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Planck scale physics and topology change through an exactly solvable model

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ABSTRACT

We consider the collapse of a charged radiation fluid in a Planck-suppressed quadratic extension of General Relativity (GR) formulated à la Palatini. We obtain exact analytical solutions that extend the charged Vaidya-type solution of GR, which allows to explore in detail new physics at the Planck scale. Starting from Minkowski space, we find that the collapsing fluid generates wormholes supported by the electric field. We discuss the relevance of our findings in relation to the quantum foam structure of space–time and the meaning of curvature divergences in this theory.

1. Introduction

An outstanding question in quantum gravitational physics is whether large metric fluctuations at the Planck scale may induce a change in topology. In fact, as suggested by Wheeler, at scales below the Planck length, the highly nonlinear and strongly interacting metric fluctuations may endow space–time with a foam-like structure [1]. Thus, this hints that the geometry, and the topology, may be constantly fluctuating, so that space–time may take on all manners of nontrivial topological structures, such as wormholes [2]. However, one does encounter a certain amount of criticism to Wheeler's notion of space–time foam, for instance, in that stability considerations may place constraints on the nature or even on the existence of Planck-scale foam-like structures [3]. Nevertheless, the notion of space–time foam is generally accepted, in that this picture leads to topology-changing quantum amplitudes and to interference effects between different space–time topologies [4], although these possibilities have met with some disagreement [5].

Due to the multiply-connected nature of wormholes, their respective creation/generation inevitably involves the problematic issue of topology change [6,7]. The possibility that inflation might provide a natural mechanism for the enlargement of Planck-size wormholes to macroscopic size has been explored [8]. In fact, the construction of general relativistic traversable wormholes, with the idealization of impulsive phantom radiation, has been considered extensively in the literature [9–11]. Another problematic aspect in wormhole physics is that these geometries violate the pointwise energy conditions [12]. However, this issue may be avoided in modified gravity, where the normal matter threading the wormholes may in principle be imposed to satisfy the energy conditions, and it is the higher order curvature terms that sustain these geometries [13]. In fact, the general approach to wormhole physics is to run the gravitational field equations in the reverse direction, namely, consider first an interesting space–time metric and then through the field equations, the distribution of the stress-energy tensor components is deduced. However, one may rightly argue that this approach in solving the field equations lacks physical justification, and that a more physical motivation would be to consider a plausible distribution of matter-energy.

In this work we follow the latter route and find that in a quadratic extension of GR formulated in the Palatini formalism wormholes can be generated dynamically. This result follows by

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probing Minkowski space–time with a charged null fluid, thus producing a Vaidya-type configuration. Now, switching off the flux, the metric settles down into a static configuration where the existence of a wormhole geometry becomes manifest. The topologically nontrivial character of the wormhole allows us to define the electric charge in terms of lines of electric field trapped in the topology. The size of the wormhole changes in such a way that the density of lines of force at the throat is given by a universal quantity. These facts allow to consistently interpret these solutions as geons in Wheeler's sense [1] and have important consequences for the issue of the foam-like structure of space–time [2].

2. Dynamical charged fluid in Ricci-squared Palatini theories

Consider a theory defined by the action [14]

$$S[g,\Gamma,\psi_m]=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R,Q)+S_m[g,\psi_m], \tag{1}$$

where $\mathcal{L}_G=f(R,Q)/(2\kappa^2)$ represents the gravity Lagrangian, κ^2 is a constant with suitable dimensions (in GR, $\kappa^2\equiv 8\pi G/c^3$), $g_{\mu\nu}$ is the space–time metric, $R=g^{\mu\nu}R_{\mu\nu}$, $Q=g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}R_{\alpha\beta}$, $R_{\mu\nu}=R^\rho{}_{\mu\rho\nu}$, $R^\alpha{}_{\beta\mu\nu}=\partial_\mu\Gamma^\alpha_{\nu\beta}-\partial_\nu\Gamma^\alpha_{\mu\beta}+\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\beta}-\Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$ is the Riemann tensor of the connection $\Gamma^\lambda_{\mu\nu}$, and $S_m[g,\psi_m]$ represents the matter action (with ψ_m the matter fields). We work in the Palatini formalism, where $g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ are regarded as independent fields. Setting the torsion to zero for simplicity, the field equations imply that $\Gamma^\lambda_{\mu\nu}$ can be written as the Levi-Civita connection of a metric $h_{\mu\nu}$ defined as [15]

$$h^{\mu\nu}=\frac{g^{\mu\alpha}\Sigma_\alpha{}^\nu}{\sqrt{\det\hat\Sigma}},\qquad h_{\mu\nu}=(\sqrt{\det\hat\Sigma})\Sigma^{-1}{}_\mu{}^\alpha g_{\alpha\nu}. \tag{2}$$

We have defined the matrices $\hat\Sigma$ and $\hat P$, whose components are $\Sigma_\alpha{}^\nu\equiv(f_R\delta^\nu_\alpha+2f_Q P_\alpha{}^\nu)$ and $P_\mu{}^\nu\equiv R_{\mu\alpha}g^{\alpha\nu}$, with $f_X\equiv df/dX$. From the metric variation, one finds

$$2f_Q\hat P^2+f_R\hat P-\frac{f}{2}\hat I=\kappa^2\hat T, \tag{3}$$

which represents a quadratic algebraic equation for $P_\mu{}^\nu$ as a function of $[\hat T]_\mu{}^\nu\equiv T_{\mu\alpha}g^{\alpha\nu}$. This implies that $R=[\hat P]_\mu{}^\mu$, $Q=[\hat P^2]_\mu{}^\mu$, and $\Sigma_\alpha{}^\nu$ are just functions of the matter sources. With elementary algebraic manipulations, Eq. (3) can be cast as

$$R_\mu{}^\nu(h)=\frac{\kappa^2}{\sqrt{\det\hat\Sigma}}\left(\mathcal{L}_G\delta^\nu_\mu+T_\mu{}^\nu\right). \tag{4}$$

