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Is ρ-meson melting compatible with chiral restoration?

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ABSTRACT

Utilizing in-medium vector spectral functions which describe dilepton data in ultra-relativistic heavy-ion collisions, we conduct a comprehensive evaluation of QCD and Weinberg sum rules at finite temperature. The starting point is our recent study in vacuum, where the sum rules have been quantitatively satisfied using phenomenological vector and axial-vector spectral functions which describe hadronic τ-decay data. In the medium, the temperature dependence of condensates and chiral order parameters is taken from thermal lattice QCD where available, and otherwise is estimated from a hadron resonance gas. Since little is known about the in-medium axial-vector spectral function, we model it with a Breit–Wigner ansatz allowing for smooth temperature variations of its width and mass parameters. Our study thus amounts to testing the compatibility of the ρ-broadening found in dilepton experiments with (the approach toward) chiral restoration, and thereby searching for viable in-medium axial-vector spectral functions.

1. Introduction

The structure of the QCD ground state is reflected in its observable hadron spectrum. In vacuum, the formation of quark and gluon condensates leads to the generation of hadron masses and the spontaneous breaking of chiral symmetry (SBCS). The latter induces mass splittings of ca. 0.5 GeV for chiral partners in the light-hadron spectrum, e.g., between π–σ and ρ–a_1. In a hot medium, chiral symmetry is restored across a region around a pseudo-critical temperature of $T_{\rm pc} \simeq 160$ MeV [1,2]. A long-standing question is how this restoration manifests itself in the hadron spectrum, i.e., what its observable consequences are. Dilepton data from ultra-relativistic heavy-ion collisions (URHICs) [3–5] are now providing strong evidence that the ρ resonance "melts" when the system passes through the pseudo-critical region [6], while experimental access to the in-medium a_1 spectral functions (e.g., via $a_1 \to \pi\gamma$) remains elusive. Thus, to test whether the ρ melting in the vector channel signals chiral restoration, a theoretical evaluation of the in-medium axial-vector spectral function is needed.

A straightforward approach to calculate the in-medium axial-vector spectral function, by using a chiral Lagrangian paralleling the treatment of the ρ meson, turns out to be challenging [7]. For example, the widely used scheme of implementing the ρ and a_1 mesons into the pion Lagrangian through a local gauging procedure causes considerable problems in describing the vacuum spectral functions as measured in hadronic τ decays [8,9], which led some groups to abandon the local gauging procedure [10,11]. In the present work, we adopt a more modest approach to this problem, by utilizing in-medium sum rules. Specifically, we adopt the well-known Weinberg sum rules (WSRs) [13,12,14] which relate (moments of) the difference between vector and axial-vector spectral functions to operators signifying SBCS. Using available calculations of the in-medium ρ spectral function together with temperature dependent order parameters as an input, we ask whether *a* (not necessarily *the*) axial-vector spectral function can be found to satisfy the in-medium sum rules. To tighten our constraints, we simultaneously employ finite-temperature QCD sum rules (QCD-SRs) [15,16] in vector and axial-vector channels, which additionally involve chirally invariant condensates. Related works have been carried out, e.g., in the low-temperature limit [17,18], for heavy-quark channels [19], or focusing on chirally odd condensates in the vector channel only [20].

The present analysis builds on our previous work [21] where QCD and Weinberg sum rules have been tested in vacuum with vector and axial-vector spectral functions that accurately fit hadronic τ-decays. The combination of four WSRs turned out be a rather sensitive probe of the spectral functions, allowing, e.g., to deduce the presence of an excited axial-vector meson, a_1'. This makes for a promising tool at finite temperature (T), aided by an experimentally tested in-medium vector spectral function and in-medium condensates from lattice QCD (lQCD). In the absence of reliable microscopic models for the a_1 and the excited states, the price to pay is the *a priori* unknown in-medium behavior of these states. However, with guidance from model-independent chiral mixing theorems to constrain the T dependence of the higher

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states, one can still hope for a sensitive test of the in-medium a_1 spectral function, and to gain novel insights into (the approach to) chiral restoration in the $IJ^P = 11^{\pm}$ chiral multiplet. This is the main objective of our work.

