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Covariance analysis of symmetry energy observables from heavy ion collision

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ABSTRACT

Using covariance analysis, we quantify the correlations between the interaction parameters in a transport model and the observables commonly used to extract information of the Equation of State of Asymmetric Nuclear Matter in experiments. By simulating ^{124}Sn + ^{124}Sn, ^{124}Sn + ^{112}Sn and ^{112}Sn + ^{112}Sn reactions at beam energies of 50 and 120 MeV per nucleon, we have identified that the nucleon effective mass splitting is most strongly correlated to the neutrons and protons yield ratios with high kinetic energy from central collisions especially at high incident energy. The best observable to determine the slope of the symmetry energy, L, at saturation density is the isospin diffusion observable even though the correlation is not very strong ($\sim$0.7). Similar magnitude of correlation but opposite in sign exists for isospin diffusion and nucleon isoscalar effective mass. At 120 MeV/u, the effective mass splitting and the isoscalar effective mass also have opposite correlation for the double n/p and isoscaling p/p yield ratios. By combining data and simulations at different beam energies, it should be possible to place constraints on the slope of symmetry energy (L) and effective mass splitting with reasonable uncertainties.

Knowledge about the isospin asymmetric nuclear equation of state (EoS) is of fundamental importance to our understanding of nature's most asymmetric objects including neutron stars and heavy nuclei composed of very different numbers of neutrons and protons. Theoretically, there are two microscopic approaches to describe the EoS of nuclear matter. One approach starts from a realistic two-body free NN interactions [1,2] as input to the relativistic Dirac–Bruckner–Hartree–Fock (DBHF) and its nonrelativistic counterpart Bruckner–Hartree–Fock (BHF) [3–7] and chiral effective field theory [8]. Another one is to use effective density-dependent many-body interactions such as the zero-range Skyrme interaction [9–11], finite-range Gogny interaction [12] and effective Lagrangian [13–16] as inputs leading to Skyrme–Hartree–Fock (SHF) [10,11], Gogny–Hartree–Fock [17] and Relativistic–Hartree–Fock (RHF) approaches [13–16]. Of all interactions, the effective Skyrme interaction is more commonly used in nuclear structure, reactions and astrophysics studies as the effective Skyrme interactions are relatively simple mathematically to make it computationally feasible [18] and contain sufficient physics to allow quantitative description of heavy nuclei. Furthermore, the Fock term in the non-relativistic SHF approach can be managed computationally while the Fock terms in Relativistic HF approaches are difficult to compute. The interactions parameters are usually obtained by fitting the properties of symmetric nuclear matter (such as the saturation density and the corresponding energy per nucleon and its incompressibility), properties of asymmetric nuclear matter (such as symmetry energy and isovector effective mass at saturation density), finite nuclei properties (such as binding energies and r.m.s radius of selected set of doubly magic nuclei) etc. [11]. In Ref. [19], 240 Skyrme parameter sets that fit the ground-state properties of stable nuclei, symmetric and asymmetric nuclear matter are compiled. The large number of parameterization arises in part because there are strong correlations between individual parameters or groups of parameters, that fit particular physical properties of the many-body nuclear system. These sets lead to very different Equation of States of pure neutron matter [20–22] which may have different incompressibility $K_0 = 9\rho^2 \frac{\partial^2 \epsilon/\rho}{\partial \rho^2}$, symmetry energy coefficient $S_0 = S(\rho_0)$, slope of symmetry energy $L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}|_{\rho_0}$, isoscalar effective mass $\frac{m}{m_s^*} = (1 + \frac{2m}{\hbar^2} \frac{\partial}{\partial \tau} \frac{E}{A})|_{\rho_0}$ [23], and isovector effective mass $m_v^* = \frac{1}{1+\kappa}$, where κ is the enhancement factor of the Thomas–Reich–Kuhn sum rule [24]. In this work, the magnitude of effective mass splitting, $(m_n^* - m_p^*)/m$, was not used as input variables since its form is much more complicated to be incorporated

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into the code. Instead we use $f_I = \frac{1}{2\delta}(\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{m_s^*} - \frac{m}{m_v^*}$, where $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, ρ_n and ρ_p are the neutron and proton density, m_n^*, m_p^* and m are the neutron, proton effective mass and free nucleon mass. In Skyrme Hartree–Fock approximation [10,11,19], f_I increases with increasing density, but is independent of the momentum and kinetic energy. In the DBHF and RHF approximations [16,25,26], f_I not only depends on the density but also on the kinetic energy of the in-medium nucleon.

Some properties of symmetric nuclear matter, such as $K_0 = 230 \pm 30$ MeV [17,19,27] and $m_s^*/m = 0.65$–0.9 [17,19,23,28–30], have been extracted from isoscalar collective vibrations, giant quadrupole resonance and heavy ion collisions measurements. In addition, constraints on $E_0(\rho)$ and pressure P have been obtained in density regions ranging from saturation density to five times normal densities using collective flow and kaon production data in energetic nucleus–nucleus collisions [31–33]. To obtain information of the symmetry energy with heavy ion collision data, the symmetry potential used in transport models is changed by varying its input parameters, corresponding to the different values of S_0 and/or L in the expression of density dependence of symmetry energy. The results of the calculations are then compared with data to find the best parameter sets. Recently, a consistent set of constraints on the symmetry energy near saturation density between S_0, and its slope, L, has been obtained from observables measured in both nuclear structure and nuclear reaction experiments [22,34–36].

