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On the black hole mass decomposition in nonlinear electrodynamics

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ABSTRACT

In the weak field limit of nonlinear Lagrangians for electrodynamics, i.e. theories in which the electric fields are much smaller than the scale (threshold) fields introduced by the nonlinearities, a generalization of the Christodoulou–Ruffini mass formula for charged black holes is presented. It proves that the black hole outer horizon never decreases. It is also demonstrated that reversible transformations are, indeed, fully equivalent to constant horizon solutions for nonlinear theories encompassing asymptotically flat black hole solutions. This result is used to decompose, in an analytical and alternative way, the total mass-energy of nonlinear charged black holes, circumventing the difficulties faced to obtain it via the standard differential approach. It is also proven that the known first law of black hole thermodynamics is the direct consequence of the mass decomposition for general black hole transformations. From all the above we finally show a most important corollary: for relevant astrophysical scenarios nonlinear electrodynamics decreases the extractable energy from a black hole with respect to the Einstein–Maxwell theory. Physical interpretations for these results are also discussed.

1. Introduction

Black hole solutions to the Einstein equations have always attracted the attention of researchers, not only due to their unusual properties, but also from the discovery that they could be one of the most abundant sources of energy in the Universe.

From conservation laws, R. Penrose [1] showed how energy could be extracted from a black hole [2]. D. Christodoulou [3] and D. Christodoulou and R. Ruffini [4], through the study of test particles in Kerr and Kerr–Newman spacetimes [5], quantified the maximum amount of energy that can be extracted from a black hole. These works deserve some comments. First, this maximum amount of energy can be obtained only by means of the there introduced, *reversible processes*. Such processes are the only ones in which a black hole can be brought back to its initial state, after convenient interactions with test particles. Therefore, reversible transformations constitute the most efficient processes of energy extraction from a black hole. Furthermore, it was also introduced in Refs. [3,4] the concept of *irreducible mass*. This mass can never be diminished by any sort of processes and hence would constitute an intrinsic property of the system, namely the *fundamental energy state of a black hole*. This is exactly the case of Schwarzschild black holes. From this irreducible mass, one can immediately verify that the area of a black hole never decreases after any infinitesimal transformation performed on it. Moreover, one can write down the total energy of a black hole in terms of this quantity [4].

Turning to effective nonlinear theories of electromagnetism, their conceptual asset is that they allow the insertion of desired effects such as quantum-mechanical, avoidance of singular solutions, and others e.g. via classical fields [6]. As a first approach, all of these theories are built up in terms of the two local invariants constructed out of the electromagnetic fields [7,8]. Notice that the field equations of nonlinear theories have the generic problem of not satisfying their hyperbolic conditions for all physical situations (see e.g. [9,10]). The aforementioned invariants are assumed to be functions of a four-vector potential in the same functional way as their classic counterparts, being therefore also gauge independent invariants. We quote for instance the Born–Infeld Lagrangian [11], conceived with the purpose of solving the problem of the infinite self-energy of an electron in the classic theory of electromagnetism. The Born–Infeld Lagrangian has gained a renewal of interest since the effective Lagrangian of string theory in its low energy limit has an analog form to it [12]. It has also been minimally coupled to general relativity, leading to an exact

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solution [13,14] and this coupling has been studied in a variety of problems [15–17]. Another worthwhile example of nonlinear electromagnetic theory is the Euler–Heisenberg Lagrangian [18,19]. This Lagrangian allows one to take effectively into account one-loop corrections from the Maxwellian Lagrangian coming from Quantum Electrodynamics (QED), and it has been extensively studied in the literature [6]. Nonlinear theories of electromagnetism have also been investigated in the context of astrophysics. For instance, they could play an important role in the description of the motion of particles in the neighborhood of some astrophysical systems [20], as a simulacrum to dark energy and as a simulacrum of dark energy [21].

In connection with the above discussion, the thermodynamics of black holes [22] in the presence of nonlinear theories of electromagnetism has also been investigated. The zeroth and first laws (see Section 7) have been studied in detail [12,23], allowing the raise of other important issues. We quote for example the difficulty in generalizing the so-called Smarr mass [24,25] (a parametrization of the Christodoulou–Ruffini mass [4]) for nonlinear theories [12]. Many efforts have been pursued in this direction, through the suggestion of systematic ways to write down this mass, which has led to some inconsistencies (see e.g. Ref. [26]). For some specific nonlinear Lagrangians, this problem has been circumvented [27].

