Parikh, Kraus and Wilczeck have provided an alternative method to drive the Hawking radiation of the black holes. Their method was based on the semi-classical tunneling [2], and received much attention [3,5–7]. In this method, derivation of the Hawking radiation is related to the imaginary part of action for classically forbidden process of emission across the horizon. The imaginary part of the action for emitted particles is calculated using different methods. In the null-geodesic method, the contribution to the imaginary part of the action comes from the integration of radial momentum of the emitted particles. Other approaches satisfy the relativistic Hamilton–Jacobi equation of the emitted particles to obtain the imaginary part of action [2,3,8].

Now the GUP and MDR have been the subject of many interesting works and a lot of papers have appeared in which the usual uncertainty principle is generalized at the framework of microphysics [9,10]. The GUP corrections to the entropy of black holes have been obtained by several authors based on the Cardy–Verlinde formula [11–13]. Also the modification of energy–momentum relations and it's applications have been investigated extensively [14]. Furthermore, the extra dimensional version of the MDR has been proposed in Ref. [15], by direct comparison with the extra dimensional form of GUP. The proposed extra dimensional MDR has been applied to obtain the first order corrections to the entropy of d-dimensional Schwarzschild black hole through the Cardy–Verlinde formula [13]. To study the quantum gravitational effects to the higher dimensional black holes tunneling rate, it is interesting to relate the entropy of the black holes with a minimal length quantum gravity scale.

The main goal of this paper is to compare the results of different approaches to quantum gravitational corrections on the Hawking quantum tunneling rate in extra dimensions. For this purpose, the corrections are calculated in the framework of (1) the generalized uncertainty principle and (2) the modified dispersion relation, separately. We believe that it can essentially lead to a deeper insight into the ultimate quantum gravity proposals as well as the physical properties of the higher dimensional black holes.

The paper is organized on the following order. Section 2 is devoted to a brief review of Hawking radiation via tunneling through the horizon of the d-dimensional Schwarzschild black hole. In Section 3, making use of the GUP, we calculate the quantum gravitational effects on the entropy and the quantum tunneling radiation through the horizon of the higher dimensional Schwarzschild black hole up to the sixth order in the Planck length. In Section 4, we

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examine the MDR to calculate the corrections to the entropy and tunneling radiation, up to the same order in the Plank length, through horizon of the black hole. In Section 5, we compare the results of these two alternative approaches and show that a suitable choice of the expansion coefficients leads to the same results for the entropy and tunneling radiation of the black holes in extra dimensions. Some concluding remarks and discussions are given in Section 6.

2. Review of d-dimensional Schwarzschild black hole and Hawking tunneling radiation

The metric of d-dimensional Schwarzschild black hole in $(t, r, \theta_1, \theta_2, \ldots, \theta_{d-2})$ coordinates is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{d-2}, \tag{2.1}$$

where $f(r) = 1 - \frac{m}{r^{d-3}}$. The mass of the black hole is $M = \frac{m(d-2)\Omega_{d-2}}{16\pi G_d}$ and $\Omega_{d-2} = \frac{\pi^{(d-1)/2}}{\Gamma[(d-1)/2]}$ is the area of a unit $(d-2)$-sphere. $d\Omega^2_{d-2}$ is the line element on the sphere S^{d-2} and G_d is d-dimensional gravitational constant.

According to tunneling picture, the radiation arises by a process similar to electron–positron pair production in a constant electric field. The idea is that the energy of a particle changes sign as it crosses the horizon, so that the pair created just inside or outside the horizon can materialize with zero total energy, after each one of the pairs has tunneled to the opposite sides [2]. This suggests that it should be possible to describe the black hole emission process in a semiclassical fashion as quantum tunneling. In the WKB approximation, the tunneling probability is a function of the imaginary part of the action [16]

$$\Gamma \sim e^{-2\mathcal{I}_m\,(I)}, \tag{2.2}$$

where $\mathcal{I}_m$ means the imaginary part and I is the classical action of the trajectory.

