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# Generalized heavy-to-light form factors in light-cone sum rules



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#### ABSTRACT

We study the form factors for a heavy meson into the S-wave  $K\pi/\pi\pi$  system with an invariant mass below 1 GeV. The mesonic final state interactions are described in terms of the scalar form factors, which are obtained from unitarized chiral perturbation theory. Employing generalized light-cone distribution amplitudes, we compute the heavy-to-light transition using light-cone sum rules. Our approach simultaneously respects constraints from analyticity and unitarity, and also takes advantage of the power expansion in the  $1/m_b$  and the strong coupling constant.

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#### 1. Introduction

B decays into a light vector meson are of particular interest as they can provide valuable information to extract the Standard Model (SM) parameters and therefore test the SM. In the case that large deviations from the SM calculations are found, these will shed light on new physics scenarios. Examples for such type of decays include e.g. the process  $B \to \rho(\to \pi\pi)l\bar{\nu}$  for the extraction of the CKM matrix element  $|V_{ub}|$ , the reaction  $B \to K^*(\to K\pi)l^+l^-$  to test the chirality structure in weak interaction, and the decay  $B_s \to J/\psi\phi(\to K\bar{K})$  to determine the  $B_s - \bar{B}_s$  mixing phase. Recent experimental data on these channels can be found in Refs. [1–4].

Due to the short lifetime, the light vector meson cannot be directly detected by experiments and must be reconstructed from the two or three pseudo-scalars  $\pi/K$  final state. Thus these decay modes are at least four-body processes and the semi-leptonic ones are refereed to as  $B_{l4}$  decays in the literature [5] (for a recent dispersion theoretical approach to this reaction, see Ref. [6]). To select candidate events and suppress the combinatorial background, experimentalists often implement kinematic cuts on the invariant mass. During this procedure various partial waves of the  $K\pi/\pi\pi$  system may get entangled and bring dilutions to physical observables. Particularly it is very likely the S-wave contributions are of great importance [7–29]. Therefore it is mandatory to have reliable and accurate predictions considering the high precision achieved or to be achieved by experiments.

Decay amplitudes for semi-leptonic *B* decays into two light-pseudo-scalar mesons show two distinctive features. On the one hand, the final state interaction of the two pseudo-scalars should

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satisfy unitarity and analyticity. On the other hand, the b mass scale is much higher than the hadronic scale, which allows an expansion of the hard-scattering kernels in terms of the strong coupling constant and the dimensionless power-scaling parameter  $\Lambda_{\rm QCD}/m_b$ . In this Letter, we aim to develop a formalism that makes use of both these advantages. It simultaneously combines the perturbation theory at the  $m_b$  scale based on the operator product expansion and the low-energy effective theory inspired by the chiral symmetry to describe the S-wave  $\pi\pi$  and  $K\pi$  scattering. For concreteness, we will choose the  $B\to K\pi$  matrix elements with the  $K\pi$  invariant mass below 1 GeV as an example in the following, while other processes including the charm meson decay can be treated in an analogous way. If the factorization can be proved, these form factors will also play an important role in the study of charmless three-body B decays [30–33].

#### 2. Generalized form factor

The matrix elements

$$\begin{split} & \left\langle (K\pi)_{0}(p_{K\pi}) \middle| \bar{s}\gamma_{\mu}\gamma_{5}b \middle| \bar{B}(p_{B}) \right\rangle \\ &= -i \frac{1}{m_{K\pi}} \left\{ \left[ P_{\mu} - \frac{m_{B}^{2} - m_{K\pi}^{2}}{q^{2}} q_{\mu} \right] \mathcal{F}_{1}^{B \to K\pi} \left( m_{K\pi}^{2}, q^{2} \right) \right. \\ & \left. + \frac{m_{B}^{2} - m_{K\pi}^{2}}{q^{2}} q_{\mu} \mathcal{F}_{0}^{B \to K\pi} \left( m_{K\pi}^{2}, q^{2} \right) \right\}, \\ & \left\langle (K\pi)_{0}(p_{K\pi}) \middle| \bar{s}\sigma_{\mu\nu}q^{\nu}\gamma_{5}b \middle| \bar{B}(p_{B}) \right\rangle \\ &= - \frac{\mathcal{F}_{T}^{B \to K\pi} \left( m_{K\pi}^{2}, q^{2} \right)}{m_{K\pi} \left( m_{B} + m_{K\pi} \right)} \left[ q^{2}P_{\mu} - \left( m_{B}^{2} - m_{K\pi}^{2} \right) q_{\mu} \right] \end{split} \tag{1}$$

define the S-wave generalized form factors  $\mathcal{F}_i$  [16]. Here,  $P=p_B+p_{K\pi}$  and  $q=p_B-p_{K\pi}$ .

