Physics Letters B 749 (2015) 153–158

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

An extraordinary mass invariant and an obstruction in a massive superspin one half model made with a chiral dotted spinor superfield

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A R T I C L E I N F O

Article history:
Received 27 May 2015
Received in revised form 15 July 2015
Accepted 22 July 2015
Available online 28 July 2015
Editor: M. Cvetič

A B S T R A C T

An action for a complex irreducible massive superspin $\frac{1}{2}$ multiplet can be constructed out of two chiral dotted spinor and two chiral undotted spinor superfields. To make this action a sensible one, additional 'reality constraints' are needed, and the notion of BRST recycling is needed to find the supersymmetry transformations of the theory with these additional constraints. This theory possesses three possible mass terms. An earlier paper examined the theory with the first mass term. This paper adds a second mass term and examines the consequences of that. This second mass invariant is 'extraordinary', which means that it is intrinsically dependent on the Zinn sources ('antifields') of the theory. This in turn implies that the action needs to be 'completed' so that it yields zero for the relevant Poisson Bracket. This 'Completion' meets an 'Obstruction', which is a ghost charge one object in the BRST cohomology space. Usually Obstructions arise from a one loop calculation, in which case they form anomalies of the theory. However this Obstruction arises at tree level from the completion. The coefficient of the Obstruction needs to be set to zero. This restores the complex irreducible massive superspin $\frac{1}{2}$ multiplet to its usual structure, except that the mass is constructed out of the two mass parameters. The construction suggests interesting possibilities for related interacting theories.

1. Although supersymmetry has undergone intense scrutiny for over 40 years, there are still profound mysteries and unsolved problems. The chief of these is that, so far, it does not seem to have any experimental relevance [1]. However that may be about to change as results at the LHC continue to be reported [5]. But it is also arguable that we do not know what SUSY predicts [1–4], because the spontaneous breaking of SUSY is well known to give rise to sum rules that are problematic for phenomenology, and a huge cosmological constant which is problematic for cosmology [4,8].

2. In particular there is still much to learn about the representation theory of SUSY, even in $3+1$ dimensions. Progress in the representation theory of SUSY is being made by the adinkra program and other investigations of Buchbinder and Gates et al. [9–14]. Massive representations of SUSY are clearly related to some of the puzzles of the superstring (see for example [6,7]). New efforts at understanding the BRST cohomology of SUSY are also under way [15–22].

3. Following along in the path of looking for new representations of SUSY, in [29], a new supersymmetric action for massive superspin $\frac{1}{2}$ was constructed using 'BRST Recycling', rather than superspace. This action contained the component fields of a chiral dotted spinor superfield, which was expected to have interesting cohomology. Indeed it does, as we shall show here.

4. In [29], it was shown that there was a mass term and that the theory there described a complex massive superspin $\frac{1}{2}$ multiplet, as set out in that paper. It is a curious fact that there are actually three possible mass terms in that theory.[1] In this paper we will examine the situation in which we include two of them, with independent coupling constants. In a nutshell, what happens in the theory with two mass terms, is that we are forced to do a number of things in the theory to ensure that the theory with two mass terms yields zero for the same BRST Poisson Bracket that we had in the original paper [29]. And when these things are done, we end up with another version of the original supermultiplet, except that the mass is now formed from the two mass terms.

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[1] This is shown in [31].

http://dx.doi.org/10.1016/j.physletb.2015.07.056

5. This paper assumes that the reader has read [29]. In this paper we will add an additional mass term to the action that we had in [29]. The new mass term $\mathcal{A}_{\rm E}$ is a BRST Extraordinary Invariant, which means that it is irrevocably dependent on Zinn sources, and that it satisfies ($\delta_{\rm Massless}$ is defined in (5):

$$\delta_{\rm Massless}\mathcal{A}_{\rm E} = 0 \tag{1}$$

This kind of object has sometimes been called 'finding a consistent extension of a BRST theory' and the papers [23–26,30] have discussed that concept in the context of various actions.

6. An unusual feature of the present Extraordinary Invariant is that an attempt to complete the action, so that the new action yields zero for the BRST Poisson Bracket, meets a 'Completion Obstruction' in the present case. Following the usual BRST reasoning [28], this ghost charge one 'Completion Obstruction' could also conceivably arise as an Anomaly, but it clearly does not do so in the present free Action.

