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On the uniqueness of supersymmetric attractors

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ABSTRACT

In this paper we discuss the uniqueness of supersymmetric attractors in four-dimensional $N = 2$ supergravity theories coupled to n vector multiplets. We prove that for a given charge configuration the supersymmetry preserving axion free attractors are unique. We generalise the analysis to axionic attractors and state the conditions for uniqueness explicitly. We consider the example of a two-parameter model and find all solutions to the supersymmetric attractor equations and discuss their uniqueness.

1. Introduction

Understanding the origin of black hole entropy has remained to be an important topic of research in gravity and string theory since the seminal work by Bekenstein [1] on this issue. One of the important developments in this area is the so called attractor mechanism, which states that, in a theory of gravity coupled to several scalar fields admitting a single centred extremal black hole, the scalar fields run into a fixed point at the horizon whose value depends only on the black hole charges [2–5]. There are several aspects of attractor mechanism which have been studied thoroughly [6,7]. Multiplicity of the attractors is one of the puzzling issues which remains to be understood better. Because of the presence of multiple basin of attractors, the near horizon geometry of the black hole is no longer uniquely determined by its charges and one needs to specify the area code in addition to the black hole charges.

The existence of multiple basin of attractors for a given set of charges has been first discussed in [8,9]. Area codes in the context of flux vacua and black hole attractors have been studied [10,11]. Subsequently, multiple supersymmetric attractors in five-dimensional $N = 2$ supergravity theory have been discussed and explicit constructions in the simple case of a two parameter model have been carried out [12]. The analysis has been extended to four-dimensional $N = 2$ supergravity [13] by using the known $4D$–$5D$ correspondence of the attractor points [14]. Further, new multiple non-supersymmetric attractors which do not have obvious five-dimensional embedding have been constructed [13]. Multiple attractors in a one parameter model in the presence of quantum corrections have already been studied [15].

The existence of multiple single centred supersymmetric attractors might at first sight appear to be in contradiction with the uniqueness results [16]. (For homogeneous moduli spaces, the solution is always unique up to a duality transformation [17].) However, as explained by Kallosh [18], this is not always the case, because the moduli space might in general possess several disconnected branches. The attractor solution in each of these branches remains unique. One might expect similar results in four-dimensional $N = 2$ supergravity. However, though there exist multiple non-supersymmetric attractors and also multiple attractors with one of the attractor points being supersymmetric in these four-dimensional supergravity theories there is no known example where both the attractor points are supersymmetric for these $N = 2$ supergravity theories in four dimensions [13]. This suggests that, unlike the five-dimensional case, the supersymmetric attractors might be unique in these four-dimensional supergravity theories. The present work aims to investigate this issue in detail.

The plan of this paper is as follows. In the following section, we will briefly overview the $N = 2$ supergravity theory. In Section 3 we will prove that the axion free attractors in four dimensions are unique. Subsequently, we will generalise this result for axionic attractors. This will be followed by an explicit construction of all supersymmetric attractors in a simple two-parameter model in Section 4. Finally, we will be summarise our results in Section 5.

2. Overview

The Lagrangian density for the bosonic part of the four-dimensional $N = 2$ supergravity theory coupled to n vector multiplet, is given by

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$$\mathcal{L} = -\frac{R}{2} + g_{a\bar{b}}\partial_\mu x^a \partial_\nu \bar{x}^{\bar{b}} h^{\mu\nu} - \mu_{\Lambda\Sigma}\mathcal{F}^\Lambda_{\mu\nu}\mathcal{F}^\Sigma_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} - \nu_{\Lambda\Sigma}\mathcal{F}^\Lambda_{\mu\nu} * \mathcal{F}^\Sigma_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} . \quad (2.1)$$

Here $h_{\mu\nu}$ is the space–time metric, R is the corresponding Ricci scalar, g_{ab} is the metric on the vector multiplet moduli space parameterised by the corresponding n complex scalar fields x^a and A^Λ_μ are the $(n+1)$ gauge fields with corresponding field strength $\mathcal{F}^\Lambda_{\mu\nu}$. The gauge couplings $\mu_{\Lambda\Sigma}$, $\nu_{\Lambda\Sigma}$ and the moduli space metric $g_{a\bar{b}}$ are uniquely determined by the $N=2$ prepotential F.

We are interested in static, spherically symmetric configurations. The line element corresponding to the space time metric $h_{\mu\nu}$ in this case is given by

$$ds^2 = e^{2U}dt^2 - e^{-2U}\gamma_{mn}dy^m dy^n . \quad (2.2)$$

The wrap factor U depends only on the radial coordinate r. For extremal black holes, the metric of the spacial section γ_{mn} must be identity. The equations of motion for these configurations simplifies and the system can now be described in terms of an effective one-dimensional theory with a potential which is extremised at the horizon.