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The  $K\pi$  system with invariant mass below 1 GeV can be treated as a light hadron and more explicitly in the kinematics region we are considering, the  $m_{K\pi}$  is small and the  $K\pi$  system moves very fast, the soft-collinear effective theory (SCET) is applicable [34–37]. As shown later this  $K\pi$  system has similar light-cone distribution amplitudes with the ones for a light hadron. The transition matrix elements for  $B\to K\pi$  may be factorized in the same way as the ordinary B-to-light ones like the  $B\to \pi$  transition. It has been demonstrated in SCET that, in the soft contribution limit, the form factors obey factorization [37–39]:

$$F_i = C_i \xi(q^2) + \Delta F_i, \tag{2}$$

where  $C_i$  are the short-distance and calculable functions, and  $\xi$  is a universal soft form factor from the large recoil symmetry in the heavy quark  $m_b \to \infty$  and large energy  $E \to \infty$  limit [40]. Symmetry breaking terms, starting at order  $\alpha_s$ , can be encoded into  $\Delta F_i$ , and can be expressed as a convolution in terms of the LCDA [37–39,41,42].

Watson's theorem implies that phases measured in the  $K\pi$  elastic scattering and in a decay channel where the  $K\pi$  system decouple with other hadrons are equal (modulo  $\pi$  radians). This leads to

$$\langle (K\pi)_0 | \bar{s} \Gamma b | \bar{B} \rangle \propto F_{K\pi} (m_{K\pi}^2),$$
 (3)

where the strangeness-changing scalar form factors are defined by

$$\langle 0|\bar{s}d|K\pi\rangle = C_X B_0 F_{K\pi}(m_{K\pi}^2). \tag{4}$$

 $C_X$  is an isospin factor and  $B_0$  is proportional to the QCD condensate parameter. For the  $K^-\pi^+$ ,  $C_X=1$ . Below the  $K+3\pi$  threshold, about 911 MeV, the  $K\pi$  scattering is strictly elastic. The inelastic contributions in the  $K\pi$  scattering comes from the  $K+3\pi$  or  $K\eta$ . In the region from 911 MeV to 1 GeV, the  $K+3\pi$  channel has a limited phase space, and thus is generically suppressed. Moreover, as a process-dependent study, it has been demonstrated the states with two additional pions will not give sizeable contributions to physical observables [43]. Though differences may be expected, some similarities might be shared. We leave the  $K+3\pi$  contributions for future work. The  $K\eta$  coupled-channel effects can be included in the unitarized approach of chiral perturbation theory [44–48].

In the following we will choose the light-cone sum rules (LCSR) to calculate the  $\mathcal{F}_i$ . An analysis in other approaches like the  $k_T$  factorization [49–53] would be similar, and for recent developments in this approach see Refs. [54–62]. As a reconciliation of the original QCD sum rule approach [63,64] and the application of perturbation theory to hard processes, LCSR exhibit several advantages in the calculation of quantities like the meson form factors [65–69]. In the hard scattering region the operator product expansion (OPE) near the light-cone is applicable. Based on the light-cone OPE, form factors are expressed as a convolution of light-cone distribution amplitudes (LCDA) with a perturbatively calculable hard kernel. The leading twist and a few sub-leading twist LCDA give the dominant contribution, while higher twist terms are suppressed.