7. The new Extraordinary Invariant $\mathcal{A}_{\rm E}$ here is written explicitly below in equations (8) to (10) in the notation of [29]. In this paper we will go through the exercise of completing the action so that the completed action still satisfies the original BRST Poisson Bracket in [29]. To do this we need to first drop the gauge and ghost fixing action that was used in [29], because we will need to change it after the Completion. Then we put the action plus the Extraordinary Invariant into the BRST Poisson Bracket, and observe that the BRST Poisson Bracket is no longer zero. There are two non-zero terms: the variation of a Completion Term and also an Obstruction. We add the Completion Term, and then also constrain the coefficient of the Obstruction to be zero. At that point we can add a new, more suitable, form of the gauge and ghost fixing action. Then we look at the equations of motion of the new theory, and we see how the Completion term and the Constraint act together to modify the action so that it again describes a massive superspin $\frac{1}{2}$ supersymmetry multiplet, but with a revised mass. Then we consider the origin and significance of the above results.

8. From [29], let us take the following action

$$\mathcal{A}_{\rm Massless} = \mathcal{A}_{{\rm Kinetic}\ \chi} + \mathcal{A}_{{\rm Kinetic}\ \phi} + \mathcal{A}_{{\rm Zinn}\ \chi} + \mathcal{A}_{{\rm Zinn}\ \phi} + \mathcal{A}_{\rm SUSY} \tag{2}$$

This is the full action from that paper,[2] but without the mass term $\mathcal{A}_{{\rm Mass}\ \chi\ \phi}$ and without the ghost and gauge fixing action $\mathcal{A}_{\rm GGF}$ of that paper.

The first two pieces of this action $\mathcal{A}_{\rm Massless}$ in (2) are

$$\mathcal{A}_{{\rm Kinetic}\ \chi} = \int d^4x\,\{\chi_L^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\chi}_L^{\alpha} + \chi_R^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\chi}_R^{\alpha} + G_{\dot\alpha\dot\beta}\overline{G}^{\dot\alpha\dot\beta} - 2B\overline{B}\} \tag{3}$$

and

$$\mathcal{A}_{{\rm Kinetic}\ \phi} = \int d^4x\,\Big\{\phi_L^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}_L^{\alpha} + \phi_R^{\dot\alpha}\partial_{\alpha\dot\alpha}\overline{\phi}_R^{\alpha} + W_{\alpha\dot\alpha}\overline{W}^{\alpha\dot\alpha} - \frac{1}{2}E\Box\overline{E} + \frac{1}{2}\eta'\left(\phi_L^{\dot\delta}\overline{C}_{\dot\delta} + \overline{\phi}_R^{\delta}C_{\delta}\right) + \frac{1}{2}\eta'\left(\overline{\phi}_L^{\delta}C_{\delta} + \phi_R^{\dot\delta}\overline{C}_{\dot\delta}\right)\Big\} \tag{4}$$

and the notation is set out in [29]. The other three pieces of (2) are $\mathcal{A}_{{\rm Zinn}\ \chi} + \mathcal{A}_{{\rm Zinn}\ \phi} + \mathcal{A}_{\rm SUSY}$ and it would be redundant to repeat them here. They are discussed at length in [29].

This gives rise to the following nilpotent BRST operator[3]:

$$\delta_{\rm Massless} = \delta_{{\rm Kinetic}\ \chi} + \delta_{{\rm Kinetic}\ \phi} + \delta_{{\rm Zinn}\ \chi} + \delta_{{\rm Zinn}\ \phi} + \delta_{{\rm Field}\ \chi} + \delta_{{\rm Field}\ \phi} + \delta_{\rm Susy} \tag{5}$$

where $\delta_{{\rm Kinetic}\ \chi}$ arises from functional derivatives of $\mathcal{A}_{{\rm Kinetic}\ \chi}$, etc. as described in [29]. It is the usual 'square root' of the BRST Poisson Bracket $\mathcal{P}_{\rm Total}[\mathcal{A}]$ from [29], evaluated with $\mathcal{A} \to \mathcal{A}_{\rm Massless}$, where $\mathcal{P}_{\rm Total}[\mathcal{A}]$ was defined by equation (6) of [29]. It is nilpotent because

$$\mathcal{P}_{\rm Total}\,[\mathcal{A}_{\rm Massless}] = 0 \Leftrightarrow \delta^2_{\rm Massless} = 0 \tag{6}$$

