



# Desingularization of the Milne Universe

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## ABSTRACT

Resolution of cosmological singularities is an important problem in any full theory of quantum gravity. The Milne orbifold is a cosmology with a big-bang/big-crunch singularity, but being a quotient of flat space it holds potential for resolution in string theory. It is known, however, that some perturbative string amplitudes diverge in the Milne geometry. Here we show that flat space higher spin theories can effect a simple resolution of the Milne singularity when one embeds the latter in  $2 + 1$  dimensions. We explain how to reconcile this with the expectation that non-perturbative string effects are required for resolving Milne. Along the way, we introduce a Grassmann realization of the İnönü–Wigner contraction to export much of the AdS technology to our flat space computation.

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## 1. Introduction and conclusion

General Relativity is expected to require modifications at short distances. The oft-stated reason for this expectation is the existence of an infinite number of perturbative UV divergent couplings when one quantizes metric fluctuations. String theory solves this problem because it has an enormous gauge symmetry, called worldsheet conformal invariance. This gauge symmetry of string theory essentially uniquely fixes the infinite number of couplings arising in perturbative gravity.

Apart from the quantum problem of divergences, there is also a purely classical reason why we expect that gravity might require modifications at short distances. This is because in gravity, spacetime singularities are ubiquitous [1]. Since string theory is expected to be perturbatively finite in the UV, it is natural to wonder whether it can also resolve spacetime singularities. Some progress along this direction, and answers in the affirmative of various degrees of strength, can be found in [2–4].

Singularities in cosmological (a.k.a. time-dependent) spacetimes are especially tricky in string theory because typically we only understand how to quantize string theory in supersymmetric backgrounds, and supersymmetric backgrounds are automatically time independent. One way forward is to consider cosmological quotients of flat space as simple examples of time dependent singular backgrounds. The idea is that since the covering space is flat, we should be able to use some of the standard tools from flat space string theory to explore these singular geometries. A simple

context where a lot of papers on this topic<sup>1</sup> have been written is the case of the Milne orbifold, which is a time dependent orbifold of flat space (see [5–12] for related work). It turns out that some tree level string scattering amplitudes are singular on the Milne orbifold [13–15], indicating that perturbative string theory breaks down. Also, because it is an exact CFT, there are no  $\alpha'$ -effects that can result in a resolution of the Milne singularity.<sup>2</sup> Together, if we take these two statements at face value, we come to the conclusion that only non-perturbative  $g_s$ -effects can come to the rescue of Milne, perhaps in a context like the AdS/CFT duality.<sup>3</sup>

In this paper, we will study the Milne orbifold from another perspective. We will consider it in the context of Chern–Simons higher spin theories in  $2 + 1$  dimensional flat space [16,17]. Our motivation is as follows. It is expected [18,19] that higher spin theories capture features of string theory in the tensionless limit,<sup>4</sup> which corresponds to  $\alpha' \rightarrow \infty$ . Therefore heuristically, the gauge symmetries of (classical) higher spin theory can be thought of as a target space realization of the worldsheet gauge symmetries of tree-level string theory in the  $\alpha' \rightarrow \infty$  limit. So in this limit it is possible to ask whether spacetime singularities are artifacts of a singular gauge, and if so, whether one can get rid of them by going to a different gauge. We will indeed see that by doing a flat space higher spin gauge transformation, we are able to remove the singularity in the Milne Universe in a very simple and natural way.

<sup>1</sup> There is a whole slew of papers written on this topic in the context of singularity resolution in string theory, so our citation list is necessarily incomplete.

<sup>2</sup> However, for a proposal that winding tachyon condensation can resolve singularities, see e.g. [3].

<sup>3</sup> We thank Ben Craps and Boris Pioline for correspondence on related questions.

<sup>4</sup> See [20] for a connection between tensionless strings and flat space.

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This is our main result. To accomplish this, we use a Grassmann trick to rewrite the flat space Chern–Simons HS theory in a form that closely resembles the AdS case. This trick in itself is pretty powerful, but seems to have gone unnoticed before.

Note that in this picture the string coupling  $g_s \sim 1/N$  never showed up. There is superficially some tension between this and the general belief from string theory that non-perturbative effects are necessary for resolving Milne.<sup>5</sup>

Perhaps the best way to understand our result is to note that the limit  $\alpha' \rightarrow \infty$  is precisely the opposite of the limit where the usual Einstein gravity emerges in string theory ( $\alpha' \rightarrow 0$ ). That is, higher spin theories are a different classical limit of string theory. In this limit, the stringy gauge invariances have a simple target space realization in terms of higher spin gauge symmetries. So what we do here amounts more precisely to a de-singularization via a gauge transformation, and not to a resolution<sup>6</sup>: the latter is usually accomplished via the addition of new degrees of freedom, and that is the situation that is envisaged in the usual discussions of the Milne orbifold. We emphasize, however, that it is not that the gauge transformation here is singular, it is that the solution (in the metric language) has interpretation as a spacetime singularity [12]. The Chern–Simons gauge field is in fact regular before and after the resolution, even though the metric is non-singular only after.<sup>7</sup>

Some recent papers dealing with cosmologies and singularities in a higher spin set up can be found in [22–24]. In [23] a cosmological singularity resolution was done, but in the context of dS<sub>3</sub> (higher spin) gravity. The reason why Milne is of much more interest than the dS quotients that we considered in the previous paper is because the geometry is locally flat here, so one can potentially consider string propagation on it. Indeed, Milne has been studied rather extensively in a stringy context as already mentioned. By contrast, the singularity we resolved in dS is an obscure and essentially unknown one, and was merely interesting as a proof of principle. We could not resolve Milne at the time, because flat space higher spin theories were constructed only afterwards [16,17].

