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On the power counting in effective field theories

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ABSTRACT

We discuss the systematics of power counting in general effective field theories, focusing on those that are nonrenormalizable at leading order. As an illuminating example we consider chiral perturbation theory gauged under the electromagnetic $U(1)$ symmetry. This theory describes the low-energy interactions of the octet of pseudo-Goldstone bosons in QCD with photons and has been discussed extensively in the literature. Peculiarities of the standard approach are pointed out and it is shown how these are resolved within our scheme. The presentation follows closely our recent discussion of power counting for the electroweak chiral Lagrangian. The systematics of the latter is reviewed and shown to be consistent with the concept of chiral dimensions. The results imply that naive dimensional analysis (NDA) is incomplete in general effective field theories, while still reproducing the correct counting in special cases.

1. Introduction

Effective field theories (EFTs) are the most efficient way of describing physics at a certain energy scale, provided there is a mass gap and the dynamical field content as well as the symmetries at that scale are known. What makes EFTs especially useful is that the operators one can build out of the fields can be organized according to their importance in a systematic expansion. The organizing principle is based on a power-counting argument. In weakly-coupled scenarios the power counting reduces to a dimensional expansion, where fields have canonical dimensions and higher-dimension operators are weighted with inverse powers of a cutoff scale Λ, whose value indicates the scale of new physics. In this case Λ can be arbitrarily large.

The situation is different in spontaneously broken strongly-coupled scenarios. Such theories are nonrenormalizable even at leading order. As a result, they are non-decoupling, *i.e.*, the scale of new physics is no longer arbitrary but required to be at $\Lambda \approx 4\pi f$, where f is the Goldstone-boson decay constant of the strong sector. Correspondingly, strongly-coupled effective theories can only be consistent if based on a loop expansion, where the loop divergences at a given order are renormalized by operators at the following order. The EFT is predictive if the size of the counterterms is of the same order as the loop contributions, to which they are related by renormalization [1]. Power counting is no longer based on the canonical dimension of fields and should be constructed instead by analyzing the loop structure of a given (leading-order) Lagrangian. Knowledge of the effective Lagrangian at leading order is therefore necessary.

This strategy for the construction of EFTs with strongly-coupled dynamics is notably simplified in specific cases. The paradigm of simplicity is chiral perturbation theory (χPT) [2,3] for massless pions, where the power counting reduces to an expansion in derivatives. When external sources are added and pion masses switched on [4,5], chiral symmetry is explicitly broken. It is common to extend the derivative counting to these new objects too. This formal assignment of derivative counting to couplings and fields goes under the name of chiral dimensional counting (χDC). It is constrained by the requirement that the terms in the leading-order Lagrangian must have the same chiral dimension. Following this method, extensions of χPT to include dynamical photons [6] and leptons [7] have been formulated.

Defined in this way, the assignment of chiral dimensions seems unsatisfactory in some respects. First, chiral dimensions may suggest a misleading interpretation of the physics of strongly-coupled dynamics. For instance, the electromagnetic coupling e is a parameter independent of chiral symmetry breaking, yet it has an assigned momentum scaling. χDC should thus be rather understood as a formal tool. However, to the best of our knowledge, χDC has never been justified in terms of a diagrammatic power counting. Second, χDC alone does not yet allow one to construct an operator basis. In other words, it is no substitute for a full-fledged power-counting formula. These points have led to some confusion in the literature, especially in studies of electroweak effective theories [8,9].

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An alternative approach is naive dimensional analysis (NDA) [1]. Using the chiral quark model as a paradigmatic example, a simple set of rules has been inferred to build power-counting formulas for generic EFTs. In a nutshell, fundamental and composite fields are simply associated with different scales, $1/\Lambda$ for the former and $1/f$ for the latter [10]. This prescription is in contrast to χDC.

Both χDC and NDA have the common objective to describe the systematics of EFTs in which strongly-coupled and weakly-coupled sectors mix. Since both methods rely on some sort of dimensional expansion encoded in a set of rules, it would be interesting to explore the relation between both approaches and understand whether they are mutually consistent. In this Letter we will clarify these issues by reassessing the χDC and NDA prescriptions in the light of a general power-counting formula for strongly-coupled theories with fermions, gauge bosons and scalars, initially derived in [11,12]. We will specialize it to the strong and electroweak interactions and compare it with the predictions of χDC and NDA.

We show that χDC is a consistent prescription and can be rephrased in terms of systematic power-counting arguments. Its formal and, strictly speaking, unphysical scaling rules turn out to be the price to pay in order to force a simple dimensional counting onto a strongly-coupled EFT. We also show that the rules of NDA are not valid in general and lead to contradictions, for instance in the electroweak interactions. We point out how they should be modified. In particular, we will find out that power counting is insensitive to the fundamental or composite nature of fermions, yet very sensitive to their couplings with the Goldstone modes.