For the $N=2$ Lagrangian (2.1), the effective black hole potential takes the form [4]:

$$V = e^K\left[g^{a\bar{b}}\nabla_a W\overline{\nabla_b W} + |W|^2\right] . \quad (2.3)$$

Here W and K are respectively the superpotential and the Kähler potential. The superpotential W is related to the central charge by $Z = e^{K/2}W$. In terms of the dyonic charges (q_Λ, p^Λ) and the prepotential F, the expression for W is given by

$$W = \sum_{\Lambda=0}^{n}(q_\Lambda X^\Lambda - p^\Lambda \partial_\Lambda F) . \quad (2.4)$$

The symplectic sections X^Λ are related to the physical scalar fields by $x^a = X^a/X^0$. The Kähler potential is given in terms of F by the relation:

$$K = -\log\left[i\sum_{\Lambda=0}^{n}(\overline{X^\Lambda}\partial_\Lambda F - X^\Lambda\overline{\partial_\Lambda F})\right] . \quad (2.5)$$

The covariant derivative is defined as $\nabla_a W = \partial_a W + \partial_a K W$. For supersymmetric attractors $\nabla_a W = 0$. In general, the attractor points are determined by $\partial_a V = 0$.

Throughout this paper, we will focus on the $N=2$ prepotential which is of the form

$$F = D_{abc}\frac{X^a X^b X^c}{X^0} . \quad (2.6)$$

The above prepotential appears as the leading term in the compactification of type *IIA* string theory on a Calabi–Yau manifold $\mathcal{M}$ in the large volume limit. In this case, D_{abc} are the triple intersection numbers $D_{abc} = \int_\mathcal{M}\alpha_a\wedge\alpha_b\wedge\alpha_c$, where the two forms α_a form a basis of $H^2(\mathcal{M},\mathbb{Z})$. In this paper, we will use string theory terminologies to describe various charge configurations irrespective of whether the coefficients D_{abc} are actually associated with a Calabi–Yau compactification or not.

In the following we will describe some of the well known supersymmetric attractor solutions. For this purpose we need explicit expressions for the Kähler and the superpotentials. The Kähler potential K corresponding to the $N=2$ prepotential F has the following simple form

$$K = -\log[-iD_{abc}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^c - \bar{x}^c)] . \quad (2.7)$$

(Now on we set the gauge $X^0 = 1$ without any loss of generality and express our formulae in terms of the physical scalar fields x^a.) The superpotential depends on the specific charge configurations. In this paper we will mainly focus on $D0$–$D4$ and $D0$–$D4$–$D6$ configurations. For the $D0$–$D4$ configuration, the superpotential is given by

$$W = q_0 - 3p^a D_{abc}x^b x^c , \quad (2.8)$$

whereas for the $D0$–$D4$–$D6$ configuration, we have

$$W = q_0 - 3p^a D_{abc}x^b x^c + p^0 D_{abc}x^a x^b x^c . \quad (2.9)$$

These configurations possess well known supersymmetric attractor solutions [19]. For the $D0$–$D4$ configuration, we have

$$\nabla_a W = -6D_{ab}x^b - \frac{3M_a}{M}W .$$

From here onwards we use the standard notations [20] $D_{ab} = D_{abc}p^c$, $D_a = D_{ab}p^b$, $D = D_a p^a$, $M_{ab} = D_{abc}(x^c - \bar{x}^c)$, $M_a = M_{ab}(x^b - \bar{x}^b)$ and $M = M_a(x^a - \bar{x}^a)$. (Note that M_a is real where as M_{ab} and M are pure imaginary.) Setting the ansatz, $x^a = p^a t$, we find

$$\nabla_a W = -\frac{3D_a}{2tD}(q_0 + t^2 D) ,$$

and hence,

$$x^a = ip^a\sqrt{\frac{q_0}{D}} ,$$

for the supersymmetric $D0$–$D4$ configuration. The entropy of the corresponding supersymmetric black hole is $S = 2\pi\sqrt{q_0 D}$.

The solution can be generalised in a straightforward manner upon adding $D6$ branes. We find

$$\nabla_a W = -6D_{ab}x^b + 3p^0 D_{abc}x^b x^c - \frac{3M_a}{M}W .$$

Setting the ansatz $x^a = p^a t$, we find the supersymmetric configuration corresponds to [19]

$$t = \frac{1}{2D}\left(p^0 q_0 \pm i\sqrt{4q_0 D - (p^0 q_0)^2}\right) . \quad (2.10)$$

The entropy for this configuration is

$$S = \pi\sqrt{4q_0 D - (p^0 q_0)^2} .$$

3. The general solution

In this section, we will focus on the supersymmetric conditions more carefully and obtain the general solution without assuming any specific ansatz. We will first focus on the $D0$–$D4$ configuration. Note that, in this case the superpotential contains only even powers of x^a. Thus we can set the axionic parts of the scalar fields to zero: $x^a = ix^a_2$. The supersymmetry condition now becomes

$$M_{ab}p^b + \frac{M_a}{M}(q_0 - \frac{3}{4}M_b p^b) = 0 . \quad (3.1)$$

Note that, for any configuration x^a_2 satisfying the above equation, we have $q_0 = -\frac{1}{4}M_a p^a$. We can see this by multiplying by $(x^a - \bar{x}^a)$ and simplifying the above equation. Thus, we can further simplify Eq. (3.1) by substituting $\frac{1}{4}M_a p^a = -q_0$ in it. We find

$$M_{ab}p^b + 4q_0\frac{M_a}{M} = 0 . \quad (3.2)$$