This representation of the metric field equations implies that $h_{\mu\nu}$ satisfies a set of GR-like second-order field equations, with the right-hand side being determined by $\hat T$. Since $h_{\mu\nu}$ and $g_{\mu\nu}$ are algebraically related, it follows that $g_{\mu\nu}$ also verifies second-order equations. We note that whenever $T_\mu{}^\nu=0$, Eq. (4) recovers the vacuum Einstein equations [15,16] (with possibly a cosmological constant, depending on the form of $\mathcal{L}_G$) which, similarly as in other Palatini theories [18,17,19], guarantees the absence of ghost-like instabilities.

Let us consider the matter sector as described by a spherically symmetric flux of ingoing charged matter with a stress-energy tensor given by

$$T^{flux}_{\mu\nu}=\rho_{in}k_\mu k_\nu, \tag{5}$$

where k_μ is a null vector, satisfying $k_\mu k^\mu=0$, and ρ_{in} is the energy density of the flux. The electric field generated by this flux contributes to the total stress-energy by means of $T^{em}_{\mu\nu}=\frac{1}{4\pi}[F_{\mu\alpha}F_\nu{}^\alpha-\frac14 F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu}]$. If we consider a line element of the form

$$ds^2=-A(x,v)e^{2\psi(x,v)}dv^2+2e^{\psi(x,v)}dv\,dx+r^2(v,x)\,d\Omega^2, \tag{6}$$

then Maxwell's equations, $\nabla_\mu F^{\mu\nu}=4\pi J^\nu$, where $J^\nu\equiv\Omega(v)k^\nu$ is the current of the ingoing flux, lead to $r^2e^{\psi(x,v)}F^{xv}=q(v)$, where $q(v)$ is an integration function. These equations also imply that $\Omega(v)\equiv q_v/4\pi r^2$.

To proceed further, we focus on a simple quadratic extension of GR,

$$f(R,Q)=R+l_P^2\left(aR^2+Q\right), \tag{7}$$

where $l_P\equiv\sqrt{\hbar G/c^3}$ is the Planck length and a is a free parameter. We note that the renormalizability of quantum fields in curved space–times requires [20] a high-energy completion of the Einstein–Hilbert Lagrangian including quadratic curvature terms such as those appearing in (7). Moreover, these higher-order curvature corrections typically appear in approaches to quantum gravity such as those based on string theory [21], and also when GR is regarded as an effective theory of quantum gravity [22]. The Palatini formulation has been particularly successful in the construction of an effective Lagrangian [23] that captures the Hamiltonian dynamics of loop quantum cosmology [24]. This theory cures the big bang singularity producing a cosmic bounce at the Planck density. The effective Palatini Lagrangian admits a power series expansion with quadratic and higher-order curvature corrections. The Lagrangian (7) is also able to replace the big bang singularity by a cosmic bounce in both isotropic and anisotropic cosmologies [25]. As a working hypothesis, we assume that the action (7) is able to capture the main effects of the quantum gravitational degrees of freedom in the form of an effective geometry, i.e., we assume that the geometrical nature of gravitation is not spoiled by the quantum gravitational effects. The model (7) is thus regarded as an effective model useful to describe dynamical processes and obtain some hints on the topology change issue, but in a purely classical continuum scenario with the quantum effects encoded in l_P^2.

Having specified the matter sources and the gravity Lagrangian, one finds $R=0$, $Q=\kappa^2q^4/4\pi r^8$, and

$$\Sigma_\mu{}^\nu=\begin{pmatrix}\sigma_- & \sigma_{in} & 0 & 0\\ 0 & \sigma_- & 0 & 0\\ 0 & 0 & \sigma_+ & 0\\ 0 & 0 & 0 & \sigma_+\end{pmatrix}, \tag{8}$$

where $\sigma_\pm=1\pm\frac{\kappa^2 l_P^2 q^2(v)}{4\pi r^4}$ and $\sigma_{in}=\frac{2\kappa^2 l_P^2\rho_{in}}{1-2\kappa^2 l_P^2q^2(v)/4\pi r^4}$. The field equations (4) can thus be written as

$$R_\mu{}^\nu(h)=\begin{pmatrix}-\frac{\kappa^2q^2(v)}{8\pi r^4\sigma_+} & \frac{e^{-\psi}\kappa^2\rho_{in}}{\sigma_+\sigma_-} & 0 & 0\\ 0 & -\frac{\kappa^2q^2(v)}{8\pi r^4\sigma_+} & 0 & 0\\ 0 & 0 & \frac{\kappa^2q^2(v)}{8\pi r^4\sigma_-} & 0\\ 0 & 0 & 0 & \frac{\kappa^2q^2(v)}{8\pi r^4\sigma_-}\end{pmatrix}. \tag{9}$$

The strategy now is to solve for $h_{\mu\nu}$ first and then use (2) and (8) to obtain $g_{\mu\nu}$. For this purpose, we define a line element for $h_{\mu\nu}$ of the form

$$d\tilde s^2=-F(v,x)e^{2\xi(v,x)}dv^2+2e^{\xi(v,x)}dv\,dx+\tilde r^2(v,x)\,d\Omega^2. \tag{10}$$