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Modified bosonic gas trapped in a generic 3-dim power law potential

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ABSTRACT

We analyze the consequences caused by an anomalous single-particle dispersion relation suggested in several quantum-gravity models, upon the thermodynamics of a Bose–Einstein condensate trapped in a generic 3-dimensional power-law potential. We prove that the condensation temperature is shifted as a consequence of such deformation and show that this fact could be used to provide bounds on the deformation parameters. Additionally, we show that the shift in the condensation temperature, described as a non-trivial function of the number of particles and the trap parameters, could be used as a criterion to analyze the effects caused by a deformed dispersion relation in weakly interacting systems and also in finite size systems.

1. Introduction

The possibility of a deformation in the dispersion relation of microscopic particles appears in connection with the quest for a quantum theory of gravity [1–9]. This entails, in some schemes, that a possible spacetime quantization has as a consequence a modification of the classical spacetime dispersion relation between energy E and (modulus of) momentum p of a microscopic particle with mass m [2,3,5]. A deformed dispersion relation emerges as an adequate tool in the search for phenomenological consequences caused by this type of quantum gravity models. Nevertheless, the principal difficulty in the search of quantum gravity manifestations in our low energy world is the smallness in the predicted effects [3,4]. If this kind of deformations is characterized, for instance, by some Planck scale, then the quantum gravity effects become very small [2,5]. In the non-relativistic limit, the deformed dispersion relation can be expressed as follows [5,6]

$$E \simeq m + \frac{p^2}{2m} + \frac{1}{2M_p}\left(\xi_1 mp + \xi_2 p^2 + \xi_3 \frac{p^3}{m}\right), \tag{1}$$

in units where the speed of light $c = 1$, with $M_p \simeq 1.2 \times 10^{28}$ eV the Planck mass. The three parameters ξ_1, ξ_2, and ξ_3 are model dependent [2,5], and should take positive or negative values close to 1. There is some evidence within the formalism of Loop quantum gravity [5–8] that indicates non-zero values for the three parameters, ξ_1, ξ_2, ξ_3, and particularly that produces a linear-momentum term in the non-relativistic limit [7,9]. Unfortunately, as it is usual in quantum gravity phenomenology, the possible bounds associated with the deformation parameters open a wide range of possible magnitudes, which is translated to a significant challenge.

On the other hand, the use of Bose–Einstein condensates, as a possible tool in the search of quantum-gravity manifestations (for instance, in the context of Lorentz violation or to provide phenomenological constrains on Planck-scale physics) has produced an enormous amount of interesting publications [10–19]. It turns out to be rather exciting to look for the effects in the thermodynamic properties associated with Bose–Einstein condensates caused by the quantum structure of space–time.

In a previous report [15], we were able to prove that the condensation temperature of the ideal bosonic gas is corrected as a consequence of the deformation in the dispersion relation. Moreover, this correction described as a non-trivial function of the number of particles and the shape associated with the corresponding trap could provide representative bounds for the deformation parameter ξ_1. We have proved that the deformation parameter ξ_1 can be bounded, under typical conditions, from $|\xi_1| \lesssim 10^6$ to $|\xi_1| \lesssim 10^2$, by using different classes of trapping potentials in the thermodynamic limit. In the case of a harmonic oscillator-type potential, we have obtained a bound up to $|\xi_1| \lesssim 10^4$. In Refs. [5,6] it was suggested the use of ultra-precise cold-atom-recoil experiments to constrain the form of the energy–momentum dispersion relation in the non-relativistic limit. There, the bound associated with ξ_1 is at least four orders of magnitude smaller than the bound associated

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with a Bose–Einstein condensate trapped in a harmonic oscillator in the ideal case obtained in [15].

In a more realistic system, finite size effects and interactions among the constituents of the gas must be taken into account. To this aim, let us propose a particularly simple modified Hartree–Fock type spectrum, in the semi-classical approximation, which basically consists in the assumption that the constituents of the gas behave like non-interacting bosons moving in a self-consistent mean field, valid when the semiclassical energy spectrum ϵ_p is bigger than the associated chemical potential μ, for dilute gases [20,21]

$$\epsilon_p = \frac{p^2}{2m} + \alpha p + U(\vec{r}) + 2U_0 n(\vec{r}), \tag{2}$$

where p is the momentum, m is the mass of the particle, and the term αp, with $\alpha = \xi_1 \frac{mc}{2M_p}$ in ordinary units, is the leading order modification in expression (1), with c the speed of light. The term $2U_0 n(\vec{r})$ is a mean field generated by the interactions with other constituents of the bosonic gas, with $n(\vec{r})$ the spatial density of the cloud [20]. The coupling constant U_0 is related to the s-wave scattering length a through the following expression

$$U_0 = \frac{4\pi\hbar^2}{m} a. \tag{3}$$

The potential term

$$U(\vec{r}) = \sum_{i=1}^{d} A_i \left| \frac{r_i}{a_i} \right|^{s_i} \tag{4}$$

is the generic 3-dimensional power-law potential, where A_i and a_i are energy and length scales associated with the trap [22]. Additionally, r_i are the d radial coordinates in the n_i-dimensional subspace of the 3-dimensional space. The sub-dimensions n_i satisfy the following expression in three spatial dimensions