We first deal with static spherically symmetric electrovacuum solutions to the Einstein equations minimally coupled to Abelian nonlinear theories of electromagnetism, i.e. nonlinear charged black holes, for electric fields that are much smaller than the scale fields introduced by the nonlinearities, i.e. *weak field* nonlinear Lagrangians. We decompose the total mass-energy of a charged black hole in terms of its characteristic parameters: charge, irreducible mass, and nonlinear scale parameter. We also show the constancy of the black hole outer horizon in the case of reversible transformations. We then generalize the previous results for a generic nonlinear theory leading to an asymptotically flat black hole solution. As an immediate consequence of this general result, we show that the first law of black hole thermodynamics (or mechanics) in the context of nonlinear electrodynamics [12] is a by-product of this mass decomposition. These results also allow us to investigate the extraction of energy from charged black holes in the framework of nonlinear theories of electromagnetism.

The article is organized as follows. In Section 2 the notation is established and the field equations are stated and solved formally in the spherically symmetric case for nonlinear electromagnetic theories that lead to null fields at infinity. In Section 3, reversible transformations are investigated. In Section 4 the field equations are solved for the weak field limit of nonlinear theories of electromagnetism. Section 5 is devoted to the deduction of the total mass-energy of a charged black hole in terms of irreducible and extractable quantities, when reversible transformations are taken into account. In Section 6 variations of the outer horizon associated with the capture of test particles in nonlinear theories of electromagnetism are analyzed. In Section 7, we shall present the way to decompose the energy of a black hole within nonlinear theories of electromagnetism and show that it leads automatically to the first law of black hole mechanics. Finally, in Section 8 we discuss the results of this work. We use geometric units with $c = G = 1$, and metric signature -2.

2. Field equations

The minimal coupling between gravity and nonlinear electrodynamics that depends only on the local invariant F can be stated mathematically through the action

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi} + \frac{L_{em}(F)}{4\pi}\right) \doteq S_g + \frac{S_{em}}{4\pi}, \tag{1}$$

where $F \doteq F^{\mu\nu}F_{\mu\nu}$, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ is the electromagnetic four-potential, R is the Ricci scalar, S_g is the action for the gravitational field, S_{em} is the action of the electromagnetic theory under interest, and g the determinant of the metric $g_{\mu\nu}$ of the spacetime. Under the variation of Eq. (1) with respect to $g^{\mu\nu}$, and applying the principle of least action, one obtains (see e.g. Ref. [8])

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T^{(em)}_{\mu\nu}, \tag{2}$$

with $R_{\mu\nu}$ the Ricci tensor and $T^{(em)}_{\mu\nu}$ the energy–momentum tensor of the electromagnetic field, defined as

$$4\pi T^{(em)}_{\mu\nu} \doteq \frac{2}{\sqrt{-g}}\frac{\delta S_{em}}{\delta g^{\mu\nu}} = 4L^{(em)}_F F_{\mu\alpha}F_{\nu\rho}g^{\alpha\rho} - L^{(em)}g_{\mu\nu}, \tag{3}$$

where $L^{(em)}_F \doteq \partial L^{(em)}/\partial F$.

Application of the principle of least action in Eq. (1) with respect to $A_\mu(x^\beta)$ gives

$$\nabla_\mu\left(L^{(em)}_F F^{\mu\nu}\right) = 0, \tag{4}$$

since we are interested in solutions to general relativity in the absence of sources.

In the static spherically symmetric case, it is possible to solve the Einstein equations minimally coupled to nonlinear electromagnetism theories [see Eqs. (2) and (4)] and due to the form of the energy–momentum tensor in this case the metric must be of the form

$$ds^2 = e^\nu dt^2 - e^{-\nu}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2, \tag{5}$$

with [8,23]

$$e^\nu = 1 - \frac{2M}{r} + \frac{8\pi}{r}\int_r^\infty r'^2 T_0{}^0(r')dr', \tag{6}$$

where the integration constant M stands for the total mass-energy of the black hole as measured by observers at infinity.

Eq. (4) in this special spherically symmetric case reduce to

$$L^{(em)}_F E_r r^2 = -\frac{Q}{4}, \tag{7}$$

where Q is an arbitrary constant representing physically the total charge of the black hole.

If one defines

$$E_r \doteq -\frac{\partial A_0}{\partial r} \quad \text{and} \quad \frac{\partial \mathcal{F}}{\partial r} \doteq -L^{(em)}r^2, \tag{8}$$

and take into account Eqs. (4), (5) and (7), then Eq. (6) can be rewritten as

$$e^\nu = 1 - \frac{2M}{r} + \frac{2QA_0}{r} - \frac{2\mathcal{F}}{r}, \tag{9}$$

where it has been imposed a gauge such that the scalar potential A_0 goes to zero when the radial coordinate goes to infinity, which also holds to $\mathcal{F}$. These conditions guarantee that the associated nonlinear black holes are asymptotically flat (Minkowskian). In this work we are not interested in Lagrangian densities which do not fulfill this condition.

The black hole horizons are given by the solutions to

$$g_{00}(r_h) = e^{\nu(r_h)} = 0. \tag{10}$$