Physics Letters B 734 (2014) 116–121

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Anisotropic plasma with a chemical potential and scheme-independent instabilities

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ARTICLE INFO

Article history:
Received 10 April 2014
Received in revised form 12 May 2014
Accepted 15 May 2014
Available online 20 May 2014
Editor: M. Cvetič

ABSTRACT

Generically, the black brane solution with planar horizons is thermodynamically stable. We find a counter-example to this statement by demonstrating that an anisotropic black brane is unstable. We present a charged black brane solution dual to a spatially anisotropic finite temperature $\mathcal{N} = 4$ super Yang–Mills plasma at finite $U(1)$ chemical potential. This static and regular solution is obtained both numerically and analytically. We uncover rich thermodynamic phase structures for this system by considering the cases when the anisotropy constant "a" takes real and imaginary values, respectively. In the case $a^2 > 0$, the phase structure of this anisotropic black brane is similar to that of Schwarzschild–AdS black hole with S^3 horizon topology, yielding a thermodynamical instability at smaller horizon radii. For the condition $a^2 \leq 0$, the thermodynamics is dominated by the black brane phase for all temperatures.

1. Introduction

The AdS/CFT correspondence provides a powerful tool in studying the strongly coupled problems of quantum field theory, ranging from nuclear physics to condensed matter theory [1,2]. This correspondence states the equivalence between type IIB superstring theory in $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang–Mills (SYM) gauge theory on the 4-dimensional boundary of AdS_5. From gravitational theory on the asymptotically anti-de Sitter view point, we are able to gain profound insights for such strongly coupled field theory. It is thus very crucial to search for the generic asymptotically AdS gravitational solutions, which are dual to interesting phase in the field theory side. The most well-known black brane solutions are the homogeneous and isotropic Schwarzschild–AdS black brane solution and Reissner–Nordström–AdS (RN–AdS) solution. The charged black brane solutions are particularly useful to study quark–gluon plasma (QGP) [3], superconductivity and superfluidity, Fermi surfaces and non-Fermi liquids in condensed matter system [4–8]. Generally, $U(1)$ gauge symmetries in the bulk correspond to conserved number operators in the dual field theory. The gauge field in the AdS space couples to a CFT current J_μ and the CFT states thus containing a plasma of charged quanta.

It is well-known that there are many strongly coupled systems which do not satisfy homogeneity and isotropy spontaneously. For example, some systems may have anisotropic Fermi-surface because of the atomic lattice effects and the QGP is anisotropic in a short time after creation. Therefore, the studies on anisotropic and inhomogeneous black brane solutions and their holographic applications have attracted more attention [9–11].

In this paper, we will present a charged and spatially anisotropic black brane solution. The neutral anisotropic black brane solution was obtained by Mateos and Trancanelli in their seminal papers [9] and its applications in QCD was discussed. One motivation comes from the fact that the QGP created in RHIC is not only anisotropic but also charged. In the QGP produced in RHIC, the escaped quark is surrounded by high density quark fluid liberated from the heavy ions. Under such conditions, the baryon density of the QGP and the overall $U(1)$ gauge field is relevant. Unlike chargeless case, the introduction of the $U(1)$ gauge field breaks the $SO(6)$ symmetry and thus leads to the excitations of the Kaluza–Klein modes. Another motivation comes from the applications of the anisotropic black brane solutions to condensed matter physics, since the many-body system at a finite $U(1)$ charge density corresponds to the charged black holes in the AdS peace.

We will consider the case the anisotropy is introduced through deforming the SYM theory by a θ-parameter of the form $\theta \propto z$, which acts as an isotropy-breaking external source that forces the system into an anisotropic equilibrium state [9]. The θ-parameter is dual to the type IIB axion χ with the form $\chi = az$. The constant a has dimensions of mass and is a measure of the anisotropy. From the five-dimensional theory viewpoint, the anisotropy can be interpreted as a non-zero number of dissolved D7-brane wrapped on S^5, extending along the xy-direction and distributed along the

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http://dx.doi.org/10.1016/j.physletb.2014.05.032

