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Propagation peculiarities of mean field massive gravity

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ABSTRACT

Massive gravity (mGR) describes a dynamical "metric" on a fiducial, background one. We investigate fluctuations of the dynamics about mGR solutions, that is about its "mean field theory". Analyzing mean field massive gravity ($\overline{m}$GR) propagation characteristics is not only equivalent to studying those of the full non-linear theory, but also in direct correspondence with earlier analyses of charged higher spin systems, the oldest example being the charged, massive spin 3/2 Rarita–Schwinger (RS) theory. The fiducial and mGR mean field background metrics in the $\overline{m}$GR model correspond to the RS Minkowski metric and external EM field. The common implications in both systems are that hyperbolicity holds only in a weak background-mean-field limit, immediately ruling both theories out as fundamental theories; a situation in stark contrast with general relativity (GR) which is at least a consistent classical theory. Moreover, even though both $\overline{m}$GR and RS theories can still in principle be considered as predictive effective models in the weak regime, their lower helicities then exhibit superluminal behavior: lower helicity gravitons are superluminal as compared to photons propagating on either the fiducial or background metric. Thus our approach has uncovered a novel, dispersive, "crystal-like" phenomenon of differing helicities having differing propagation speeds. This applies both to $\overline{m}$GR and mGR, and is a *peculiar* feature that is also problematic for consistent coupling to matter.

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1. Introduction

Consistency is a powerful tool for studying field theories. Already classically, there are stringent conditions that are extremely difficult to fulfill for systems with spin $s > 1$, the most important exception being ($s = 2$, $m = 0$) general relativity. Key consistency requirements are

(i) Correct degree of freedom (DoF) counts.
(ii) Non-ghost kinetic terms.
(iii) Predictability.
(iv) (Sub)luminal propagation.

Requirements (i) and (ii) are closely related (as are (iii) and (iv)). Models whose constraints do not single out the correct propagating DoF suffer from relatively ghost kinetic terms: the relevant example here is the sixth ghost excitation that plagues generic massive gravity (mGR) theories [1]. The discovery that a class of mGR models satisfied requirements (i) and (ii) generated a revival of interest in massive spin 2 theories [2–7] even though failure of the propagation requirements (iii) and (iv) were long known to bedevil higher spin theories [8,9].

The predictability requirement is that initial data can be propagated to the future of spacetime hypersurfaces. In PDE terms, this means that the underlying equations must be hyperbolic [10]. The final requirement, that signals cannot propagate faster than light, can be imposed once the hyperbolicity requirement is satisfied. The classic example of a model that obeys requirements (i) and (ii) as well as (iii) but only in a weak field region, is the charged, massive, $s = 3/2$ RS theory. Curiously enough, the propagation problems of this model were first discovered in a quantum setting by Johnson and Sudarshan [11] who studied the model's canonical field commutators (this is easy to understand in retrospect, because field commutators and propagators are directly related [12]). The first detailed analysis of the model's propagation characteristics was carried out by Velo and Zwanziger; our aim is

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to reproduce their RS results in $\overline{\text{m}}$GR, so we quote their 1971 abstract verbatim [8]:

> The Rarita–Schwinger equation in an external electromagnetic potential is shown to be equivalent to a hyperbolic system of partial differential equations supplemented by initial conditions. The wave fronts of the classical solutions are calculated and are found to propagate faster than light. Nevertheless, for sufficiently weak external potentials, a consistent quantum mechanics and quantum field theory may be established. These, however, violate the postulates of special relativity.

In previous works we and other authors have shown that similar conclusions hold for the full non-linear mGR models [5,6,13,14,7,15]. These investigations rely on the method of characteristics, which amounts to studying leading kinetic terms and is thus essentially equivalent to an analysis of linear fluctuations around a mean field background. Since this mean field massive gravity ($\overline{\text{m}}$GR) fluctuation model depends both on a background and a fiducial metric, it is in direct correspondence with the charged RS model. Hence, without any computation at all, one can readily predict that: (a) mGR loses hyperbolicity in some strong field regime and (b) in the weak field hyperbolic regime where predictability is restored, lower helicity modes have propagation characteristics differing from maximal helicity ± 2; thus superluminality with respect to (luminal) photons is inevitable. Apart from confirming earlier conclusions in a very simple setting, our results give a precise description of mGR's effective, weak field, regime.

2. Massive gravity

At its genesis, the first known non-linear mGR model of [16] was originally formulated in terms of dynamical and fiducial vierbeine e^m and f^m. It took some forty years for researchers — independently in an effective field theory-inspired metric formulation — to discover that this model was one of a three-parameter family [2] that avoided the sixth, ghost-like excitation of [1]. The action describing these fiducial mGR models is given by[1]

$$S_{\rm mGR}[e,\omega;f] = -\int \epsilon_{mnrs} e^m \left\{ \frac{1}{4} e^n \left[d\omega^{rs} + \omega^r{}_t \omega^{ts} \right] - m^2 \left[\frac{\beta_0}{4} e^n e^r e^s + \frac{\beta_1}{3} e^n e^r f^s + \frac{\beta_2}{2} e^n f^r f^s + \beta_3 f^n f^r f^s \right] \right\} .$$

The parameter β_0 governs a standard cosmological term; this is required to obtain the Fierz–Pauli (FP) linearized limit when both the fiducial and mGR backgrounds are Minkowski. When both the fiducial and mGR backgrounds are Einstein with cosmological constant $\bar{\Lambda}$, the model's parameters must obey $\frac{\bar{\Lambda}}{3!} = m^2(\beta_0+\beta_1+\beta_2+\beta_3)$ and the linearized theory is FP with mass $m_{\rm FP}^2 := m^2(\beta_1+2\beta_2+3\beta_3)$.

