Physics Letters B 734 (2014) 7–12

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Structure of longitudinal chromomagnetic fields in high energy collisions

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ARTICLE INFO

Article history:
Received 23 January 2014
Received in revised form 29 April 2014
Accepted 4 May 2014
Available online 9 May 2014
Editor: J.-P. Blaizot

ABSTRACT

We compute expectation values of spatial Wilson loops in the forward light cone of high-energy collisions. We consider ensembles of gauge field configurations generated from a classical Gaussian effective action as well as solutions of high-energy renormalization group evolution with fixed and running coupling. The initial fields correspond to a color field condensate exhibiting domain-like structure over distance scales of order the saturation scale. At later times universal scaling emerges at large distances for all ensembles, with a nontrivial critical exponent. Finally, we compare the results for the Wilson loop to the two-point correlator of magnetic fields.

1. Introduction

Heavy ion collisions at high energies involve non-linear dynamics of strong QCD color fields [1]. These soft fields correspond to gluons with light-cone momentum fractions $x \ll 1$, which can be described in the "Color Glass Condensate" (CGC) framework. Because of the high gluon occupation number the gluon field can be determined from the classical Yang–Mills equations with a static current on the light cone [2]. It consists of gluons with a transverse momentum on the order of the density of valence charges per unit transverse area, Q_s^2 [3]. Parametrically, the saturation momentum scale Q_s separates the regime of non-linear color field interactions from the perturbative (linear) regime. It is commonly defined using a two-point function of electric Wilson lines, the "dipole scattering amplitude" evaluated in the field of a single hadron or nucleus [4] as described below.

Before the collision the individual fields of projectile and target are two dimensional pure gauges; in light cone gauge,

$$\alpha_m^i = \frac{i}{g} V_m \partial^i V_m^\dagger \tag{1}$$

where $m = 1, 2$ labels the projectile and target, respectively. Here V_m are light-like SU(N_c) Wilson lines, which correspond to the eikonal phase of a high energy projectile passing through the classical field shockwave [5,6].

The field in the forward light cone after the collision up to the formation of a thermalized plasma is commonly called the "glasma" [7]. Immediately after the collision longitudinal chromoelectric and magnetic fields $E_z, B_z \sim 1/g$ dominate [7,8]. They fluctuate according to the random local color charge densities of the valence sources. The magnitude of the color charge fluctuations is related to the saturation scale Q_s^2. The transverse gauge potential at proper time $\tau \equiv \sqrt{t^2 - z^2} \to 0$, is given by [9]

$$A^i = \alpha_1^i + \alpha_2^i. \tag{2}$$

Note that while the fields of the individual projectiles α_m^i are pure gauges, for a non-Abelian gauge theory A^i is not. Hence, spatial Wilson loops evaluated in the field A^i are not equal to 1. The field at later times is then obtained from the classical Yang–Mills equations of motion, which can be solved either analytically in an expansion in the field strength [9,10] or numerically on a lattice [11–13]. The Wilson loop, and the magnetic field correlator, provide an explicitly gauge-invariant method to study the nonperturbative dynamics of these fields, complementary to studies of the gluon spectrum [14].

Spatial Wilson loops at very early times τ have recently been studied numerically in Ref. [15], using the MV model [7,8]. for the colliding color charge sheets. It was observed that the loops effectively satisfy area law scaling for radii $\gtrsim 1/Q_s$, up to a few times this scale. Furthermore, Ref. [16] found that two-point correlators of B_z over distances $\lesssim 1/Q_s$ correspond to two dimensional screened propagators with a magnetic screening mass a few

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http://dx.doi.org/10.1016/j.physletb.2014.05.005
0370-2693/

times Q_s. This indicates that the initial fields exhibit *structure* such that magnetic flux does not spread uniformly over the transverse plane (like in a Coulomb phase) but instead is concentrated in small domains.

