Physics Letters B 734 (2014) 111–115

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Challenges of $D=6$ $\mathcal{N}=(1,1)$ SYM theory

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ARTICLE INFO

Article history:
Received 11 May 2014
Accepted 12 May 2014
Available online 16 May 2014
Editor: L. Alvarez-Gaumé

Keywords:
Amplitudes
Extended supersymmetry
UV divergences
Regge behaviour

ABSTRACT

Maximally supersymmetric Yang–Mills theories have several remarkable properties, among which are the cancellation of UV divergences, factorization of higher loop corrections and possible integrability. Much attention has been attracted to the $\mathcal{N}=4$ $D=4$ SYM theory. The $\mathcal{N}=(1,1)$ $D=6$ SYM theory possesses similar properties but is nonrenormalizable and serves as a toy model for supergravity. We consider the on-shell four point scattering amplitude and analyze its perturbative expansion within the spin-helicity and superspace formalism. The integrands of the resulting diagrams coincide with those of the $\mathcal{N}=4$ $D=4$ SYM and obey the dual conformal invariance. Contrary to 4 dimensions, no IR divergences on mass shell appear. We calculate analytically the leading logarithmic asymptotics in all loops. Their summation leads to a Regge trajectory which is calculated exactly. The leading powers of s are calculated up to six loops. Their summation is performed numerically and leads to a smooth function of s. The leading UV divergences are calculated up to 5 loops. The result suggests the geometrical progression which ends up in a finite expression. This leads us to a radical point of view on nonrenormalizable theories.

1. Introduction

In the last decade there has been considerable activity on the calculation of the amplitudes in maximally supersymmetric Yang–Mills theories (SYM) [1,2] and maximally supersymmetric gravity [3]. Gauge and gravity SUSY theories in $D=4$ such as the $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ SUGRA are the most important examples. These theories are believed to possess several remarkable properties, among which are total or partial cancelation of UV divergences, factorization of higher loop corrections and possible integrability. The success of factorization leading to the BDS ansatz [1] for the amplitudes in $D=4$ $\mathcal{N}=4$ SYM stimulated similar activity in other models and dimensions. Many magnificent insights in the structure of amplitudes (the S-matrix) of gauge theories in various dimensions (for review see, for example, [4]) were obtained. It was understood that the structure of the integrands in all these theories is the same and has an imprint of conformal and dual conformal invariance [5–7]. As a result, the structure of the UV divergences is also similar, in particular, the boundary where the first divergences in SYM appear happens to be given by the universal formula [8–10]

$$D=4+6/L, \tag{1}$$

where D is the dimension and L is the number of loops. The structure of the amplitudes (and divergences) in SUGRA was also found to be linked to the SYM [11]. This renewed attempts to check the finiteness of the $D=4$ $\mathcal{N}=8$ SUGRA [3,12].

All this activity became possible with the development of new techniques: the spinor helicity and momentum twistor formalisms, different sets of recurrence relations for the tree level amplitudes, the unitarity based methods for loop amplitudes and different realizations of the on-shell superspace technique for theories with supersymmetry [4]. These techniques were generalized to a space–time dimension greater than $D=4$ [13–15].

In this note, we consider one of these theories, namely, the $D=6$ $\mathcal{N}=(1,1)$ SYM. This is a maximal supersymmetric theory in $D=6$ dimensions, after additional compactification on two-torus it is reduced to the $D=4$ $\mathcal{N}=4$ SYM. It can also be considered as a special low energy limit (the effective actions on the 5-branes [16]) of the string/M theory. It is believed that this theory is also exceptional; at the same time, it is nonrenormalizable by power counting, the coupling constant has a dimension -2 in mass units like in $D=4$ gravity. Therefore, this theory serves as a toy model for quantum gravity.

Investigation of this theory which we performed within the spinor-helicity and superfield formalism has led us to some far-reaching conclusions. We first present our calculations which we

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http://dx.doi.org/10.1016/j.physletb.2014.05.022

performed up to 5 and 6 loops and then make some speculations concerning nonrenormalizable theories.

