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Generalized supersymmetry and sigma models

Rabin Banerjee, Sudhaker Upadhyay *

S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake City, Kolkata-700 098, India

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ABSTRACT

In this paper, we discuss the generalizations of exact supersymmetries present in the supersymmetrized sigma models. These generalizations are made by making the supersymmetric transformation parameter field-dependent. Remarkably, the supersymmetric effective actions emerge naturally through the Jacobian associated with the generalized supersymmetry transformations. We explicitly demonstrate these for two different supersymmetric sigma models, namely, one-dimensional sigma model and topological sigma model for hyperinstantons on quaternionic manifold.

1. Introduction

Supersymmetry is one of the most important concepts in modern theoretical physics, especially in the search of unified theories beyond the standard model [1]. In particle physics, for example, the supersymmetric standard model predicts the existence of a superpartner for every particle in the standard model. However, theoretical understanding of supersymmetry is quite far from complete. To examine the non-perturbative aspects of supersymmetric standard model, the utilization of the so-called space–time lattice simulation method is quite obscure as the theory involves many different scales. Supersymmetry is also relevant in string theories though it is quite far from the real experimental world. The advantage of superstring theories (those string models which also incorporate supersymmetry) is that it does not predict the existence of a bad behaving particle called the Tachyon. In particle theory, supersymmetry finds a way to stabilize the hierarchy between the unification scale and the electroweak scale or the Higgs boson mass. Supersymmetry models are also considered as a natural dark matter candidate [2].

Since it encompasses both theoretical and phenomenological interests, some serious attempts have been made to study supersymmetric theories [3,4]. But these attempts encountered some problems like supersymmetry breaking or fine-tuning. Recent developments have been made in the construction of lattice actions which possess a subset of the supersymmetries of the continuum theory and have a Poincaré invariant continuum limit [5]. The presence of the exact supersymmetry provides a way to obtain the continuum limit with no fine tuning or fine tuning much smaller than conventional lattice constructions. The remarkable feature of the presence of exact supersymmetry is that it reduces and in some cases eliminates the need for fine tuning to achieve a continuum limit invariant under the full supersymmetry of the target theory [5–7]. However, the construction of the supersymmetric non-linear sigma model with $O(N)$ target manifold was first made by Witten [8] and then by P. Di Vecchia and S. Ferrara [9] who describe the spontaneous breaking of chiral symmetry and the dynamical generation of particle masses [10–13]. Subsequently, the geometric interpretation of supersymmetric sigma models was classified in terms of BRST operator [14,15]. These sigma models are described by maps between a two-dimensional space called the world-sheet and some target space, taken to be a manifold in this setting. The connections of supersymmetry and geometry became more stronger after Witten's seminal construction of the so-called topological twist [16]. The motivation behind the twist is that in a topological field theory one can compute certain physical quantities more easily than in the original theory, where we sometimes lack the tools to compute them exactly. The topological sigma models in four dimensions are also used in the study of triholomorphic maps on hyper-Kähler manifolds [17]. A naive discussion of gauge invariant topological field theory is presented in BRST-BV framework [18].

On the other hand, generalization of BRST transformation by making the infinitesimal parameter finite and field-dependent was first developed in [19] and is known as finite field-dependent BRST (FFBRST) transformation. Such generalizations have found various applications in gauge field theories as well as in M-theory [19–30]. However, this generalization of BRST technique has, as yet, not been done for supersymmetry. Considering the deep connection between BRST and supersymmetry, we feel that this is a glaring omission.

* Corresponding author.
E-mail addresses: rabin@bose.res.in (R. Banerjee), sudhakerupadhyay@gmail.com, sudhaker@bose.res.in (S. Upadhyay).

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The aim of the present paper is to investigate the features of generalized supersymmetry in the framework of FFBRST formulation. Specifically, we consider supersymmetric sigma model and supersymmetric topological sigma model in a gauge invariant framework. Further, we discuss the generalizations of supersymmetries present in the theory in a detailed way. These generalizations are made by making the infinitesimal transformation parameter finite and field-dependent. Further, we stress the significant features of this generalized supersymmetry. For instance, we find that while the effective actions are invariant under generalized supersymmetry, the measures of path integrals are not. The obvious reason for this is that the path integral measure changes non-trivially. This non-trivial Jacobian plays a significant role in the formation of supersymmetric actions for sigma models. We show that the path integral measure under generalized supersymmetry transformation with some specific choices of parameter reproduces exactly the same effective actions as the original theories. In other words, the supersymmetric actions proposed in the literature [6,17] may be systematically obtained within the framework of FFBRST transformations. We analyse results in one-dimensional supersymmetric sigma model and in supersymmetric topological sigma model where the gauge-fixing is provided by the triholomorphic instanton condition. Even though we establish the results with the help of specific examples but this works for a general supersymmetric invariant theory.

The paper is organized in four sections. First, we provide the mechanism to generalize the supersymmetry in FFBRST framework in Section 2. In Section 3, which is the main section of the paper, we show that the Jacobians of the functional measures for FFBRST transformations with judicious choices of the transformation parameters naturally yield the supersymmetric actions for sigma models. We draw concluding remarks in the last section.