To describe the tunneling phenomena, we need the coordinates which, unlike the coordinates given in Eq. (2.1), are regular on the horizon. For this purpose, we use the following coordinate transformations in (2.1),

$$dt \to dt - \frac{\sqrt{1-f(r)}}{f(r)}dr. \tag{2.3}$$

This coordinate transformation leads to the non-singular nature of the space–time with the following line element [17]

$$ds^2 = -f(r)\,dt^2 + 2\sqrt{1-f(r)}drdt + dr^2 + r^2 d\Omega^2_{d-2}. \tag{2.4}$$

In the null geodesic method, the imaginary part of the action for an outgoing positive energy particle which crosses the horizon outward from r_{in} to r_{out} comes from the radial part of the momentum

$$\mathcal{I}_m\,(I) = \mathcal{I}_m \int_{r_{in}}^{r_{out}} p_r dr = \mathcal{I}_m \int_{r_{in}}^{r_{out}}\int_0^{p_r} dp_r dr$$
$$= \mathcal{I}_m \int_M^{M-\omega}\int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH, \tag{2.5}$$

where the Hamilton's equation $\dot{r} = dH/dp_r$ is used. The radial null geodesic for an outgoing massless particle is given by [4]

$$\dot{r} = 1 - \sqrt{1-f(r)}. \tag{2.6}$$

If we fix the total mass and let the black hole mass to fluctuate, a shell of energy ω travels on the geodesic given by line element (2.4) with $M \to M - \omega$. Using Eq. (2.6) in Eq. (2.5) and switch the order of integration we have

$$\mathcal{I}_m\,(I) = \int_M^{M-\omega}\int_{r_{in}}^{r_{out}} \frac{dr}{1-\sqrt{1-f(r)}} d\omega' \tag{2.7}$$

which can be calculated by deforming the contour according to Feynman's $\omega' \to \omega' + i\epsilon$ prescription. This results the black hole radiation probability as

$$\Gamma \sim e^{-2\mathcal{I}_m\,(I)} = e^{\Delta S}, \tag{2.8}$$

where $\Delta S = S(M-\omega) - S(M)$ is the difference between the initial and the final values of the Beckenstein–Hawking entropy of the black hole.

The horizon has an associated entropy (S) and Hawking temperature (T) as

$$S = \frac{A}{4G_d}, \quad \text{and} \quad A = d\, r_h^{d-2}\Omega_{d-2}, \tag{2.9}$$

$$T = \frac{d-3}{4\pi r_h}, \quad \text{and} \quad r_h = m^{\frac{1}{d-3}}. \tag{2.10}$$

The mass is related to horizon radius as

$$M = \frac{(d-2)\Omega_{d-2}}{16\pi G_d} r_h^{d-3}. \tag{2.11}$$

In the following, we calculate the corrections to the tunneling rate (2.8) using the quantum corrected entropy of the black hole based on GUP and MDR analysis. Finally we compare the results obtained from these two alternative approaches.

3. The GUP corrections to the tunneling rate

To study the quantum gravity effects on the thermodynamics and tunneling probability, we employ the GUP. It is shown that usual uncertainty principle receives a modification at the micro-physics regime [18],

$$\delta x \geq \frac{\hbar}{\delta p} + \alpha L_P^2 \frac{\delta p}{\hbar}, \tag{3.1}$$

where $L_p = (\hbar G_d/c^3)^{1/(d-2)}$ is the Planck length. The term $\alpha L_P^2 \delta p/\hbar$ in Eq. (3.1) shows the gravitational effects on the usual uncertainty principle. Inverting Eq. (3.1) we obtain

$$\frac{\delta x}{2\alpha L_P^2}\left(1 - \sqrt{1 - \frac{4\alpha L_P^2}{(\delta x)^2}}\right) \leq \frac{\delta p}{\hbar} \leq \frac{\delta x}{2\alpha L_P^2}\left(1 + \sqrt{1 - \frac{4\alpha L_P^2}{(\delta x)^2}}\right). \tag{3.2}$$

From Eq. (3.2), one can write

$$\left(\frac{\delta p}{\hbar}\right)_{min} = \frac{1}{\delta x}\left[\frac{(\delta x)^2}{2\alpha L_P^2}\left(1 - \sqrt{1 - \frac{4\alpha L_P^2}{(\delta x)^2}}\right)\right]$$
$$= \frac{1}{\delta x} F_{(GUP)}((\delta x)^2), \tag{3.3}$$

where

$$F_{(GUP)}((\delta x)^2) = \frac{(\delta x)^2}{2\alpha L_P^2}\left(1 - \sqrt{1 - \frac{4\alpha L_P^2}{(\delta x)^2}}\right), \tag{3.4}$$