The Letter is organized as follows. We recall the in-medium QCDSRs and WSRs in Section 2 and specify the T dependence of their "right-hand sides" (condensates) in Section 3. The finite-T axial-vector spectral functions ("left-hand sides") are detailed in Section 4, followed by quantitative sum rule analyses in Section 5. We conclude in Section 6.

2. Finite temperature sum rules

The basic quantity figuring into WSRs and QCDSRs is the isovector current–current correlator in the vector (V) and axial-vector (A) channels,

$$\Pi^{\mu\nu}_{V,A}(q^2) = -i\int d^4x e^{ixq}\langle T\vec{J}^{\mu}_{V,A}(x)\vec{J}^{\nu}_{V,A}(0)\rangle. \tag{1}$$

In the quark basis with two light flavors, the currents read $\vec{J}^{\mu}_V = \bar{q}\vec{\tau}\gamma^{\mu}q$ and $\vec{J}^{\mu}_A = \bar{q}\vec{\tau}\gamma^{\mu}\gamma_5 q$, ($\vec{\tau}$: isospin Pauli matrices). From here on, we focus on charge-neutral states (isospin $I_3 = 0$) and drop isospin indices. In vacuum, the currents can be decomposed into 4D transverse and longitudinal components as

$$\Pi^{\mu\nu}_{V,A}(q^2) = \Pi^T_{V,A}(q^2)\left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) + \Pi^L_{V,A}(q^2)\frac{q^{\mu}q^{\nu}}{q^2}. \tag{2}$$

Vector-current conservation implies $\Pi^L_V(q^2) = 0$, while the pion pole induces the partial conservation of the axial-vector current (PCAC),

$$\Pi^L_A(q^2) = f_\pi^2 q^2 \delta(q^2 - m_\pi^2). \tag{3}$$

Lorentz symmetry breaking at finite T splits the 4D-transverse polarization functions into 3D-transverse and 3D-longitudinal parts. From here on, we focus on vanishing 3-momentum ($\vec{q} = 0$), for which the 3D components are degenerate. We define pertinent spectral functions as

$$\rho_{V,A} = -\frac{\mathrm{Im}\,\Pi^T_{V,A}}{\pi}, \qquad \rho_{\bar{A}} = \rho_A - \frac{\mathrm{Im}\,\Pi^L_A}{\pi}. \tag{4}$$

The QCDSRs equate a dispersion integral on the left-hand-side (LHS) to an operator product expansion (OPE) on the right-hand-side (RHS); for the axial-vector channels they read [22–24]

$$\begin{aligned}\frac{1}{M^2}&\int_0^\infty ds\,\frac{\rho_{V,\bar{A}}(s)}{s}e^{-s/M^2}\\ &= \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) + \frac{m_q\langle\bar{q}q\rangle}{M^4} + \frac{1}{24M^4}\left\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right\rangle\\ &\quad - \frac{\pi\alpha_s}{M^6}\frac{(56,-88)}{81}\langle\mathcal{O}_4^{V,A}\rangle + \sum_h\frac{\langle\mathcal{O}_h^{d=4,\tau=2}\rangle_T}{M^4}\\ &\quad + \frac{\langle\mathcal{O}_h^{d=6,\tau=2}\rangle_T}{M^6} + \frac{\langle\mathcal{O}_h^{d=6,\tau=4}\rangle_T}{M^6}\ldots,\end{aligned} \tag{5}$$

where the space-like q^2 is traded for the Borel mass M^2 by a standard Borel transform. On the RHS, we include all operators up to dimension-6, i.e., the common scalar operators already present in the vacuum (quark, gluon, and 4-quark condensates, $\langle\bar{q}q\rangle$, $\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle$, and $\langle\mathcal{O}_4^{V,A}\rangle$, respectively), as well as non-scalar operators induced by thermal hadrons (h), organized by dimension (d) and twist (τ). The T dependencies are detailed in Section 3.