The calculation begins with the correlation function:

$$\Pi(p_{K\pi}, q) = i \int d^4x \, e^{iq \cdot x} \langle (K\pi)_0(p_{K\pi}) | T\{j_{\Gamma_1}(x), j_{\Gamma_2}(0)\} | 0 \rangle, \tag{5}$$

where  $j_{\Gamma_1}$  is one of the currents in Eq. (1) defining the form factors:  $j_{\Gamma_1} = \bar{s}\gamma_\mu\gamma_5 b$  for  $\mathcal{F}_1$  and  $\mathcal{F}_0$ , and  $j_{\Gamma_1} = \bar{s}\sigma_{\mu\nu}\gamma_5 q^{\nu}b$  for  $\mathcal{F}_T$ . We choose  $j_{\Gamma_2} = \bar{b}i\gamma_5 d$  to interpolate the B meson, whose matrix element gives the decay constant  $f_B$ :

$$\langle \bar{B}(p_B) | \bar{b}i\gamma_5 d|0\rangle = \frac{m_B^2}{m_b + m_d} f_B. \tag{6}$$

The hadronic representation of the correlation function consists in the contribution of the *B* meson and of the higher resonances and the continuum state:

$$\Pi^{\text{HAD}}(p_{K\pi}, q) = \frac{\langle (K\pi)_0(p_{K\pi})|j_{\Gamma_1}|\bar{B}(p_{K\pi} + q)\rangle\langle\bar{B}(p_{K\pi} + q)|j_{\Gamma_2}|0\rangle}{m_B^2 - (p_{K\pi} + q)^2} + \int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_{K\pi} + q)^2}, \tag{7}$$

where higher resonances and the continuum of states are described in terms of the spectral function  $\rho^h(s,q^2)$  and start from the threshold  $s_0$ .

The correlation function in Eq. (5) can also be evaluated in the deep Euclidean region in QCD at the quark level. The quark–hadron duality guarantees the equality of the two calculations and thus we obtain the sum rules

$$\langle (K\pi)_{0}(p_{K\pi}) | j_{\Gamma_{1}} | \bar{B}(p_{B}) \rangle \langle \bar{B}(p_{B}) | j_{\Gamma_{2}} | 0 \rangle \exp \left[ -\frac{m_{B}^{2}}{M^{2}} \right]$$

$$= \frac{1}{\pi} \int_{(m_{b}+m_{c})^{2}}^{s_{0}} ds \exp \left[ -s/M^{2} \right] \operatorname{Im} \Pi^{QCD}(s, q^{2}). \tag{8}$$

In the above, a Borel transformation has been performed to improve the convergence of the OPE series, and to enhance the contribution of the low-lying states to the correlation function for suitably chosen values of  $M^2$ .

The calculation of  $\Pi^{\rm QCD}$  is based on the expansion of the T-product in the correlation function near the light-cone, which produces matrix elements of non-local quark-gluon operators. These quantities are in terms of the generalized LCDA of increasing twist [71–74]:

$$\left\langle (K\pi)_0 \middle| \bar{s}(x) \gamma_\mu d(0) \middle| 0 \right\rangle = N p_{K\pi\mu} \frac{1}{m_{K\pi}} \int_0^1 du \, e^{iup_{K\pi} \cdot x} \Phi_{K\pi}(u),$$

$$\langle (K\pi)_0 | \bar{s}(x)d(0) | 0 \rangle = N \int_0^1 du \, e^{iup_{K\pi} \cdot x} \Phi_{K\pi}^s(u),$$

 $\langle (K\pi)_0 | \bar{s}(x) \sigma_{\mu\nu} d(0) | 0 \rangle$ 

$$= -N \frac{1}{6} (p_{K\pi\mu} x_{\nu} - p_{K\pi\nu} x_{\mu}) \int_{0}^{1} du \, e^{iup_{K\pi} \cdot x} \Phi_{K\pi}^{\sigma}(u), \tag{9}$$

where  $N=C_XB_0F_{K\pi}$ . Due to the Watson's theorem, the above matrix elements are proportional to the  $K\pi$  scalar form factors which have been absorbed into the normalization constant N. As a result, the distribution amplitudes,  $\Phi_{K\pi}$  and  $\Phi_{K\pi}^{s,\sigma}$ , are real.

The LCDA  $\Phi_{K\pi}$  is twist-2, and the other two are twist-3. Their normalizations are given as

$$\int_{0}^{1} du \, \Phi_{K\pi}(u) = \frac{m_{s} - m_{d}}{m_{K\pi}},$$

$$\int_{0}^{1} du \, \Phi_{K\pi}^{s}(u) = \int_{0}^{1} du \, \Phi_{K\pi}^{\sigma}(u) = 1.$$
(10)

The use of conformal symmetry in QCD [70] indicates that the twist-3 LCDA have the asymptotic form [71–74]:

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