2. Generalized supersymmetric BRST transformation

In this section, we briefly review the generalized supersymmetric BRST formulation of pure gauge theories by making the infinitesimal parameter finite and field-dependent. It is a supersymmetric generalization of finite field-dependent BRST (FFBRST) transformation originally advocated in [19] for the non-supersymmetric cases. We first present the general methodology for the standard Maxwell theory in Euclidean space–time. For this purpose, let us start by defining the partition function for BRST invariant Maxwell theory in four dimensions as following

$$Z_M=\int \mathcal{D}A_\mu \mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}B e^{-S_M}, \tag{1}$$

where the effective action S_M in Lorentz gauge is defined by

$$S_M=\int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}B^2-B\partial_\mu A^\mu+\partial_\mu\bar{c}\partial^\mu c\right]. \tag{2}$$

Here B, c and $\bar{c}$ are Nakanishi–Lautrup, ghost and anti-ghost fields, respectively. This effective action as well as the partition function are invariant under usual BRST transformations

$$\begin{aligned}
&\delta_b A_\mu(x)=\partial_\mu c(x)\delta\Lambda,\\
&\delta_b c(x)=0,\\
&\delta_b \bar{c}(x)=B(x)\delta\Lambda,\\
&\delta_b B(x)=0,
\end{aligned} \tag{3}$$

where $\delta\Lambda$ is an infinitesimal, anticommuting and global parameter. Most of the features of the BRST transformation do not depend on whether the parameter $\delta\Lambda$ is (i) finite or infinitesimal, (ii) field-dependent or not, as long as it is anticommuting and space–time independent. These observations give us a freedom to generalize the BRST transformation by making the parameter, $\delta\Lambda$, finite and field-dependent without affecting its properties. To generalize such transformation we start by making the infinitesimal parameter field-dependent with introduction of an arbitrary parameter κ $(0\le\kappa\le 1)$. We allow the generic fields, $\Phi(x,\kappa)$, to depend on κ in such a way that $\Phi(x,\kappa=0)=\Phi(x)$ and $\Phi(x,\kappa=1)=\Phi'(x)$, the transformed field.

The usual infinitesimal transformation, thus can be written generically as [19]

$$\begin{aligned}
&\frac{dA_\mu(x,\kappa)}{d\kappa}=\partial_\mu c(x)\Theta'\big[\Phi(x,\kappa)\big],\\
&\frac{dc(x,\kappa)}{d\kappa}=0,\\
&\frac{d\bar{c}(x,\kappa)}{d\kappa}=B(x)\Theta'\big[\Phi(x,\kappa)\big],\\
&\frac{dB(x,\kappa)}{d\kappa}=0,
\end{aligned} \tag{4}$$

where the $\Theta'[\Phi(x,\kappa)]$ is the infinitesimal but field-dependent parameter. The FFBRST transformation (δ_f) then can be constructed by integrating such infinitesimal transformation from $\kappa=0$ to $\kappa=1$, as

$$\begin{aligned}
&\delta_f A_\mu(x)=A_\mu(x,\kappa=1)-A_\mu(x,\kappa=0)=\partial_\mu c(x)\Theta\big[\Phi(x)\big],\\
&\delta_f c(x)=c(x,\kappa=1)-c(x,\kappa=0)=0,\\
&\delta_f \bar{c}(x)=\bar{c}(x,\kappa=1)-\bar{c}(x,\kappa=0)=B(x)\Theta\big[\Phi(x)\big],\\
&\delta_f B(x)=B(x,\kappa=1)-B(x,\kappa=0)=0,
\end{aligned} \tag{5}$$

where [19]

$$\Theta\big[\Phi(x)\big]=\int_0^1 d\kappa'\,\Theta'\big[\Phi\big(x,\kappa'\big)\big] \tag{6}$$

is the finite field-dependent parameter. Such a generalized transformation with finite field-dependent parameter is a symmetry of the effective action S_M, i.e.,

$$\delta_f S_M=(s_b S_M)\Theta=0, \tag{7}$$

where s_b is Slavnov variation. Let us explicitly show the invariance of the Maxwell term. Under the transformations (5), the Maxwell pieces changes as,

$$\begin{aligned}
\delta_f\big(F_{\mu\nu}F^{\mu\nu}\big)&=4F_{\mu\nu}\delta_f\partial^\mu A^\nu,\\
&=4F_{\mu\nu}\partial^\mu\big[\partial^\nu c\Theta\big],\\
&=0.
\end{aligned} \tag{8}$$

Since the FFBRST parameter Θ is space–time independent, the derivative acts only on the variable c. By symmetry this term vanishes. Hence the Maxwell piece remains invariant. Although the action remains invariant, the functional measure is not invariant under such a transformation as the Grassmann parameter is field-dependent in nature. The Jacobian, $J(\kappa)$, of path integral measure changes non-trivially and can be replaced as [19]

$$J(\kappa)\longmapsto e^{-S_1[\Phi(x,\kappa)]}, \tag{9}$$

if and only if the following condition is satisfied as we do not want any numerical change in the path integral measure [19]

$$\int\mathcal{D}\Phi(x)\left[\frac{d}{d\kappa}\ln J(\kappa)+\frac{dS_1[\Phi(x,\kappa)]}{d\kappa}\right]e^{-S_1[\Phi(x,\kappa)]}=0, \tag{10}$$