The WSRs relate moments of the difference between the vector and axial-vector spectral functions to chiral order parameters. Their formulation at finite T was first carried out in Ref. [14]. Subtracting the two channels of the finite-T QCDSRs from one another, Taylor-expanding the Borel exponential, and equating powers of M^2 on each side of the sum rule yields

$$\text{(WSR1)}\quad \int_0^\infty ds\,\frac{\Delta\rho(s)}{s} = f_\pi^2, \tag{6}$$

$$\text{(WSR2)}\quad \int_0^\infty ds\,\Delta\rho(s) = f_\pi^2 m_\pi^2 = -2m_q\langle\bar{q}q\rangle, \tag{7}$$

$$\text{(WSR3)}\quad \int_0^\infty ds\,s\Delta\rho(s) = -2\pi\alpha_s\langle\mathcal{O}_4^{\mathrm{SB}}\rangle, \tag{8}$$

where $\Delta\rho = \rho_V - \rho_A$. The chiral breaking 4-quark condensate is given by the axial-vector ones as

$$\langle\mathcal{O}_4^{\mathrm{SB}}\rangle = \frac{16}{9}\left(\frac{7}{18}\langle\mathcal{O}_4^V\rangle + \frac{11}{18}\langle\mathcal{O}_4^A\rangle\right). \tag{9}$$

Since the WSRs only contain chiral order parameters, they are particularly sensitive to chiral symmetry restoration, whereas the QCDSRs are channel specific thus providing independent information.

3. In-medium condensates

We now turn to the T dependence of each condensate figuring into the QCDSRs. To leading order in the density of a hadron h in the heat bath, the in-medium condensate associated with a given operator $\mathcal{O}$ can be approximated by

$$\langle\mathcal{O}\rangle_T \simeq \langle\mathcal{O}\rangle_0 + d_h\int\frac{d^3k}{(2\pi)^3 2E_h}\langle h(\vec{k})|\mathcal{O}|h(\vec{k})\rangle n_h(E_h), \tag{10}$$

where $\langle\mathcal{O}\rangle_0$ is the vacuum value of the operator, $\langle h(\vec{k})|\mathcal{O}|h(\vec{k})\rangle$ its hadronic matrix element, $E_h^2 = m_h^2 + \vec{k}^2$, and d_h, m_h, and n_h are the hadron's spin–isospin degeneracy, mass, and thermal distribution function (Bose (n_b) or Fermi (n_f)), respectively. Working at zero baryon chemical potential ($\mu_B = 0$), we absorb anti-baryons into the degeneracy factor of baryons. Corrections to Eq. (10) figure via multi-hadron matrix elements of the operator.

We approximate the medium by a hadron resonance gas (HRG) including all confirmed states with mass $m_h \leqslant 2$ GeV [25]. For the temperatures of interest here, $T \lesssim 170$ MeV, the HRG is known to reproduce the equation of state from lQCD quite well [26]. Since the calculation of the in-medium ρ spectral function is also based on HRG degrees of freedom, the OPE and spectral function sides of the sum rules are evaluated in the same basis. For the subsequent discussion, we define the integrals

$$I_n^h = d_h\int\frac{d^3k}{(2\pi)^3 E_h}k^{2n-2}n_h(E_h). \tag{11}$$

Note that $m_h I_1^h$ is the scalar density, ϱ_s^h.

3.1. Quark condensate

The HRG correction to the quark condensate is [27,28]

$$\frac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_0} = 1 - \frac{\varrho_s^\pi}{2m_\pi f_\pi^2} - \frac{\varrho_s^K}{4m_K f_K^2} - \frac{\varrho_s^\eta}{6m_\eta f_\eta^2} - \frac{\varrho_s^{\eta'}}{3m_{\eta'} f_{\eta'}^2}$$