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Singularity avoidance in classical gravity from four-fermion interaction

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ABSTRACT

We derive the dynamics of the gravitational collapse of a homogeneous and spherically symmetric cloud in a classical set-up endowed with a topological sector of gravity and a non-minimal coupling to fermions. The effective theory consists of the Einstein–Hilbert action plus Dirac fermions interacting through a four-fermion vertex. At the classical level, we obtain the same picture that has been recently studied by some of us within a wide range of effective theories inspired by a super-renormalizable and asymptotically free theory of gravity. The classical singularity is replaced by a bounce, beyond which the cloud re-expands indefinitely. We thus show that, even at a classical level, if we allow for a non-minimal coupling of gravity to fermions, event horizons may never form for a suitable choice of some parameters of the theory.

In the previous work, some of us have studied the gravitational collapse in a wide class of asymptotically free theories of gravity [1]. There was found a picture that substantially differs from the standard scenario. The central singularity that appears in classical general relativity is replaced by a bounce, after which the collapsing body starts expanding. It was argued that, strictly speaking, black holes never form, in the sense that there are no regions causally disconnected to future null infinity. The collapse can only produce a temporary trapped surface, which looks like an event horizon for an observational timescale much shorter than the one of the collapse. While this time interval is of order a dynamical timescale for a comoving observer, it is definitively long for an observer in the exterior metric that is far away from the collapsing body. For all practical purposes these objects are therefore like black holes. Similar studies have been presented in Ref. [2]. In the present work, we show that the same picture can be found in classical general relativity, when we extend the gravitational sector to include topological terms and we consider an experimentally allowed non-minimal coupling of fermions in the Dirac action.

We can start from the non-minimal Einstein–Cartan–Holst (ECH) action, as cast by Bojowald and Das in [3]

$$S[e,A,\psi]=S_G[e,A]+S_F[e,A,\psi]$$
$$=\frac{1}{2\kappa}\int d^4x\,|e|e_I^\mu e_J^\nu P^{IJ}{}_{KL}F_{\mu\nu}{}^{KL}(A)$$
$$+\frac{i}{2}\int d^4x|e|\left[\overline{\psi}\gamma^I e_I^\mu\left(1-\frac{i}{\alpha}\gamma_5\right)\nabla_\mu\psi + im\overline{\psi}\psi+\text{h.c.}\right], \qquad (1)$$

where $\kappa=8\pi G_{\mathrm{N}}$ is the reduced Planck length square. Notice the presence of a non-minimal coupling parameter $\alpha\in\mathbb{R}$, which has been first introduced by Freidel, Minic and Takeuchi in [4], but without γ_5. This γ_5 turns out to be crucial for parity invariance and was introduced by Mercury in [5]. The experimental bounds for α and γ arising from lepton-quark contact interactions are discussed in [4]. The operator

$$P^{IJ}{}_{KL}=\delta_K^{[I}\delta_L^{J]}-\frac{1}{2\gamma}\epsilon^{IJ}{}_{KL}, \qquad (2)$$

where ϵ_{IJKL} is the Levi-Civita symbol, is defined in terms of the Barbero–Immirzi parameter γ, and can be inverted for $\gamma^2\neq -1$. As shown in [5], the Einstein–Cartan action is recovered for $\alpha=\gamma$, with a term that reduces to the Nieh–Yan invariant when the second Cartan structure equation holds. This case is referred to as minimal coupling in the Einstein–Cartan theory. From the point of view of the Holst action, minimal coupling is met in the limit $\alpha\to\pm\infty$.

The covariant derivative ∇_μ of Dirac spinors and the field-strength of the Lorentz connection are defined by

$$\nabla_\mu\equiv\partial_\mu+\frac{1}{4}A_\mu^{IJ}\gamma_{[I}\gamma_{J]},\qquad [\nabla_\mu,\nabla_\nu]=\frac{1}{4}F_{\mu\nu}^{IJ}\gamma_{[I}\gamma_{J]}. \qquad (3)$$

Because of the presence of fermions, a torsional part of the connection enters the non-minimal ECH action. Nevertheless, we can

