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A note on the resolution of the entropy discrepancy

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ABSTRACT

It was found by Hung, Myers and Smolkin that there is entropy discrepancy for the CFTs in 6-dimensional space–time, between the field theoretical and the holographic analyses. Recently, two different resolutions to this puzzle have been proposed. One of them suggests to utilize the anomaly-like entropy and the generalized Wald entropy to resolve the HMS puzzle, while the other one initiates the use of the entanglement entropy which arises from total derivative terms in the Weyl anomaly to explain the HMS mismatch. We investigate these two proposals carefully in this note. By studying the CFTs dual to Einstein gravity, we find that the second proposal cannot solve the HMS puzzle. Moreover, the Wald entropy formula is not well-defined on horizon with extrinsic curvatures, in the sense that, in general, it gives different results for equivalent actions.

1. Introduction

Hung, Myers and Smolkin (HMS) found that the field theoretical and holographic logarithmic terms of entanglement entropy do not match for 6d CFTs [1]. For simplicity, we denote this entropy discrepancy as 'HMS puzzle' or 'HMS mismatch' in this note. Recently, two different approaches were proposed to resolve this entropy discrepancy. One of them suggests to utilize the anomaly-like entropy and the generalized Wald entropy derived from the Weyl anomaly to solve the HMS puzzle [2]. While the other one initiates to use the entropy which arises from total derivative terms in the Weyl anomaly to explain the HMS mismatch [3,4]. The question as to which proposal is correct is an important problem. We clarify this issue in this note.

It is worth to point out that the results in [3,4] are crucially based on the regularization given in [5]. If the Lewkowycz–Maldacena regularization [6,7] is applied instead, the entropy of covariant total derivatives vanishes [8]. This implies that the proposal of [3,4] is unreliable. In this note, we give a solid proof that the approach in [3,4] actually fails in solving the HMS puzzle.

It is counterintuitive that total derivative terms in the Weyl anomaly, arising from cohomologically trivial solutions to the Wess–Zumino consistency conditions, contribute to non-zero entropy. Given this fact, the logarithmic term of entanglement entropy of CFTs would depend on the approaches of regularization [3,4]. However, entropy is physical and thus should be independent of the choices of regularization. The authors of [3,4] argued that this is not a problem for 4d CFTs, since no total derivative term appears in the holographic Weyl anomaly in 4d space–time [9]. Nevertheless, total derivatives do appear in the holographic Weyl anomaly in 6d space–time. The authors of [3,4] propose to utilize the entropy arising from these total derivative terms to explain the HMS mismatch. They did not take into account all the total derivative terms but only part of them to resolve the HMS mismatch [4].

In this note, we apply the method of [3,4] to investigate the logarithmic term of entanglement entropy for 6d CFTs dual to Einstein gravity. In contrast to [4], we examine all the total derivative terms in the holographic Weyl anomaly and find that the field theoretical result does not match the holographic analysis. Thus, the proposal of [3,4] does not resolve the HMS puzzle [1]. This is the main new result of this note.

We also find that the Wald entropy formula

$$S_{\text{Wald}} = -2\pi \int_{\Sigma} dx^{D-2}\sqrt{h}\frac{\delta L}{\delta R_{ijkl}}\epsilon_{ij}\epsilon_{kl} \tag{1}$$

is not well-defined on the horizon with non-zero extrinsic curvatures. In general, it is inconsistent with the Bianchi identities. It turns out that only the total gravitational entropy, which consists

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of Wald entropy [16], the anomaly-like entropy [5,7,13] and the generalized Wald entropy [2], is well-defined. Similar to the Weyl anomaly, the anomaly-like entropy arises from the would-be logarithmic terms in the gravitational action [7]. Notice that it only appears in the higher curvature gravity rather than Einstein gravity. In addition to Wald entropy [16] and the anomaly-like entropy [5,7,13], a new component of entropy appears in general higher derivative gravity. It is named as 'generalized Wald entropy' in [2] because of its similarity to Wald entropy. In this note, we mainly focus on the total gravitational entropy and denote it as the total entropy below for simplicity.