2. Color decomposition, spinor helicity and superfield formalism in $D = 6$ $\mathcal{N} = (1, 1)$ SYM

The aim is to calculate the multiparticle amplitudes on mass shell. For this purpose, we first perform the color decomposition extracting the color ordered partial amplitude [4]

$$\mathcal{A}_n^{a_1\ldots a_n}\left(p_1^{\lambda_1}\ldots p_n^{\lambda_n}\right) = \sum_{\sigma\in S_n/Z_n} Tr\left[\sigma\left(T^{a_1}\ldots T^{a_n}\right)\right] A_n\left(\sigma\left(p_1^{\lambda_1}\ldots p_n^{\lambda_n}\right)\right) + \mathcal{O}(1/N_c). \tag{2}$$

The color ordered amplitude A_n is evaluated in the planar limit which corresponds to $N_c \to \infty$, $g_{YM}^2 \to 0$ and $g_{YM}^2 N_c$ – fixed.

The next step is to use the spinor helicity formalism and on-shell methods [4]. Their advantage is that one calculates explicitly the physical amplitude with external states of a given helicity without unphysical degrees of freedom, gauge fixing, ghosts, etc., the usual attributes of a gauge theory. The description of the spinor helicity formalism can be found in [14,17]. Applying it one can rewrite the on-shell amplitudes in a compact form. For example, using the six dimensional version of the BCFW recurrence relation the tree level 4 gluon color ordered amplitude A_4 can be written as

$$\mathcal{A}_4^{(0)}(1_{a\dot a}2_{b\dot b}3_{c\dot c}4_{d\dot d}) = -ig_{YM}^2\frac{\langle 1_a2_b3_c4_d\rangle[1_{\dot a}2_{\dot b}3_{\dot c}4_{\dot d}]}{st}, \tag{3}$$

where 1, 2, 3 and 4 are external momenta, $\langle 1_a2_b3_c4_d\rangle \doteq \epsilon_{ABCD} \times \lambda_1^{Aa}\lambda_2^{Bb}\lambda_3^{Cc}\lambda_4^{Dd}$ and $[1_{\dot a}2_{\dot b}3_{\dot c}4_{\dot d}] \doteq \epsilon^{ABCD}\tilde\lambda^{\dot a}_{A,1}\tilde\lambda^{\dot b}_{B,2}\tilde\lambda^{\dot c}_{C,3}\tilde\lambda^{\dot d}_{D,4}$, λ_i^{Aa} and $\tilde\lambda^{\dot a}_{A,i}$ being the spinors associated with momenta p_i^{AB} of ith particle. $A, B = 1, \ldots, 4$ are the fundamental representation of the $Spin(SO(5,1)) \simeq SU(4)^*$ indices, $a = 1, 2$ and $\dot a = \dot 1, \dot 2$ are the $D = 6$ little group $SO(4) \simeq SU(2) \times SU(2)$ indices. Note that in $D = 6$ for the massless states helicity is no longer conserved in contrast to the $D = 4$ case.

The superfield formalism allows one to take into account the full strength of the $\mathcal{N} = (1, 1)$ supersymmetry. The self-consistent way of constructing the superamplitude comprises the harmonic superspace techniques developed in [17]. It results in the following form of the color ordered n-particle superamplitude:

$$A_n\left(\{\lambda_a^A, \tilde\lambda_A^{\dot a}, \eta_a, \bar\eta_{\dot a}\}\right) = \delta^6\left(p^{AB}\right)\delta^4\left(q^A\right)\delta^4(\bar q_A)\mathcal{P}_n\left(\{\lambda_a^A, \tilde\lambda_A^{\dot a}, \eta_a, \bar\eta_{\dot a}\}\right), \tag{4}$$

$$p^{AB} = \sum_i^n \lambda_i^{Aa}\lambda_{a,i}^B, \qquad q^A = \sum_i^n \lambda_a^{A,i}\eta_i^a, \qquad \bar q_A = \sum_i^n \tilde\lambda^{\dot a}_{A,i}\bar\eta_{\dot a,i}, \tag{5}$$

λ_a^A, $\tilde\lambda_A^{\dot a}$ and η^a, $\bar\eta_{\dot a}$ being the bosonic and fermionic coordinates of $\mathcal{N} = (1, 1)$ on-shell momentum superspace, and $\mathcal{P}_n$ is a polynomial with respect to η and $\bar\eta$ of degree of $2n - 8$.