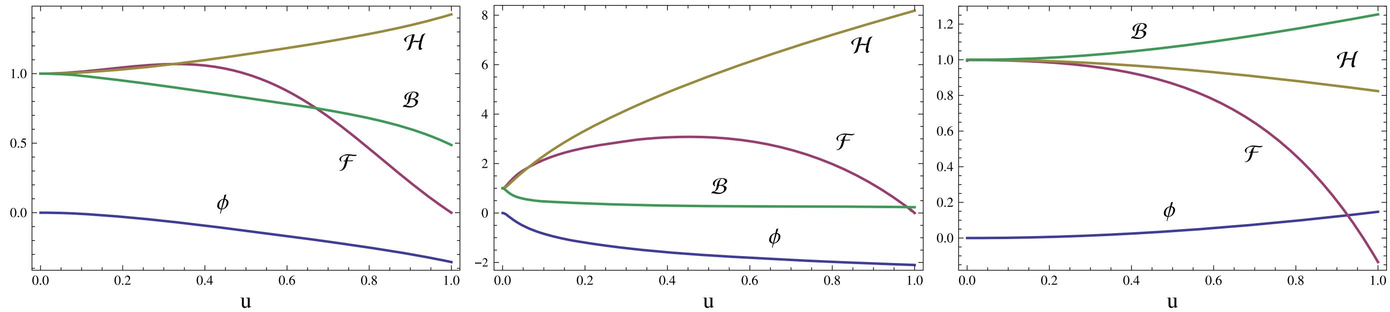

Fig. 1. (Color online.) The metric functions for $a = 1.86$, $Q = 6.23$ (left), $a = 64.06$, $Q = 9.76$ (middle) and $a = 1.2i$, $Q = 1/10$ (right), with $u_{\rm H} = 1$.

z-direction with density n_{D7} [9]. So a can be regarded as "charge density" and should not be imaginary-valued.

However, we will consider both cases with $a^2 > 0$ and $a^2 < 0$ which have different thermodynamic properties, although imaginary axion field might be unphysical in type IIB supergravity theory. If the axion field merely plays the role of providing the appropriate source to support a spatially anisotropic spacetime, then the imaginary-valued a could be acceptable. Furthermore, we will see later that imaginary a can be understood as a consequence of the tachyon condensate of the dilaton field. A careful analysis in the following will disclose that the anisotropic black brane solution corresponding to $a^2 > 0$ is actually a "prolate" version of the solution because it has a z-axis longer than the x- and y-axes (i.e. $\mathcal{H}(u_{\rm H}) > 1$), while the "oblate" version of the anisotropic black brane solution requires $a^2 < 0$ (i.e. $\mathcal{H}(u_{\rm H}) < 1$) and thus the z-axis is shorter than the x- and y-axes. As what we will uncover, the "prolate" black brane solution suffers thermodynamic instabilities, similar to those of the Schwarzschild–AdS with a spherical horizon, but the "oblate" solution is stable.

2. Numerical solution

The charged anisotropic black brane solution can be derived from the effective action after S^5 reduction of type IIB supergravity [12,13]. In Einstein frame, the type IIB supergravity Lagrangian which have been truncated out NS–NS and R–R 2-form potentials is

$$\mathcal{L} = \hat{R} * 1 - \frac{1}{2} d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{2} e^{2\hat{\phi}} \hat{F}_1 \wedge *\hat{F}_1 - \frac{1}{4} \hat{F}_5 \wedge *\hat{F}_5, \qquad (1)$$

where $\hat{\phi}$ and $\hat{F}_1 = d\hat{\chi}$ are the dilaton and the axion field-strength in ten-dimensions respectively. The 5-form field $\hat{F}_5$ should satisfy the self-duality condition and be imposed at the level of equations of motion. The theory can be reduced on to minimal supergravity and the corresponding five-dimensional axion–dilaton–Maxwell–gravity action is given by

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(R + 12 - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} e^{2\phi} (\partial\chi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) + S_{GH}, \qquad (2)$$

where we have set the AdS radius $L = 1$, $\kappa^2 = 4\pi^2/N_c^2$ and S_{GH} is the Gibbons–Hawking boundary term.