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follow here the procedure used by Perez and Rovelli in [6], and integrate out of the theory the torsional part of the connection through the Cartan equation, which is found by varying the total action with respect to the connection A_μ^{IJ}. The variation of the action with respect to the connection A gives

$$P^{IJ}_{KL}\nabla_\mu\left(e e^\mu_I e^\nu_J\right)=\kappa e J^\nu_{KL}. \tag{4}$$

This equation can then be solved for the connection. For this purpose, we write the connection in the form

$$A_\mu^{IJ}=\omega(e)^{IJ}_\mu+C^{IJ}_\mu, \tag{5}$$

where C^{IJ}_μ is the contorsion tensor and $\omega(e)$ is the torsion free spin connection determined by e, namely the solution of $\widetilde{\nabla}_{[\mu}e^I_{\nu]}=0$. Note that we have introduced a new definition for the covariant derivative compatible with the tetrad e^I_μ,

$$\widetilde{\nabla}_\mu\equiv\partial_\mu+\frac{1}{4}\omega^{IJ}_\mu\gamma_{[I}\gamma_{J]}. \tag{6}$$

Replacing the definition (5) in (4) we find

$$C^\mu_{\mu[I}e^\nu_{J]}+C^\nu_{[IJ]}=\kappa\left(P^{-1}\right)^{KL}_{IJ}eJ^\nu_{KL}, \tag{7}$$

in which

$$J^\nu_{KL}=e\frac{1}{4}e^\nu_I\epsilon^I{}_{KLM}\overline{\psi}\gamma_5\gamma^M\psi-\frac{1}{2\alpha}e^{\nu I}\eta_{I[K}\overline{\psi}\gamma_5\gamma_{L]}\psi. \tag{8}$$

Note that we transform internal and spacetime indices into one another, using the tetrad field, and preserving the horizontal order of the indices. Then the Cartan equation expresses the contortion tensor C^{IJ}_μ in terms of the fermionic fields and tetrads

$$e^\mu_I C_{\mu JK}=\frac{\kappa}{4}\frac{\gamma}{\gamma^2+1}\left(\beta\epsilon_{IJKL}J^L-2\theta\eta_{I[J}J_{K]}\right), \tag{9}$$

having introduced the flat metric η_{IJ}, the fermionic axial current $J^L=\overline{\psi}\gamma^L\gamma_5\psi$, and the coefficients, functions of the free parameters within the non-minimal ECH theory, $\beta=\gamma+1/\alpha$ and $\theta=1-\gamma/\alpha$. Thanks to (9) the non-minimal ECH action recasts in terms of the metric compatible connection, as a sum of the Einstein–Hilbert action and the Dirac action. The latter is now written in terms of metric compatible variables, and is further provided with novel interaction terms, which capture the new physics within the non-minimal ECH theory. Consequences of this new interaction term in cosmology have been investigated by Alexander, Biswas and Calcagni in [7]. The theory can be rewritten as

$$\begin{aligned}S[e,A,\psi]&=S_G[e,\omega]+S_F[e,\omega,\psi]+S_{\text{int}}[e,\psi]\\&=\frac{1}{2\kappa}\int d^4x|e|e^\mu_I e^\nu_J F^{IJ}_{\mu\nu}(\omega)\\&\quad+\frac{i}{2}\int d^4x\,|e|\left(\overline{\psi}\gamma^I e^\mu_I\nabla_\mu\psi-\overline{\nabla_\mu\psi}\gamma^I e^\mu_I\psi+im\overline{\psi}\psi\right)\\&\quad-\kappa\xi\int d^4x\,|e|\left(\bar{\psi}\gamma_5\gamma^L\psi\right)\left(\bar{\psi}\gamma_5\gamma_L\psi\right),\end{aligned} \tag{10}$$

where

$$\xi=\frac{3}{16}\frac{\gamma^2}{\gamma^2+1}\left(1+\frac{2}{\alpha\gamma}-\frac{1}{\alpha^2}\right). \tag{11}$$

Einstein equations $G_{\mu\nu}=\kappa T_{\mu\nu}$ provide the dynamics for the gravitational field e^I_μ, and must be coupled to the equations of motion for fermionic matter and radiation. We have denoted with $G_{\mu\nu}$ the Einstein tensor and the stress-energy tensor is

$$T_{\mu\nu}=\frac{e_{\mu I}}{|e|}\frac{\delta(|e|\mathcal{L}_{\text{matt}})}{\delta e^\nu_I}. \tag{12}$$