$$\sum_{i=1}^{d} n_i = 3. \tag{5}$$

If $d = 3$, $n_1 = n_2 = n_3 = 1$, then the potential becomes the Cartesian trap. If $d = 2$, $n_1 = 2$ and $n_2 = 1$, then we obtain the cylindrical trap. If $d = 1$, $n_1 = 3$, then we have the spherical trap. If $s_i \to \infty$, we have a free gas in a box. The external potential included in (2) is quite general. Different combinations of these parameters give different classes of potentials, according to (4). It is noteworthy to mention that the use of these generic potentials opens the possibility to adiabatically cool the system in a reversible way, by changing the shape of the trap [20]. The analysis of a Bose–Einstein condensate in the ideal case, weakly interacting, and with a finite number of particles, trapped in different potentials shows that the main properties associated with the condensate, and in particular the condensation temperature, strongly depend on the trapping potential under consideration [22–35]. Additionally, the characteristics of the potential (in particular, the parameter that defines the shape of the potential) have a strong impact on the dependence of the condensation temperature with the number of particles (or the associated density).

The main goal of this work is to analyze the shift in the condensation temperature caused by a deformed dispersion relation in weakly interacting systems and also in systems containing a finite number of particles. We stress that these systems could be used, in principle, to obtain criteria of viability for possible signals coming from Planck scale regime, by analyzing some relevant thermodynamic variables, for instance, the number of particles, and the frequency associated with the trap, when $|\xi_1| \lesssim 1$.

2. Condensation temperature in the thermodynamic limit; $U_0 = 0$

Due to an extensive use of some results, let us briefly summarize the results obtained in [15]. From (2), the case $U_0 = 0$ leads to

$$\epsilon_p = \frac{p^2}{2m} + \alpha p + U(\vec{r}). \tag{6}$$

In the semiclassical approximation, the single-particle phase-space distribution may be written as [20,21]

$$n(\vec{r}, \vec{p}) = \frac{1}{e^{\beta(\epsilon_p - \mu)} - 1}, \tag{7}$$

where $\beta = 1/\kappa T$, κ is the Boltzmann constant, T is the temperature, and μ is the chemical potential. The number of particles in the 3-dimensional space obeys the normalization condition [20,21],

$$N = \frac{1}{(2\pi\hbar)^3} \int d^3\vec{r}\, d^3\vec{p}\, n(\vec{r}, \vec{p}), \tag{8}$$

where

$$n(\vec{r}) = \int d^3\vec{p}\, n(\vec{r}, \vec{p}) \tag{9}$$

is the spatial density. Using expression (6), and integrating expression (7) over momentum, with the help of (9), we get the spatial distribution associated with our modified semi-classical spectrum (6)

$$n(\vec{r}) = \lambda^{-3} g_{3/2}\left(e^{\beta(\mu_{eff} - U(\vec{r}))}\right) - \alpha\lambda^{-2}\left(\frac{m}{\pi\hbar}\right) g_1\left(e^{\beta(\mu_{eff} - U(\vec{r}))}\right) + \alpha^2\lambda^{-1}\left(\frac{m^2}{2\pi\hbar^2}\right) g_{1/2}\left(e^{\beta(\mu_{eff} - U(\vec{r}))}\right) \tag{10}$$

where $\lambda = \left(\frac{2\pi\hbar^2}{m\kappa T}\right)^{1/2}$ is the de Broglie thermal wavelength, $\mu_{eff} = \mu + m\alpha^2/2$ is an effective chemical potential, and $g_\nu(z)$ is the so-called Bose–Einstein function defined by [36]

$$g_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}\, dx}{z^{-1}e^x - 1}. \tag{11}$$

If we set $\alpha = 0$ in Eq. (10) we recover the usual result for the spatial density in the semiclassical approximation [20,21]. By using the properties of the Bose–Einstein functions [36], assuming that $m\alpha^2/2 \ll \kappa T$ and integrating the normalization condition (8), we obtain an expression for the number of particles N to first order in α

$$N - N_0 = C \prod_{l=1}^{d} A_l^{-\frac{n_l}{s_l}} a_l^{n_l} \Gamma\left(\frac{n_l}{s_l} + 1\right) \left[\left(\frac{m}{2\pi\hbar^2}\right)^{3/2} g_\gamma(z)(\kappa T)^\gamma - \alpha\left(\frac{m^2}{2\pi^2\hbar^3}\right) g_{\gamma-1/2}(z)(\kappa T)^{\gamma-1/2}\right], \tag{12}$$

where

$$\gamma = \frac{3}{2} + \sum_{l=1}^{d} \frac{n_l}{s_l} \tag{13}$$

is the parameter that defines the shape of the potential (4). In (12), N_0 are the particles in the ground state, $\Gamma(y)$ is the Gamma function, and C is a constant associated with the potential in question. In the case of Cartesian traps, and in consequence, for a three dimensional harmonic oscillator potential $\gamma = 3$ and $C = 8$. If we set $\alpha = 0$ in (12) then, we recover the result given in [22]. In the