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Two-loop evolution equations for light-ray operators

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ABSTRACT

QCD in non-integer $d = 4 - 2\epsilon$ space–time dimensions possesses a nontrivial critical point and enjoys *exact* scale and conformal invariance. This symmetry imposes nontrivial restrictions on the form of the renormalization group equations for composite operators in physical (integer) dimensions and allows to reconstruct full kernels from their eigenvalues (anomalous dimensions). We use this technique to derive two-loop evolution equations for flavor-nonsinglet quark–antiquark light-ray operators that encode the scale dependence of generalized hadron parton distributions and light-cone distribution amplitudes in the most compact form.

1. Studies of hard exclusive reactions contribute significantly to the research program at all major existing and planned accelerator facilities. The relevant nonperturbative input in such processes involves operator matrix elements between states with different momenta, dubbed generalized parton distributions (GPDs), or vacuum-to-hadron matrix elements related to light-front hadron wave functions at small transverse separations, the distribution amplitudes (DAs). Scale dependence of these distributions is governed by the renormalization group (RG) equations for the corresponding (nonlocal) operators and has to be calculated to a sufficiently high order in perturbation theory in order to make the QCD description of exclusive reactions fully quantitative. At present, the evolution equations for GPDs (and DAs) are known to the two-loop accuracy [1–3], one order less compared to the corresponding "inclusive" distributions that involve forward matrix elements [4,5] and closing this gap is desirable. The direct calculation is very challenging, and also finding a suitable representation for the results may become a problem as the two-loop evolution equations for GPDs are already very cumbersome.

It has been known for some time [6] that conformal symmetry of the QCD Lagrangian allows one to restore full evolution kernels at given order of perturbation theory from the spectrum of anomalous dimensions at the same order, and the calculation of the special conformal anomaly at one order less. This result was used to calculate the complete two-loop mixing matrix for twist-two operators in QCD [7–9], and derive the two-loop evolution kernels in momentum space for the GPDs [1–3]. In Ref. [10] we have suggested an alternative technique, the difference being that instead of studying conformal symmetry *breaking* in the physical theory [7–9] we make use of the *exact* conformal symmetry of a modified theory – QCD in $d = 4 - 2\epsilon$ dimensions at critical coupling. Exact conformal symmetry allows one to use algebraic group-theory methods to resolve the constraints on the operator mixing and also suggests the optimal representation for the results in terms of light-ray operators. In this way a delicate procedure of the restoration of the evolution kernels from the results for local operators is completely avoided. We expect that these features will become increasingly advantageous in higher orders.

This modified approach was illustrated in [10] on several examples to the two- and three-loop accuracy for scalar theories. Application to gauge theories, in particular QCD, involves several subtleties that are considered in this work. The main new result are the two-loop evolution equations for flavor-nonsinglet quark–antiquark light-ray operators that encode the scale dependence of generalized hadron parton distributions and light-cone distribution amplitudes in the most compact form.

2. Before going over to technical details, let us first describe the general structure of the approach and the results on a more qualitative level. In order to make use of the (approximate) conformal symmetry of QCD it is natural to use a coordinate-space representation in which the symmetry transformations have a simple form [11]. The relevant objects are light-ray operators that can be understood as generating functions for the renormalized leading-twist local operators:

$$[\mathcal{O}](x; z_1, z_2) \equiv \big[\bar{q}(x + z_1 n)\not{n} q(x + z_2 n)\big]$$
$$\equiv \sum_{m,k} \frac{z_1^m z_2^k}{m!k!}\big[\big(D_n^m \bar{q}\big)(x)\not{n}\big(D_n^k q\big)(x)\big]. \qquad (1)$$

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Here $q(x)$ is a quark field, the Wilson line is implied between the quark fields on the light-cone, $D_n = n_\mu D^\mu$ is a covariant derivative, n^μ is an auxiliary light-like vector, $n^2 = 0$, that ensures symmetrization and subtraction of traces of local operators. The square brackets $[\ldots]$ stand for the renormalization using dimensional regularization and MS subtraction. We will tacitly assume that the quark and antiquark have different flavor so that there is no mixing with gluon operators. In most situations the overall coordinate x^μ is irrelevant and can be put to zero; we will often abbreviate $\mathcal{O}(z_1, z_2) \equiv \mathcal{O}(0; z_1, z_2)$.

Light-ray operators satisfy a renormalization-group equation of the form [12]

$$\big(M\partial_M + \beta(g)\partial_g + \mathbb{H}\big)\big[\mathcal{O}(z_1, z_2)\big] = 0, \tag{2}$$

where $\mathbb{H}$ is an integral operator acting on the light-cone coordinates of the fields. It can be written as

$$\mathbb{H}[\mathcal{O}](z_1, z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha, \beta)[\mathcal{O}]\big(z_{12}^\alpha, z_{21}^\beta\big), \tag{3}$$

where

$$z_{12}^\alpha \equiv z_1\bar\alpha + z_2\alpha, \quad \bar\alpha = 1 - \alpha, \tag{4}$$

and $h(\alpha, \beta)$ is a certain weight function (kernel).

One can show, see e.g. [10], that the powers $[\mathcal{O}](z_1, z_2) \mapsto (z_1 - z_2)^N$ are eigenfunctions of the evolution operator $\mathbb{H}$, and the corresponding eigenvalues

$$\gamma_N = \int d\alpha d\beta\, h(\alpha, \beta)(1 - \alpha - \beta)^{N-1} \tag{5}$$

are nothing else as the anomalous dimensions of local operators of spin N (with $N - 1$ derivatives).