The note is organized as follows. In Section 2, we briefly review the HMS entropy discrepancy [1] and two possible resolutions [2–4]. In Section 3, the method of [3,4] is employed to calculate the logarithmic term of entanglement entropy for 6d CFTs dual to Einstein gravity, while in Section 4 we apply the method of [2]. It turns out that it is the proposal of [2] rather than the one of [3,4] that can resolve the HMS puzzle. Further evidences for this conclusion are provided in Section 5. In Section 6, we show that, in general, Wald entropy gives different results for equivalent actions, while the total entropy is indeed well-defined. We conclude with some discussions in Section 7.

2. The HMS entropy discrepancy

In this section, we briefly review the HMS entropy discrepancy [1]. It was found by Hung, Myers and Smolkin that the logarithmic term of entanglement entropy derived from the field theoretical approach does not agree with the holographic result for 6d CFTs [1]. For simplicity, they only focus on the case with zero extrinsic curvature.

In the field theoretical approach, the logarithmic term of entanglement entropy can be derived by taking the Weyl anomaly as a gravitational action and then calculating the 'entropy' of this 'action' [1,5]. It turns out that this 'entropy' equals to the logarithmic term of entanglement entropy for CFTs [1,5]. In 6-dimensional space–time, the Weyl anomaly of CFTs takes the following form

$$\langle T^i{}_i \rangle = \sum_{n=1}^{3} B_n\, I_n + 2A\, E_6 + \nabla_i \hat{J}^i, \tag{2}$$

where B_i, A are central charges, E_6 is the Euler density, $\nabla_i \hat{J}^i$ are total derivative terms and I_i are conformal invariants given by

$$I_1 = C_{kijl} C^{imnj} C_m{}^{kl}{}_n\,, \qquad I_2 = C_{ij}{}^{kl} C_{kl}{}^{mn} C_{mn}{}^{ij}\,, \tag{3}$$

$$I_3 = C_{iklm}(\nabla^2\, \delta^i_j + 4R^i{}_j - \frac{6}{5}\, R\, \delta^i_j) C^{jklm}\,. \tag{4}$$

For the entangling surfaces with the rotational symmetry, only Wald entropy contributes to holographic entanglement entropy. Thus, we have [1]

$$S = \log(\ell/\delta) \int d^4y\sqrt{h} \left[2\pi \sum_{n=1}^{3} B_n \frac{\partial I_n}{\partial R^{ij}{}_{kl}}\, \tilde{\varepsilon}^{ij}\, \tilde{\varepsilon}_{kl} + 2\, A\, E_4 \right]_{\Sigma}, \tag{5}$$

where

$$\frac{\partial I_1}{\partial R^{ij}{}_{kl}}\, \tilde{\varepsilon}^{ij}\, \tilde{\varepsilon}_{kl} = 3\left(C^{jmnk}\, C_m{}^{il}{}_n \tilde{\varepsilon}_{ij}\, \tilde{\varepsilon}_{kl} - \frac{1}{4}\, C^{iklm}\, C^j{}_{klm}\, \tilde{g}^{\perp}_{ij} + \frac{1}{20}\, C^{ijkl}\, C_{ijkl} \right), \tag{6}$$

$$\frac{\partial I_2}{\partial R^{ij}{}_{kl}}\, \tilde{\varepsilon}^{ij}\, \tilde{\varepsilon}_{kl} = 3\left(C^{klmn}\, C_{mn}{}^{ij}\, \tilde{\varepsilon}_{ij}\, \tilde{\varepsilon}_{kl} - C^{iklm}\, C^j{}_{klm}\, \tilde{g}^{\perp}_{ij} + \frac{1}{5}\, C^{ijkl}\, C_{ijkl} \right), \tag{7}$$

$$\begin{aligned}\frac{\partial I_3}{\partial R^{ij}{}_{kl}}\, \tilde{\varepsilon}^{ij}\, \tilde{\varepsilon}_{kl} = {}& 2\left(\Box\, C^{ijkl} + 4\, R^i{}_m C^{mjkl} - \frac{6}{5}\, R\, C^{ijkl} \right) \tilde{\varepsilon}_{ij}\, \tilde{\varepsilon}_{kl} \\ & - 4\, C^{ijkl}\, R_{ik}\, \tilde{g}^{\perp}_{jl} + 4\, C^{iklm}\, C^j{}_{klm}\, \tilde{g}^{\perp}_{ij} - \frac{12}{5}\, C^{ijkl}\, C_{ijkl}.\end{aligned} \tag{8}$$