Varying the model's dynamical fields (e^m, ω^{mn}) gives equations of motion

$$\nabla e^m \approx 0 \approx G_m - m^2 t_m , \tag{1}$$

where $t_m := \epsilon_{mnrs}\big[\beta_0 e^n e^r e^s + \beta_1 e^n e^r f^s + \beta_2 e^n f^r f^s + \beta_3 f^n f^r f^s\big]$. Also, the Einstein three-form is defined by $G_m := \frac{1}{2}\epsilon_{mnrs} e^n R^{rs}$ and $R^{mn} := d\omega^{mn} + \omega^m{}_r \omega^{rn}$ is the Riemann curvature; ∇ is the connection of ω^{mn}. The forty equations above are subject to thirty constraints that are spelled out in detail in [7]. In particular, these include the covariant algebraic relations[2]

$$e^m f_m \approx 0 \approx K_{mn} e^m f^n \approx \epsilon_{mnrs} M^{mn} K^{rs} ,$$

where the tensor $K^{mn} := \omega^{mn} - \chi^{mn}$ denotes the contorsion and $M^{mn} := \beta_1 e^m e^n + 2\beta_2 e^{[m} f^{n]} + 3\beta_3 f^m f^n$.

3. Mean field massive gravity

Consider mGR propagating in an arbitrary fiducial (pseudo-) Riemannian manifold $(M, \bar{g}_{\mu\nu})$ with corresponding vierbeine and spin connections (f^m, χ^{mn}). Now let (e^m, ω^{mn}) be a solution to the mGR equations of motion (1). We wish to study fluctuations $(\varepsilon^m, \lambda^{mn})$ about this configuration:

$$\tilde{e}^m = e^m + \varepsilon^m , \quad \tilde{\omega}^{mn} = \omega^{mn} + \lambda^{mn} .$$

The action governing these is the quadratic part of $S_{\rm mGR}[\tilde{e},\tilde{\omega};f] - S_{\rm mGR}[e,\omega;f]$, namely

$$\begin{aligned} S[h,\lambda;e,f] := -\frac{1}{2}\int \epsilon_{mnrs}\Big[& e^m \varepsilon^n \nabla \lambda^{rs} \\ & + \frac{1}{2}\left(e^m e^n \lambda^r{}_t \lambda^{ts} + R^{mn}\varepsilon^r \varepsilon^s\right) \\ & - m^2\big(3\beta_0 e^m e^n \varepsilon^r \varepsilon^s + 2\beta_1 e^m f^n \varepsilon^r \varepsilon^s \\ & + \beta_2 f^m f^n \varepsilon^r \varepsilon^s\big)\Big] . \end{aligned}$$

The mean field model is a theory of forty dynamical fields $(\varepsilon^m, \lambda^{mn})$. In the above, ∇ is the Levi-Civita connection of e^m, and R^{mn} its Riemann tensor; we stress that henceforth the fiducial field $\big(f^m, \chi^{mn}(f), \bar{g}_{\mu\nu}(f)\big)$ and mGR background fields $\big(e^m, \omega^{mn}(e), g_{\mu\nu}(e)\big)$ are *non-dynamical*; all index manipulations will be carried out using the mGR background metric and vierbein.

The $\overline{\text{m}}$GR equations of motion are

$$\begin{aligned} \mathcal{T}^m &:= \nabla \varepsilon^m + \lambda^{mn} e_n \approx 0 , \\ \mathcal{G}_m &:= \frac{1}{2}\epsilon_{mnrs}\big[e^n \nabla \lambda^{rs} + \varepsilon^n R^{rs}\big] - m^2\, \tau_m \approx 0 , \end{aligned} \tag{2}$$

where $\tau_m := \epsilon_{mnrs}\Big[3\beta_0\, e^n e^r \varepsilon^s + 2\beta_1\, e^n f^r \varepsilon^s + \beta_2\, f^n f^r \varepsilon^s\Big]$.

4. Mean field degrees of freedom

In principle, since we are describing the linearization of a model whose constraints have been completely analyzed in [7], we know *a priori* that $\overline{\text{m}}$GR describes five propagating degrees of freedom. However, for completeness and our causality study, we reanalyze its constraints.

The first step is to introduce a putative choice of time coordinate t, which for now need not rely in any way on either the fiducial or background metric, and use this to decompose any p-form θ (with $p < 4$) as

$$\theta := \boldsymbol{\theta} + \mathring{\theta} , \tag{3}$$

where $\mathring{\theta} \wedge dt = 0$. Thus $\boldsymbol{\theta}$ is the purely spatial part of the form θ. Hence for any on-shell relation $\mathcal{P} \approx 0$ polynomial in $(\nabla, \varepsilon, \lambda)$,

[1] Here d is the exterior derivative and the dynamical vierbiene and spin connection (e, ω), are one-forms. We suppress wedge products unless necessary for clarity.

[2] The first of these assumes invertibility of the operator M^{mn} as a map from two-forms to antisymmetric Lorentz tensors; we shall always work on the model's branch where this holds.

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