This Letter is organized as follows. We revisit χPT with dynamical photons in Section 2 and derive the relevant power-counting formula. The latter is a new result, in spite of the fact that this EFT has been widely used. The formula allows us to prove that the definition of chiral dimensions employed in [6] is both consistent and unique. In Section 3 we turn to the electroweak interactions and discuss the power counting that applies when the spontaneous symmetry breaking is induced by strongly-coupled dynamics. We show how the general results derived in [11,12] can be reinterpreted in the language of chiral dimensions. Section 4 comments on the implications for the NDA prescription. We conclude in Section 5.

2. Chiral perturbation theory with photons

2.1. Lagrangian at leading order

Many of the essential features in the power counting of strongly-coupled effective field theories are already present in the case of chiral perturbation theory of pions and kaons coupled to electromagnetism. Due to its relative simplicity this case will serve as an instructive example for our discussion.

Under $SU(3)_L \times SU(3)_R$ the Goldstone boson matrix U transforms as

$$U \to g_L U g_R^\dagger, \quad g_{L,R} \in SU(3)_{L,R} \tag{1}$$

The explicit relation between the matrix U and the Goldstone fields φ^a is

$$U = \exp(2i\Phi/f), \quad \Phi = \varphi^a T^a \tag{2}$$

where $T^a = T_a = \lambda^a/2$ are the generators of $SU(3)$ and $f \approx 93$ MeV is the Goldstone-boson decay constant.

The vectorial subgroup of $SU(3)_L \times SU(3)_R$ is gauged under the electromagnetic $U(1)$, so that the covariant derivative is given by

$$D_\mu U = \partial_\mu U + ieA_\mu[Q, U] \tag{3}$$

where $Q = \mathrm{diag}(2/3, -1/3, -1/3)$.

The full chiral symmetry $SU(3)_L \times SU(3)_R$ is broken by the quark-mass term (χ) and by electromagnetism (Q). This can be implemented in the standard way through the corresponding spurions transforming as

$$\chi \to g_L \chi g_R^\dagger, \qquad Q_L \to g_L Q_L g_L^\dagger, \qquad Q_R \to g_R Q_R g_R^\dagger \tag{4}$$

with the identification $Q_L = Q_R = Q$. Similarly, $\chi = 2B\mathcal{M}$ with $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$.

The leading-order Lagrangian can then be written as [6,7,13–16]

$$\mathcal{L}_{\mathrm{LO}} = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle + \frac{f^2}{4}\langle U^\dagger\chi + \chi^\dagger U\rangle - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2\Delta\langle U^\dagger Q U Q\rangle \tag{5}$$

where $\langle\cdots\rangle$ denotes the trace.

Eq. (5) is the lowest-order approximation to the theory of pseudo-Goldstone bosons and photons with typical energies of the order of f. The expansion parameter governing higher-order corrections is f^2/Λ^2, with $\Lambda = 4\pi f$ the scale of chiral symmetry breaking. All terms in (5) are indeed of leading order ($\sim f^4$) in this expansion. This follows from the fact that in a scattering process involving Goldstone bosons and photons at a typical energy $\sim f \ll \Lambda$, the relevant quantities scale as

$$\partial_\mu \sim f, \qquad \varphi^a \sim f, \qquad \chi \sim f^2, \qquad A_\mu \sim f, \qquad e \sim 1 \tag{6}$$

The spurion χ is proportional to the pseudo-Goldstone masses squared, which are counted in the standard way as $\sim f^2$, consistent with the homogeneous scaling of the meson propagator. The coupling e is an independent parameter, which can be viewed as a quantity of order one. A further expansion for $e^2 \ll 1$ can always be performed if desired.

The scaling $\sim f^4$ follows immediately for the first three terms in (5). The last term has no derivatives and amounts to a potential for the Goldstone bosons, induced by virtual photons. It is proportional to the cut-off squared, but carries a loop suppression. It scales as [17]

$$\Delta \sim f^2\frac{\Lambda^2}{16\pi^2} \sim f^4 \tag{7}$$

and is consistently included in $\mathcal{L}_{\mathrm{LO}}$. We note that the leading-order Lagrangian has terms with canonical dimension zero (second and fourth term), two (first term) and four (third term). As is well known, the Lagrangian is not ordered by canonical dimension of operators in the case of a strongly-interacting system. This is hardly surprising, since already the first term in $\mathcal{L}_{\mathrm{LO}}$ contains operators of arbitrarily large canonical dimension when expanded out in powers of the field Φ.

Rather than by dimensional counting, the higher-order terms correcting the Lagrangian in (5) are governed by a loop expansion, which corresponds to a series in powers of $1/(16\pi^2) = f^2/\Lambda^2$. The systematics of this construction can be described by a power-counting formula, which we discuss in the following section.

2.2. Power counting and the Lagrangian at NLO

The leading-order Lagrangian (5) is nonrenormalizable. Corrections can be organized in the form of a loop expansion. The relevant power counting has been discussed in [11,12] for the electroweak chiral Lagrangian. It makes use of the assumption that the loop effects in the strong sector $\sim 1/(16\pi^2)$ are actually of the same order of magnitude as the corresponding coefficients of NLO operators $\sim f^2/\Lambda^2$ [1,10]. This implies the identification $\Lambda = 4\pi f$.