Here C_{ijkl} are the Weyl tensors, l is the length scale of the entangling surface Σ and δ is the short-distance cut-off that we use to regulate the calculations. h_{ij} and y^a are the induced metric and coordinates on the entangling surface Σ, respectively. $\tilde{\varepsilon}_{ij}$ and $\tilde{g}^{\perp}_{ij}$ are the two-dimensional volume form and metric in the space transverse to Σ, respectively.

The logarithmic term of entanglement entropy can also be derived from the holographic entanglement entropy. We call this method as the holographic approach. Taking Einstein gravity as an example, the logarithmic term of entanglement entropy is given by [1]

$$S = 4\pi\, \log(\ell/\delta) \int_{\Sigma} d^4y\sqrt{h} \left[\frac{1}{2} h^{ij} \overset{(2)}{g}_{ij} + \frac{1}{8} (h^{ij} \overset{(1)}{g}_{ij})^2 - \frac{1}{4} \overset{(1)}{g}_{ij}\, h^{jk}\, \overset{(1)}{g}_{kl}\, h^{li} \right] \tag{9}$$

where we have set Newton's constant $G = \frac{1}{16\pi}$ the AdS radius $L = 1$. The definitions of $g^{(n)}_{ij}$ can be found in the Fefferman–Graham expansion, i.e., $g_{ij} = g^{(0)}_{ij} + \rho g^{(1)}_{ij} + \rho^2 g^{(2)}_{ij} + \ldots$, for the asympotically Anti-de Sitter space

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j. \tag{10}$$

Note that x^i with $(i = 1, 2, \ldots, 6)$ are the coordinates on the boundary of AdS and y^a with $(a = 1, 2, \ldots, 4)$ are the coordinates on the entangling surface Σ.

The mismatch between holographic result, eq. (9), and field theoretical result, eq. (5), becomes

$$\begin{aligned}\Delta S = -4\pi\, B_3 \log(\ell/\delta) \int_{\Sigma} d^4y\sqrt{h} (& C_{mn}{}^{rs} C^{mnkl} \tilde{g}^{\perp}_{sl}\, \tilde{g}^{\perp}_{rk} \\ & - C_{mnr}{}^{s} C^{mnrl} \tilde{g}^{\perp}_{sl} + 2C_m{}^n{}_r{}^s C^{mkrl} \tilde{g}^{\perp}_{ns} \tilde{g}^{\perp}_{kl} \\ & - 2C_m{}^n{}_r{}^s C^{mkrl} \tilde{g}^{\perp}_{nl} \tilde{g}^{\perp}_{ks}).\end{aligned} \tag{11}$$

This is the HMS mismatch [1]. Note that the above equations are derived in the case of zero extrinsic curvatures.

It is proposed to use the anomaly-like entropy and the generalized Wald entropy to explain the HMS mismatch in [2]. When the extrinsic curvatures vanish, only $C^2_{ijkl} C^{ijkl} \simeq -\nabla_m C_{ijkl} \nabla^m C^{ijkl}$ in I_3 contributes to non-zero anomaly-like entropy. Taking into account this contribution, the field theoretical and the holographic results match exactly. Note that the entropy of total derivative terms vanishes by applying the Lewkowycz–Maldacena regularization [6,7]. However, the authors of [3,4] claim that, in addition to $-\nabla_m C_{ijkl} \nabla^m C^{ijkl}$, the total derivative terms $B_3 \nabla_m (C_{ijkl} \nabla^m C^{ijkl}) + \nabla_i \hat{J}^i$ also contribute to the logarithmic term of entanglement entropy. They find that the entropy from total derivative terms is non-zero by applying the regularization of [5]. And they suggest to utilize the entropy from total derivative terms to explain the HMS puzzle [3,4]. Whether total derivative terms contribute to non-zero