In order to obtain an anisotropic D3-brane with an asymmetry between the xy- and z-directions, we assume the Einstein-frame metric takes the form

$$ds_5^2 = \frac{e^{-\frac{1}{2}\phi}}{u^2} \left(-\mathcal{F}\mathcal{B} dt^2 + dx^2 + dy^2 + \mathcal{H} dz^2 + \frac{du^2}{\mathcal{F}} \right). \qquad (3)$$

$$A = A_t(u) dt, \quad \text{and} \quad \chi = az. \qquad (4)$$

The functions ϕ, $\mathcal{F}$, $\mathcal{B}$ and $\mathcal{H} = e^{-\phi}$ depend only on the radial coordinate u, which we solved numerically [13]. The electric potential A_t can be obtained via $A_t(u) = -\int_{u_{\rm H}}^{u} du\, Q \sqrt{\mathcal{B}} e^{\frac{3}{4}\phi} u$ from the Maxwell equations, where Q is an integral constant related to the charge. The horizon locates at $u = u_{\rm H} \equiv 1/r_H$ with $\mathcal{F}(u_H) = 0$ and the boundary is at $u = 0$ where $\mathcal{F} = \mathcal{B} = \mathcal{H} = 1$. The asymptotic AdS_5 boundary condition requires the boundary condition $\phi(0) = 0$. The Hawking temperature is given by $T = -\frac{\mathcal{F}'(u_{\rm H})\sqrt{\mathcal{B}_{\rm H}}}{4\pi}$ through the Euclidean method.

Fig. 1 depicts the metric functions corresponding to different initial conditions. The first two plots in Fig. 1 reflect that the profile for $\mathcal{B}$ is seriously suppressed at the horizon as the charge Q increases. The Hawking temperature depends strongly on Q and the anisotropy a is sensitive to the initial condition $\phi(u_{\rm H})$. We can also see from Fig. 1 (right) that even the anisotropy constant a takes imaginary value, the black brane solution is still regular. Note that the metric functions $\mathcal{H}(u_{\rm H}) > 1$ and $\mathcal{B}(u_{\rm H}) < 1$ for $a^2 > 0$, corresponding to the "prolate" solution, but $\mathcal{H}(u_{\rm H}) < 1$, $\mathcal{B}(u_{\rm H}) > 1$ for $a^2 < 0$, corresponding to the "oblate" solution.

Note that the temperature is determined by the inverse horizon radius $u_{\rm H} = 1/r_{\rm H}$ and the charge Q. As can be seen from Fig. 2 (left), for a given temperature there are two branches of allowed black brane solutions, a branch with larger radii and one with smaller. This intriguing behavior is similar to the case of Schwarzschild–AdS black holes with a spherically horizon [15]. The smaller branch of the black brane is unstable with negative specific heat.

It is well-known that for black brane solutions with horizon topology R^3, there is only one branch of black brane solutions and the free energy is negative definite, so that the black brane structure is trivial and the thermodynamics is dominated by the black brane for all temperatures [14]. However, the anisotropic black brane solution obtained here provides a counter example to the above statement. We notice that even in the absence of $U(1)$ gauge field, two branches of black brane solution still exist, reflecting that it is mainly caused by the anisotropy. This behavior was not noticed in [9] and all the numerical computation was carried out at the stable black brane branch.

As to the "oblate" solution with $a^2 < 0$, the behavior of the solution differs sharply from the real anisotropy situation, which is qualitatively the same as the planar black brane case [14]: In that situation, there is only one stable branch of black brane solution and the thermodynamics is dominated by this solution for all temperatures (see Fig. 2(c)). The entropy density decreases as temperature goes down so that the specific heat is positive $c_\rho = T(\partial s/\partial T)_\rho > 0$.

Extremal limit As shown in Fig. 2(a), for the "prolate" solution with anisotropy parameter $a^2 > 0$, the temperature T cannot reach zero and thus this charged anisotropic configuration has no extremal limit, which is consistent with [16]. This is further supported by Fig. 2(b), which plots the entropy density as a function