The present paper extends this previous work as follows. We perform lattice measurements of spatial Wilson loops over a much broader range of radii to analyze their behavior at short ($R \ll 1/Q_s$) and long ($R \gg 1/Q_s$) distances. We also implement the so-called JIMWLK [3,17,18] high-energy functional renormalization group evolution which resums observables to all orders in $\alpha_s \log(1/x)$. High-energy evolution modifies the classical ensemble of gauge field configurations (4), (5) to account for nearly boost invariant quantum fluctuations at rapidities far from the sources. Finally, we also solve the Yang–Mills equations in the forward light cone to study the time evolution of magnetic flux loops.

The calculation of the initial conditions and the numerical solution of the classical boost-invariant[1] Yang–Mills fields in the initial stages of a heavy ion collision have been documented in the references given below, so here we will only describe them very briefly in Section 2 before moving on to show our results in Sections 3 and 4.

2. Lattice implementation

We work on a two dimensional square lattice of $N_\perp^2$ points with periodic boundary conditions and consider color sources that fill the whole transverse plane. The lattice spacing is denoted as a, thus the area of the lattice in physical units is $L^2 = N_\perp^2 a^2$. The calculations are performed for $N_c = 3$ colors. In this work we only consider symmetric collisions, where the color charges of both colliding nuclei are taken from the same probability distribution.

In this work we compare three different initial conditions for the classical Yang–Mills equations: the classical MV model (5) as well as fixed and running coupling JIMWLK evolution. We define the saturation scale $Q_s(Y)$ at rapidity Y through the expectation value of the dipole operator as

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, V^\dagger(\mathbf{x}_T) V(\mathbf{y}_T)\right\rangle_{Y,|\mathbf{x}_T-\mathbf{y}_T|=\sqrt{2}/Q_s} = e^{-1/2}. \tag{3}$$

Throughout this paper we shall use Q_s defined in this way from the light-like Wilson lines $V(\mathbf{x}_T)$ in the fundamental representation. The saturation scale is the only scale in the problem and we attempt to construct the various initial conditions in such a way that the value of $Q_s a$ is similar, to ensure a similar dependence on discretization effects.

In the MV model the Wilson lines are obtained from a classical color charge density ρ as

$$V(\mathbf{x}_T) = \mathbb{P}\exp\left\{ i\int \mathrm{d}x^-\, g^2 \frac{1}{\boldsymbol{\nabla}_T^2}\rho^a\left(\mathbf{x}_T, x^-\right)\right\}, \tag{4}$$

where $\mathbb{P}$ denotes path-ordering in x^-. The color charge density is a random variable with a local Gaussian probability distribution

$$P\left[\rho^a\right] \sim \exp\left\{ -\int \mathrm{d}^2\mathbf{x}_T\, \mathrm{d}x^-\, \frac{\rho^a(\mathbf{x}_T, x^-)\rho^a(\mathbf{x}_T, x^-)}{2\mu^2(x^-)}\right\}. \tag{5}$$

The total color charge $\int \mathrm{d}x^-\, \mu^2(x^-) \sim Q_s^2$ is proportional to the thickness of a given nucleus.

In the numerical calculation the MV model initial conditions have been constructed as described in Ref. [13], discretizing the longitudinal coordinate Y in $N_y = 100$ steps. For the calculations using the MV model directly for the initial conditions (1), (2) we have performed simulations on lattices of two different sizes: $N_\perp = 1024$, with the MV model color charge parameter $g^2\mu L = 156$ which translates into $Q_s a = 0.119$; and with $N_\perp = 2048$, using $g^2\mu L = 550$, which results in $Q_s a = 0.172$.

The MV model also provides the configurations used as the initial condition for quantum evolution in rapidity via the JIMWLK renormalization group equation, starting at $Y = \log x_0/x = 0$. Performing a step ΔY in rapidity opens phase space for radiation of additional gluons which modify the classical action (4), (5). This process can be expressed as a "random walk" in the space of light-like Wilson lines $V(\mathbf{x}_T)$ [18–20]:

$$\begin{aligned}\partial_Y V(\mathbf{x}_T) = {} & V(\mathbf{x}_T)\frac{i}{\pi}\int \mathrm{d}^2\mathbf{u}_T\, \frac{(\mathbf{x}_T-\mathbf{u}_T)^i\eta^i(\mathbf{u}_T)}{(\mathbf{x}_T-\mathbf{u}_T)^2} \\ & - \frac{i}{\pi}\int \mathrm{d}^2\mathbf{v}_T\, V(\mathbf{v}_T)\frac{(\mathbf{x}_T-\mathbf{v}_T)^i\eta^i(\mathbf{v}_T)}{(\mathbf{x}_T-\mathbf{v}_T)^2} V^\dagger(\mathbf{v}_T)V(\mathbf{x}_T),\end{aligned} \tag{6}$$

where the Gaussian white noise $\eta^i = \eta^i_a t^a$ satisfies $\langle \eta^a_i(\mathbf{x}_T)\rangle = 0$ and, for fixed coupling,

$$\left\langle \eta^a_i(\mathbf{x}_T)\eta^b_j(\mathbf{y}_T)\right\rangle = \alpha_s \delta^{ab}\delta_{ij}\delta^{(2)}(\mathbf{x}_T-\mathbf{y}_T). \tag{7}$$

Here the equation is written in the left–right symmetric form introduced in [20,21].

The fixed coupling JIMWLK equation is solved using the numerical method developed in [19,20,22]. For the smaller lattice size $N_\perp = 1024$ we start with the MV model with $g^2\mu L = 31$ and without a mass regulator, which corresponds to an initial $Q_s a = 0.0218$. After $\Delta y = 1.68/\alpha_s$ units of evolution in rapidity[2] this results in $Q_s a = 0.145$. For an $N_\perp = 2048$-lattice we again start with $g^2\mu L = 31$, corresponding to $Q_s a = 0.0107$, and after $\Delta y = 1.8/\alpha_s$ units of evolution end up with $Q_s a = 0.141$.

For running coupling the evolution is significantly slower. We use the running coupling prescription introduced in [20], where the scale of the coupling is taken as the momentum conjugate to the distance in the noise correlator in Eq. (7). For the smaller $N_\perp = 1024$ lattice we again start with $g^2\mu L = 31$, i.e. $Q_s a = 0.0218$ and evolve for $\Delta Y = 10$ units in rapidity, arriving at $Q_s a = 0.118$. For the larger $N_\perp = 2048$ lattice we test a configuration that is farther from the IR cutoff, starting the JIMWLK evolution with $g^2\mu L = 102.4$, i.e. $Q_s a = 0.0423$ and evolve for $\Delta Y = 10$ units in rapidity, arriving at $Q_s a = 0.172$. In the rc-JIMWLK simulations the QCD scale is taken as $\Lambda_{\mathrm{QCD}} a = 0.00293$ and the coupling is frozen to a value $\alpha_0 = 0.76$ in the infrared below $2.5\Lambda_{\mathrm{QCD}}$.

As already mentioned above, RG evolution in rapidity resums quantum corrections to the fields α^μ_m of the individual charge sheets to all orders in $\alpha_s \log 1/x$, with leading logarithmic accuracy. In other words, the effective action at Y is modified from that at $Y = 0$, written in Eq. (5).

Once an ensemble of Wilson lines $V(\mathbf{x}_T)$ at a rapidity Y is constructed, separately for both projectile and target, these configurations define α^i_1 and α^i_2 in light-cone gauge as written in Eq. (1); the initial field A^i of produced soft gluons at proper time $\tau = +0$ corresponds to their sum, Eq. (2). The evolution to $\tau > 0$ follows from the real-time Hamiltonian evolution described in Ref. [11]. This has been used in many classical field calculations,

[1] The YM equations are solved in terms of the coordinates $\tau = \sqrt{t^2 - z^2}$, $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$ and $\mathbf{x}_T$; hence $\mathrm{d}s^2 = \mathrm{d}\tau^2 - \tau^2\,\mathrm{d}\eta^2 - \mathrm{d}\mathbf{x}_T^2$.

[2] For fixed coupling the evolution variable is $\alpha_s y$, so we do not need to specify a particular value of α_s separately.