We further concentrate on the four point amplitude. In this case, the degree of Grassmannian polynomial $\mathcal{P}_4$ is 0; hence $\mathcal{P}_4$ is a function of bosonic variables only

$$A_4\left(\{\lambda_a^A, \tilde\lambda_A^{\dot a}, \eta_a, \bar\eta_{\dot a}\}\right) = \delta^6\left(p^{AB}\right)\delta^4\left(q^A\right)\delta^4(\bar q_A)\mathcal{P}_4\left(\{\lambda_a^A, \tilde\lambda_A^{\dot a}\}\right). \tag{6}$$

Comparing this expression with (3) one concludes that the tree level 4-point superamplitude can be written in a very compact form:

$$A_4^{(0)} = -ig_{YM}^2\delta^6\left(p^{AB}\right)\frac{\delta^4(q^A)\delta^4(\bar q_A)}{st}. \tag{7}$$

What is essential, at any order of PT the amplitude is proportional to the bosonic and fermionic δ-function of reflecting the (super)momentum conservation as in (6). This means that the tree level amplitude always factorizes and we get a universal expression for the color ordered superamplitude with the radiative corrections:

$$A_4(s,t) = A_4^{(0)}(s,t)[1 + \text{loop corrections}]. \tag{8}$$

For the loop corrections one has expansion which due to a universal form of the integrands in any SYM theory coincides with the one in $D = 4$ $\mathcal{N} = 4$ SYM up to dimensional factors since in $D = 6$ dimensions the coupling has a mass dimension equal to -2 [7]. This is the consequence of the dual conformal invariance in momentum space [6] equally valid in $D = 4$ and in $D = 6$. A remarkable property of this expansion is that all the bubble and triangle diagrams cancel and one is left with the sequence of scalar box diagrams shown in Fig. 1.

3. Perturbation expansion for the amplitudes

Our task here is to calculate the radiative corrections to the four point amplitude. In what follows we proceed loop by loop. The first question is: is there any kind of factorization similar to the BDS formula? The answer is negative for general values of Mandelstam variables s and t as it was shown in [18] where the two loop box diagram was calculated. While the one loop box gives the double logarithm of s/t, the two loop one contains the Polylog functions. In this situation, we concentrate on the Regge asymptotic behaviour when $s \to \infty$ and $t < 0$ is fixed. Then, all the integrals are expressed in terms of powers of s and t and $\log^{2n}(s/t)$.

Since the coupling g_{YM}^2 in $D = 6$ has dimension -2, the expansion parameter is either $g_{YM}^2 s$ or $g_{YM}^2 t$ and one can consider separately the series with the leading powers of s. In what follows we consider the infinite vertical series of diagrams of Fig. 1 summing up the leading powers of s, the leading logarithms and the leading UV divergences. Note that the first UV divergence, in accordance with Eq. (1), is encountered in three loops.

3.1. The leading logarithms

In the Regge limit the main contribution to the leading logs comes from the vertical multiple boxes, the so-called ladder diagrams. For the vertical n-loop ladder diagram $B_n(t,s)$, which is UV and IR finite, the leading contribution was found in [18] and takes the form

$$B_n(t,s) \simeq \frac{1}{s}\frac{L^{2n}(x)}{n!(n+1)!}, \quad L \equiv \log(s/t). \tag{9}$$

Combined with the combinatorial factor $s(-\frac{t}{2})^n$ this leads to the series or the leading logarithmical contributions ($L.L.$) to the amplitude

$$\left.\frac{A_4}{A_4^{(0)}}\right|_{L.L.} = \sum_{n=0}^\infty \frac{(-g^2t/2)^n L^{2n}(x)}{n!(n+1)!}, \quad \text{where } g^2 \equiv \frac{g_{YM}^2 N_c}{64\pi^3}. \tag{10}$$

This series can be summed and represents the Bessel function of the imaginary argument

$$\sum_{n=0}^\infty \frac{(-g^2t/2)^n L^{2n}(x)}{n!(n+1)!} = \frac{I_1(2y)}{y}, \quad y \equiv \sqrt{g^2|t|/2}\, L(x). \tag{11}$$