In general the function $h(\alpha, \beta)$ is a function of two variables and therefore the knowledge of the anomalous dimensions γ_N is not sufficient to fix it. However, if the theory is conformally invariant then to the one loop accuracy $\mathbb{H}$ must commute with the generators of the $SL(2)$ transformations $[\mathbb{H}, S_\alpha^{(0)}] = 0$, where

$$S_+^{(0)} = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2(z_1 + z_2),$$
$$S_0^{(0)} = z_1\partial_{z_1} + z_2\partial_{z_2} + 2, \qquad S_-^{(0)} = -\partial_{z_1} - \partial_{z_2}. \tag{6}$$

In this case it can be shown that the function $h(\alpha, \beta)$ (up to trivial terms $\sim\delta(\alpha)\delta(\beta)$ that correspond to the unit operator) takes the form [13]

$$h(\alpha, \beta) = \bar h(\tau), \quad \tau = \frac{\alpha\beta}{\bar\alpha\bar\beta} \tag{7}$$

and is effectively a function of one variable τ called the conformal ratio. This function can easily be reconstructed from its moments (5), alias from the anomalous dimensions.

Conformal symmetry of QCD is broken by quantum corrections which implies that the symmetry of the evolution equations is lost at the two-loop level. In other words, writing the evolution kernel as an expansion in the coupling constant

$$\mathbb{H} = a_s\mathbb{H}^{(1)} + a_s^2\mathbb{H}^{(2)} + \ldots$$
$$\mapsto \quad h(\alpha, \beta) = a_s h^{(1)}(\alpha, \beta) + a_s^2 h^{(2)}(\alpha, \beta) + \ldots, \tag{8}$$

where $a_s = \alpha_s/(4\pi)$, we expect that $h^{(1)}(\alpha, \beta)$ only depends on the conformal ratio whereas higher-order contributions remain to be nontrivial functions of two variables α and β.

This prediction is confirmed by the explicit calculation [12]:

$$\mathbb{H}^{(1)} f(z_1, z_2)$$
$$= 4C_F\Bigg\{\int_0^1 d\alpha\, \frac{\bar\alpha}{\alpha}\big[2f(z_1, z_2) - f\big(z_{12}^\alpha, z_2\big) - f\big(z_1, z_{21}^\beta\big)\big]$$
$$- \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, f\big(z_{12}^\alpha, z_{21}^\beta\big) + \frac{1}{2}f(z_1, z_2)\Bigg\}. \tag{9}$$

The corresponding one-loop kernel $h^{(1)}(\alpha, \beta)$ can be written in the following, remarkably simple form [13]

$$h^{(1)}(\alpha, \beta) = -4C_F\bigg[\delta_+(\tau) + \theta(1 - \tau) - \frac{1}{2}\delta(\alpha)\delta(\beta)\bigg], \tag{10}$$

where the regularized δ-function, $\delta_+(\tau)$, is defined as

$$\int d\alpha d\beta\, \delta_+(\tau) f\big(z_{12}^\alpha, z_{21}^\beta\big)$$
$$\equiv \int_0^1 d\alpha \int_0^1 d\beta\, \delta(\tau)\big[f\big(z_{12}^\alpha, z_{21}^\beta\big) - f(z_1, z_2)\big]$$
$$= -\int_0^1 d\alpha\, \frac{\bar\alpha}{\alpha}\big[2f(z_1, z_2) - f\big(z_{12}^\alpha, z_2\big) - f\big(z_1, z_{21}^\alpha\big)\big]. \tag{11}$$

Taking appropriate matrix elements and making a Fourier transformation to the momentum fraction space one can check that the expression in Eq. (10) reproduces all classical leading-order (LO) QCD evolution equations: DGLAP equation for parton distributions, ERBL equation for the meson light-cone DAs, and the general evolution equation for GPDs.

The two-loop kernel $h^{(2)}(\alpha, \beta)$ contains contributions of two color structures and a term proportional to the QCD beta function,

$$h^{(2)}(\alpha, \beta) = 8C_F^2 h_1^{(2)}(\alpha, \beta) + 4C_F C_A h_2^{(2)}(\alpha, \beta)$$
$$+ 4b_0 C_F h_3^{(2)}(\alpha, \beta). \tag{12}$$

Let us explain how it can be calculated. The idea of Ref. [10] is to consider a modified theory, QCD in non-integer $d = 4 - 2\epsilon$ dimensions. In this theory the β-function has the form

$$\beta(a) = M\partial_M a = 2a\big(-\epsilon - b_0 a + \mathcal{O}\big(a^2\big)\big),$$
$$b_0 = \frac{11}{3}N_c - \frac{2}{3}n_f, \tag{13}$$

and for a large number of flavors n_f there exists a critical coupling $a_s^* = \alpha_s^*/(4\pi) \sim \epsilon$ such that $\beta(a_s^*) = 0$. The theory thus enjoys exact scale invariance [14,15] and one can argue (see below) that full conformal invariance is also present.[1] As a consequence, the renormalization group equations are exactly conformally invariant: the evolution kernels commute with the generators of the conformal group. The generators are, however, modified by quantum corrections as compared to their canonical expressions (6):

$$S_\alpha = S_\alpha^{(0)} + a_s^*\Delta S_\alpha^{(1)} + \big(a_s^*\big)^2\Delta S_\alpha^{(2)} + \ldots \tag{14}$$

[1] Formally the gauge-fixed QCD Lagrangian contains two charges, the coupling and the gauge parameter. The corresponding β-function, $\beta_\xi = M\partial_M\xi$, vanishes in the Landau gauge, $\xi = 0$, so that *all* Green functions are scale-invariant at the critical point in this gauge; β_ξ also drops out of the RG equations for the correlation functions of gauge-invariant operators.