In [29], we noted that the following 'Ordinary' mass invariant is in the cohomology space[4] of $\delta_{\rm Massless}$:

$$\mathcal{A}_{\rm O} = \int d^4x\,\{m_1\phi_{L\dot\alpha}\chi_R^{\dot\alpha} + m_1\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha} + m_1E\overline{B} + m_1W_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha} + m_1\eta'\overline{\omega}\} + * \tag{7}$$

Now we claim that there is another kind of mass term here. The following 'Extraordinary' mass invariant is **also** in the cohomology space[5] of $\delta_{\rm Massless}$:

$$\mathcal{A}_{\rm E} = \int d^4x\Big\{2m_2\Upsilon\overline{\omega} - \frac{m_2}{2}\partial_{\alpha\dot\alpha}\overline{V}^{\alpha\dot\alpha}E - m_2Z_L^{\dot\alpha}C^{\alpha}\overline{V}_{\alpha\dot\alpha} \tag{8}$$

$$+ m_2\overline{Z}_R^{\alpha}\overline{C}^{\dot\alpha}\overline{V}_{\alpha\dot\alpha} + m_2\phi_{L\dot\alpha}\chi_R^{\dot\alpha} - m_2\overline{\phi}_{R\alpha}\overline{\chi}_L^{\alpha} \tag{9}$$

$$- m_2\Sigma^{\alpha\dot\alpha}\overline{C}_{\dot\beta}\overline{\chi}_{L\alpha} + m_2\Sigma^{\alpha\dot\alpha}\chi_{R\dot\alpha}C_{\alpha} + 2m_2J'\overline{B}\Big\} + * \tag{10}$$

Like the mass term $\mathcal{A}_{\rm O}$, the existence of $\mathcal{A}_{\rm E}$ is indicated by spectral sequence techniques applied to the massless BRST operator $\delta_{\rm Massless}$. This somewhat technical analysis will be presented in a third paper [31], where we find even more cohomology than is discussed here.[6]

9. Note the following:

1. The 'Ordinary' mass invariant $\mathcal{A}_{\rm O}$ does not contain any Zinn sources. It contains only fields and Fadeev–Popov ghosts.
2. The 'Extraordinary' mass invariant $\mathcal{A}_{\rm E}$ does contain Zinn sources, namely Υ, $Z_L^{\dot\alpha}$, $\overline{Z}_R^{\alpha}$, $\Sigma^{\alpha\dot\alpha}$ and J'.
3. Note that all the Zinn sources in $\mathcal{A}_{\rm E}$ are ϕ type Zinn sources. There are no χ type Zinn sources present in $\mathcal{A}_{\rm E}$.
4. Each term of each invariant contains one χ field.
5. Each term of each invariant contains one ϕ field or one ϕ Zinn source.

[2] $\mathcal{A}_{\rm SUSY}$ is discussed in footnote 4 of [29].

[3] This operator will be written in full detail in [31].

[4] The relevant operator in [29] was simply what we called $\delta_{\rm First}$ in equation (15) of that paper. Whether we included the Zinn variation terms of δ that arise from equations of motion from the two actions $\mathcal{A}_{{\rm Kinetic}\ \chi}$ and $\mathcal{A}_{{\rm Kinetic}\ \phi}$ was irrelevant, because $\mathcal{A}_{\rm O}$ does not contain any Zinns. But it is important to note that these do not give rise to $\mathcal{A}_{\rm O}$ as a boundary. However for the case of $\mathcal{A}_{\rm E}$ we need to be more careful, and so we define the new operator $\delta_{\rm Massless}$ explicitly in the foregoing.

[5] Finding this term $\mathcal{A}_{\rm E}$ is more tricky than finding the mass term above, as is obvious from its complicated form.

[6] In fact this theory contains three independent supersymmetric mass terms and five obstructions. Discussion of the other mass term and the other obstructions would needlessly complicate the present paper. They do ultimately need analysis of course.