In Skyrme parametrizations, the symmetry energy are correlated to other parameters such as those associated with the nucleon effective mass m_s^* and isovector effective mass m_v^*. Such correlations would affect the uncertainties of symmetry energy constraints obtained from heavy ion collision data. In order to achieve the goal of obtaining precise and accurate symmetry energy constraints, one has to identify what experimental observables are crucial for better constraining the interested physical quantities in the theoretical models [37]. Ideally, one would do a global chi-square analysis using existing data to obtain the best set of model parameters. Then a covariance matrix can be obtained between any two model parameters using a chi-square fit [38]. Currently, this is not feasible considering the intensive CPU time needed to do transport model calculations. As a start to tackle this issue, we propose to use 12 parameters sets to perform covariance analysis to quantitatively examine the correlations between model parameters A and observables B commonly used in experiments. The linear-correlation coefficient C_{AB} between variable A and observable B is calculated as follows [38]:

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)} \tag{1}$$

$$cov(A,B) = \frac{1}{N-1}\sum_i (A_i - <A>)(B_i - <B>) \tag{2}$$

$$\sigma(X) = \sqrt{\frac{1}{N-1}\sum_i (X_i - <X>)^2}, X = A, B \tag{3}$$

$$<X> = \frac{1}{N}\sum_i X_i, i = 1, N. \tag{4}$$

$cov(A,B)$ is the covariance, $\sigma(X)$ is the variance. $C_{AB} = \pm 1$ means there is a linear dependence between A and B, and $C_{AB} = 0$ means no correlations.

We use the Improved Quantum Molecular Dynamic Model which incorporates the effective Skyrme interactions (ImQMD-Sky) [39] to simulate the collisions of heavy ions with parameter sets listed in Table 1. A_i represents the ith parameter set of the transport variable A where $A = K_0$, S_0, L, m_s^*, m_v^* or f_I used as input

Table 1
List of twelve parameter sets used in the ImQMD calculations. $\rho_0 = 0.16$ fm^{-3}, $E_0 = -16$ MeV, and $g_{sur} = 24.5$ MeV fm^2, $g_{sur,iso} = -4.99$ MeV fm^2.

Para.	K_0 (MeV)	S_0 (MeV)	L (MeV)	m_s^*/m	f_I
1	**230**	**32**	**46**	**0.7**	**−0.238**
2	**280**	32	46	0.7	−0.238
3	**330**	32	46	0.7	−0.238
4	230	**30**	46	0.7	−0.238
5	230	**34**	46	0.7	−0.238
6	230	32	**60**	0.7	−0.238
7	230	32	**80**	0.7	−0.238
8	230	32	**100**	0.7	−0.238
9	230	32	46	**0.85**	−0.238
10	230	32	46	**1.00**	−0.238
11	230	32	46	0.7	**0.0**
12 (SLy4)	230	32	46	0.7	**0.178**

to the ImQMD-Sky. In ImQMD-Sky, the nucleonic potential energy density is $u_{loc} + u_{md}$, where

$$\begin{aligned} u_\rho = &\frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 \\ &+ \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 \\ &+ A_{sym}\rho^2\delta^2 + B_{sym}\rho^{\eta+1}\delta^2 \end{aligned} \tag{5}$$

and the energy density of Skyrme-type momentum dependent interaction are written based on its interaction form $\delta(r_1 - r_2)(p_1 - p_2)^2$ [9,10],

$$\begin{aligned} u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \\ &= C_0 \int d^3p d^3p' f(\vec{r},\vec{p}) f(\vec{r},\vec{p}')(\vec{p} - \vec{p}')^2 \\ &\quad + D_0 \int d^3p d^3p' [f_n(\vec{r},\vec{p}) f_n(\vec{r},\vec{p}')(\vec{p} - \vec{p}')^2 \\ &\quad + f_p(\vec{r},\vec{p}) f_p(\vec{r},\vec{p}')(\vec{p} - \vec{p}')^2]. \end{aligned} \tag{6}$$

The 9 parameters α, β, η, A_{sym}, B_{sym}, C_0, D_0, g_{sur}, $g_{sur,iso}$ used in ImQMD-Sky can be derived from standard Skyrme parameter sets with 9 parameters $\{t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \sigma\}$ [39,40]. The coefficients of the surface terms are set as $g_{sur} = 24.5$ MeV fm^2 and $g_{sur;iso} = -4.99$ MeV fm^2 which are the same values derived from SLy4 parameter set [11]. Varying g_{sur} and $g_{sur,iso}$ for different Skyrme interactions have negligible effects on the experimental observables at intermediate energy. By using the relationship which was derived in reference [41,42], the reduced 7 Skyrme parameter sets $\{\alpha, \beta, \eta, A_{sym}, B_{sym}, C_0, D_0\}$ can be replaced by the parameter sets $\{\rho_0, E_0, K_0, S_0, L, m_s^*, m_v^*\}$ which is directly related to the properties of nuclear matter at saturation density ρ_0. Choosing the experimental values of $\rho_0 = 0.16$ fm^{-3}, $E_0 = -16$ MeV, the parameter sets are further reduced to 5 variables, $A = K_0, S_0$, L, m_s^*, m_v^*. As explained above, to simplify the coding, the input variables used in ImQMD-Sky are $A = K_0$, S_0, L, m_s^*, $f_I(m_s^*, m_v^*)$.

For HIC observables, we adopt the ratios constructed from nucleon spectra. Most transport models cannot describe accurately the absolute yield of free nucleons due to models inadequacies in describing light clusters [43–45]. This problem can be largely alleviated by constructing "coalescence invariant" quantities, i.e., nucleon observables summed over all light clusters, which show much better agreement between theory and experiment [36,46–48]. We construct the coalescence invariant (CI) nucleon yield spectra and their ratios in the same way as in Refs. [46–48] by combining the nucleons in the light particles and free nucleons at given kinetic energy per nucleon as follows,