The fermionic Lagrangian including the interaction reads

$$\mathcal{L}_{\text{fer}}=|e|\left[\frac{1}{2}\left(\overline{\psi}\gamma^I e^\mu_I i\widetilde{\nabla}_\mu\psi-m\overline{\psi}\psi\right)+\text{h.c.}-\kappa\xi J^L J_L\right],$$

which yields the energy–momentum tensor

$$T^{\text{fer}}_{\mu\nu}=\frac{1}{4}\left(\overline{\psi}\gamma_I e^I_\mu i\widetilde{\nabla}_\nu\psi+\overline{\psi}\gamma_I e^I_\nu i\widetilde{\nabla}_\mu\psi\right)+\text{h.c.}-g_{\mu\nu}\mathcal{L}_{\text{fer}}. \tag{13}$$

The Dirac equations on curved background for the interacting system are the following,

$$\gamma^I e^\mu_I i\widetilde{\nabla}_\mu\psi-m\psi=2\xi\kappa\left(\overline{\psi}\psi+\overline{\psi}\gamma_5\psi\gamma_5+\overline{\psi}\gamma_I\psi\gamma^I\right)\psi, \tag{14}$$

in which we have used the Fierz-decomposition

$$\left(\overline{\psi}\gamma_5\gamma^I\psi\right)\left(\overline{\psi}\gamma_5\gamma_I\psi\right)=(\overline{\psi}\psi)^2+(\overline{\psi}\gamma_5\psi)^2+\left(\overline{\psi}\gamma^I\psi\right)\left(\overline{\psi}\gamma_I\psi\right). \tag{15}$$

In what follows, we study the dynamics of the collapse of a homogeneous and spherically symmetric body. In the comoving gauge, the tetrad e^I_μ for the Friedmann–Lemaître–Robertson–Walker (FLRW) type metrics is

$$e^I_0=\delta^I_0\quad\text{and}\quad e^I_j=a(t)\delta^I_j, \tag{16}$$

where a is the FLRW scale factor and t is the comoving time. Solutions of the Dirac equations on curved backgrounds that are suitable to develop cosmological analyses have been studied by Armendariz-Picon and Greene [8]. They resorted to a form of the spinor which allows for the vanishing of the spatial components of the vector (but not of the axial) fermionic current

$$\psi=\left(\psi_0(t),0,0,0\right). \tag{17}$$

This ensures homogeneity and isotropy on spatial hyper-surfaces for theories in which a cooling between vector current and any other observable vector quantity is present. We then simplify the Dirac equation using their *ansatz*, which still holds in our framework due to the appearance of only quadratic powers of J_L. Within the comoving gauge, the only non-vanishing spin connection components for $\omega^{IJ}{}_K=\omega^{IJ}_\mu e^\mu_K$ are $\omega_{0ij}=-\omega_{i0j}=-H\delta_{ij}$. This implies $\widetilde{\nabla}_0=\partial_0$ and $\widetilde{\nabla}_i=\partial_i+aH/2\delta_{ij}\text{diag}(\sigma^j,-\sigma^j)$. The Dirac equation then reads

$$\dot{\psi}_0+\frac{3}{2}H\psi_0+i\left(m+4\kappa\xi\psi^*_0\psi_0\right)\psi_0=0, \tag{18}$$

where * denotes complex conjugation. The equation of motion for the bilinear $\psi^*_0\psi_0$ is

$$\frac{d}{dt}\psi^*_0\psi_0+3H\psi^*_0\psi_0=0, \tag{19}$$

and yields the familiar a^{-3} scaling for the particle number density

$$\psi^*_0\psi_0=n_0/a^3, \tag{20}$$

where n_0 is a constant. With the use of Eq. (20), the first Friedmann equation reads

$$H^2=\frac{\kappa m}{3}\frac{n_0}{a^3}+\frac{\kappa^2\xi}{3}\frac{n_0^2}{a^6}. \tag{21}$$