Indeed, after the first version of this paper appeared, we have revisited the 2-to-2 string scattering amplitude on the Milne orbifold, and exhaustively scanned for divergences [25]. The result is that *all* the singularity-related divergences arise when the  $\alpha'$  (made dimensionless by multiplication with appropriate momenta) is less than some numerical value. The remaining divergences all arise when the tower of intermediate string states go on-shell, and are physical IR divergences. Since higher spin theories are morally the  $\alpha' \rightarrow \infty$  limit of string theory, we believe the fact that we are able to resolve the geometry at infinite  $\alpha'$  and the fact that string amplitudes are UV finite when  $\alpha'$  is large enough, is at the very least, suggestive.

<sup>5</sup> The connection between string theory and higher spin theory (in particular C–S higher spin theory in flat 2 + 1 D space) has not been made precise, and is mostly heuristic at present. So it is not entirely clear that our result is a bona-fide challenge to the string theory expectations.

<sup>6</sup> Loosely, we will use the two terms interchangeably.

<sup>7</sup> It is perhaps also worth emphasizing that the Chern–Simons formulation of gravity and higher spin gravity that we are using is best suited for classical questions. Note that in the spin-2 theory (the usual C–S gravity) it is currently believed that the quantization of the gravity is *not* the same as the quantization of the C–S theory [21]. It is best thought of as only a classical equivalence. This is because invertibility of the vielbeins is required for the CS interpretation, so in the path integral one is integrating over different field configurations. Another relevant observation is that the existence of black holes (at least in the AdS<sub>3</sub> context) indicates a huge degeneracy of states which is surprising in a Chern–Simons theory. We will be using the holonomies to distinguish classical solutions, not define observables in the quantum theory.

## 2. Flat 2 + 1 D (higher spin) gravity

We will work with the spin-3 theory in this paper. In 2 + 1 dimensions, working with higher spin theories is easily accomplished via the Chern–Simons formulation of gravity [26], but with a higher rank version of  $ISO(2, 1)$  as discussed in [16,17].

A lot of work on higher spin theories has been done in the context of AdS<sub>3</sub> theories, and we will make an observation that enables us to translate a lot of this AdS machinery to the flat space theory. This observation is that if one makes the replacement

$$\frac{1}{\ell} \rightarrow \epsilon \quad (1)$$

where  $\ell$  is the AdS<sub>3</sub> radius and  $\epsilon$  is a Grassmann parameter defined by the condition that  $\epsilon^2 = 0$ , then the AdS Chern–Simons action<sup>8</sup> written in terms of the triad and the spin connection (and their higher spin cousins) reduces to the flat space Einstein–Hilbert action (and its higher spin cousin) times  $\epsilon$ , provided one takes the Newton constant to be

$$\frac{1}{16G} = k. \quad (2)$$

Since we are only concerned about classical equations and their solutions, the overall Grassmann factor in the action will not affect our discussions. The basic reason why this trick works is because of the fact that  $ISO(2, 1)$  is the İnönü–Wigner contraction of  $SL(2, R) \times SL(2, R)$  (and similarly, for the higher spin generalizations). This approach and some of its applications are further explored in [32].

There are two basic reasons why this trick is useful.

- We can adopt the notations and the  $SL(3)$  generator matrices of [27] without modifications as long as we make sure that  $1/\ell \rightarrow \epsilon$  squares to zero. Without the Grassmann approach, we would be faced with the task of constructing an explicit set of matrices for the generators in [16,17], such that they have a non-degenerate trace form.
- The non-degenerate trace form of the AdS Chern–Simons theory descends to give us a non-degenerate trace form for the flat space theory with this trick.

The upshot is that we can work with flat space (higher spin) gravity in 2 + 1 dimensions by starting with the AdS Chern–Simons theory, writing the gauge field in terms of the vielbein and spin connection, and setting  $1/\ell \rightarrow \epsilon$  with  $\epsilon^2 = 0$ .

## 3. Milne: metric and connection

We will take the Milne metric in 2 + 1 dimensions in the form [28,29]

$$ds^2 = -dT^2 + r_C^2 dX^2 + \alpha^2 T^2 d\varphi^2, \quad (3)$$

where for comparison with [30] we define the metric parameters  $\alpha$ ,  $r_C$  in terms of the “mass”,  $M$ , and “spin”,  $J$ , by

$$\alpha = \sqrt{M}, \quad r_C = \sqrt{\frac{J^2}{4M}}. \quad (4)$$

(We are following the convention, where  $8G = 1$ .)  $X$ ,  $\varphi$  directions are compact and closed, both with period  $2\pi$ . The spacetime looks like a double cone. There is a causal structure singularity at  $T = 0$  which is where the  $\varphi$ -circle crunches before re-expanding in a

<sup>8</sup> See for example Section 2 and Appendix A of [27].

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